## 3.2 KARNAUGH MAP:

The simplification of the functions using Boolean laws and theorems becomes complex with the increase in the number of variables and terms. The map method, first proposed by Veitch and slightly improvised by Karnaugh, provides a simple, straightforward procedure for the simplification of Boolean functions. The method is called **Veitch diagram** or **Karnaugh map**, which may be regarded as a pictorial representation of a truth table.

The Karnaugh map technique provides a systematic method for simplifying and manipulation of Boolean expressions. **A** K-map is a diagram made up of squares, with each square representing one minterm of the function that is to be minimized. For *n* variables on a Karnaugh map there are 2n numbers of squares. Each square or cell represents one of the minterms. It can be drawn directly from either minterm (sum-of- products) or maxterm (product-of-sums) Boolean expressions.

Two- Variable, Three Variable and Four Variable Maps

Karnaugh maps can be used for expressions with two, three, four and five variables. The number of cells in a Karnaugh map is equal to the total number of possible input variable combinations as is the number of rows in a truth table. For three variables, the number of cells is $2^3 = 8$. For four variables, the number of cells is $2^4 = 16$.

1-Variable map

	A
$\overline{A}$	$\overline{A}$
A	A

2-Variable map

A$\diagdown$B	$\overline{B}$	B
$\overline{A}$	$\overline{A}\,\overline{B}$	$\overline{A}\,B$
A	$A\,\overline{B}$	$A\,B$

3-Variable map

A$\diagdown$BC	$\overline{B}\,\overline{C}$	$\overline{B}\,C$	$B\,C$	$B\,\overline{C}$
$\overline{A}$	$\overline{A}\,\overline{B}\,\overline{C}$	$\overline{A}\,\overline{B}\,C$	$\overline{A}\,B\,C$	$\overline{A}\,B\,\overline{C}$
A	$A\,\overline{B}\,\overline{C}$	$A\,\overline{B}\,C$	$A\,B\,C$	$A\,B\,\overline{C}$

4-Variable map

AB$\diagdown$CD	$\overline{C}\,\overline{D}$	$\overline{C}\,D$	$C\,D$	$C\,\overline{D}$
$\overline{A}\,\overline{B}$	$\overline{A}\,\overline{B}\,\overline{C}\,\overline{D}$	$\overline{A}\,\overline{B}\,\overline{C}\,D$	$\overline{A}\,\overline{B}\,C\,D$	$\overline{A}\,\overline{B}\,C\,\overline{D}$
$\overline{A}\,B$	$\overline{A}\,B\,\overline{C}\,\overline{D}$	$\overline{A}\,B\,\overline{C}\,D$	$\overline{A}\,B\,C\,D$	$\overline{A}\,B\,C\,\overline{D}$
$A\,B$	$A\,B\,\overline{C}\,\overline{D}$	$A\,B\,\overline{C}\,D$	$A\,B\,C\,D$	$A\,B\,C\,\overline{D}$
$A\,\overline{B}$	$A\,\overline{B}\,\overline{C}\,\overline{D}$	$A\,\overline{B}\,\overline{C}\,D$	$A\,\overline{B}\,C\,D$	$A\,\overline{B}\,C\,\overline{D}$

Product terms are assigned to the cells of a K-map by labeling each row and each column of a map with a variable, with its complement or with a combination of variables & complements. The below figure shows the way to label the rows & columns of a 1, 2, 3 and 4- variable maps and the product terms corresponding to each cell.

It is important to note that when we move from one cell to the next along any row or from one cell to the next along any column, one and only one variable in the product term changes (to a complement or to an uncomplemented form). Irrespective of number of variables the labels along each row and column must

conform to a single change. Hence gray code is used to label the rows and columns of K-map as shown ow.


1 -Variable map   2 -Variable map   3 -Variable map

4- Variable map
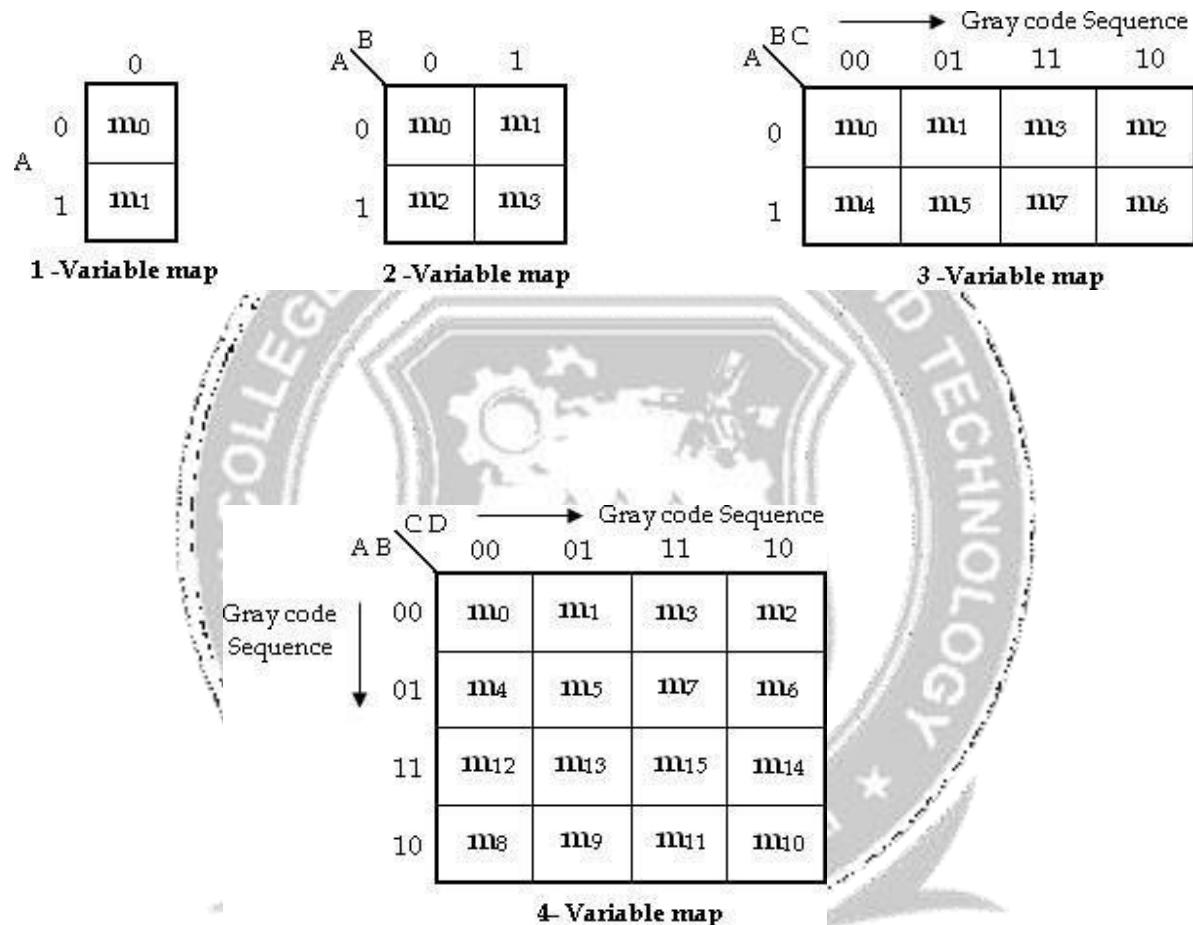
Grouping cells for Simplification:

The grouping is nothing but combining terms in adjacent cells. The simplification is achieved by grouping adjacent 1's or 0's in groups of 2i, where i = 1, 2, …, n and n is the number of variables. When adjacent 1's are grouped then we get result in the sum of product form; otherwise we get result in the product of sum form.

Grouping Two Adjacent 1's: (Pair)

In a Karnaugh map we can group two adjacent 1's. The resultant group is called Pair.
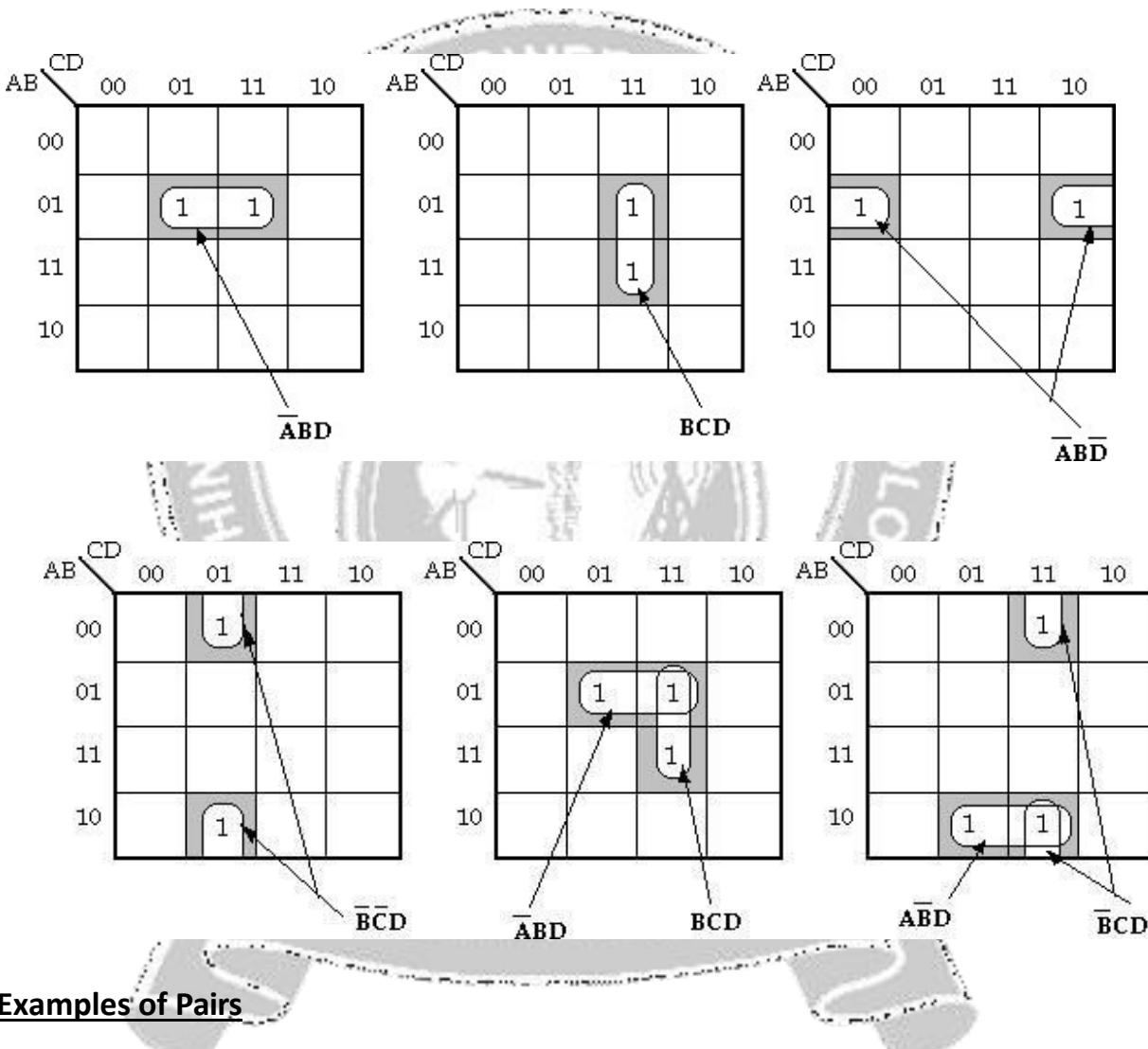
Examples of Pairs

Grouping Four Adjacent 1's: (Quad)

In a Karnaugh map we can group four adjacent 1's. The resultant group is called Quad. Fig (a) shows the four 1's are horizontally adjacent and Fig (b) shows

they are vertically adjacent. Fig (c) contains four 1's in a square, and they are considered adjacent to each other.

Examples of Quads
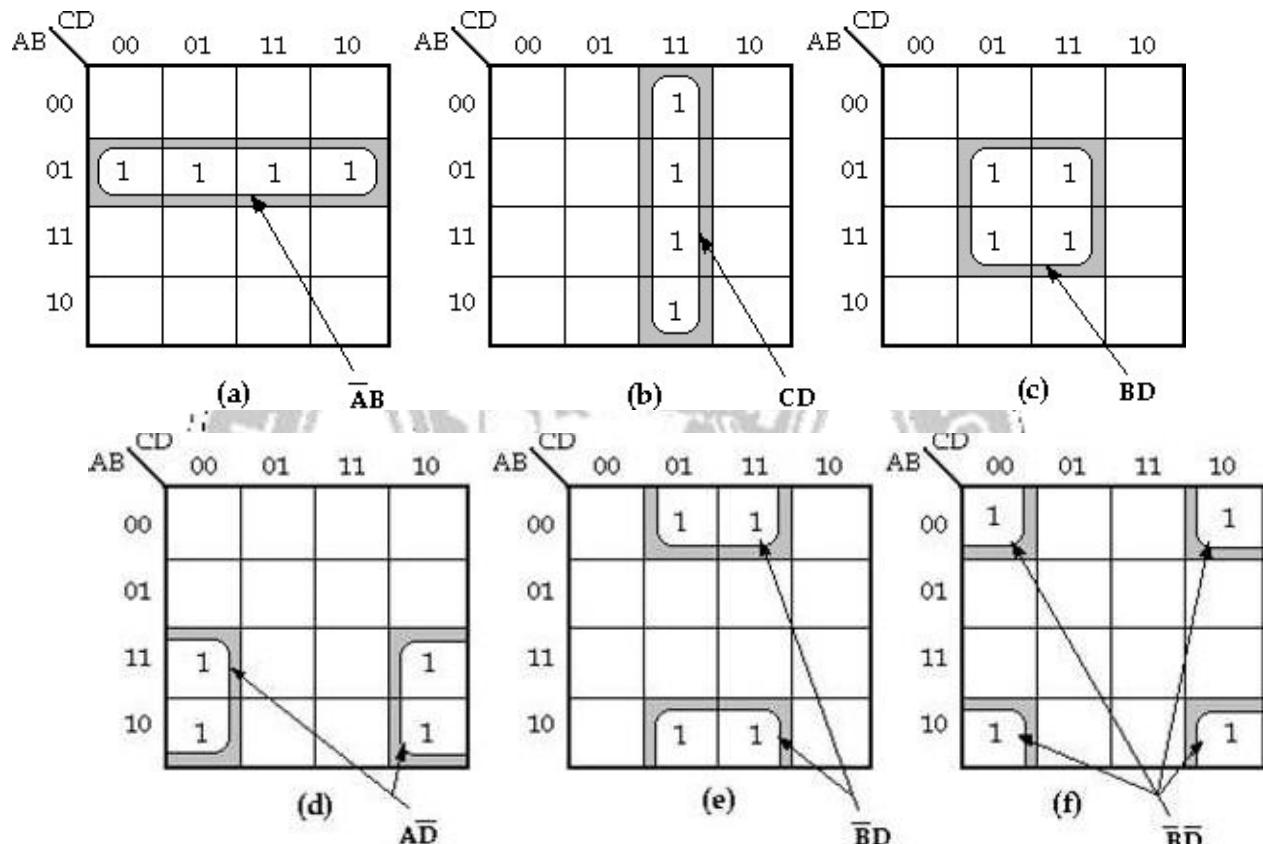
The four 1's in fig (d) and fig (e) are also adjacent, as are those in fig (f) because, the top and bottom rows are considered to be adjacent to each other and the leftmost and rightmost columns are also adjacent to each other.

Grouping Eight Adjacent 1's: (Octet)

In a Karnaugh map we can group eight adjacent 1's. The resultant group is called Octet.
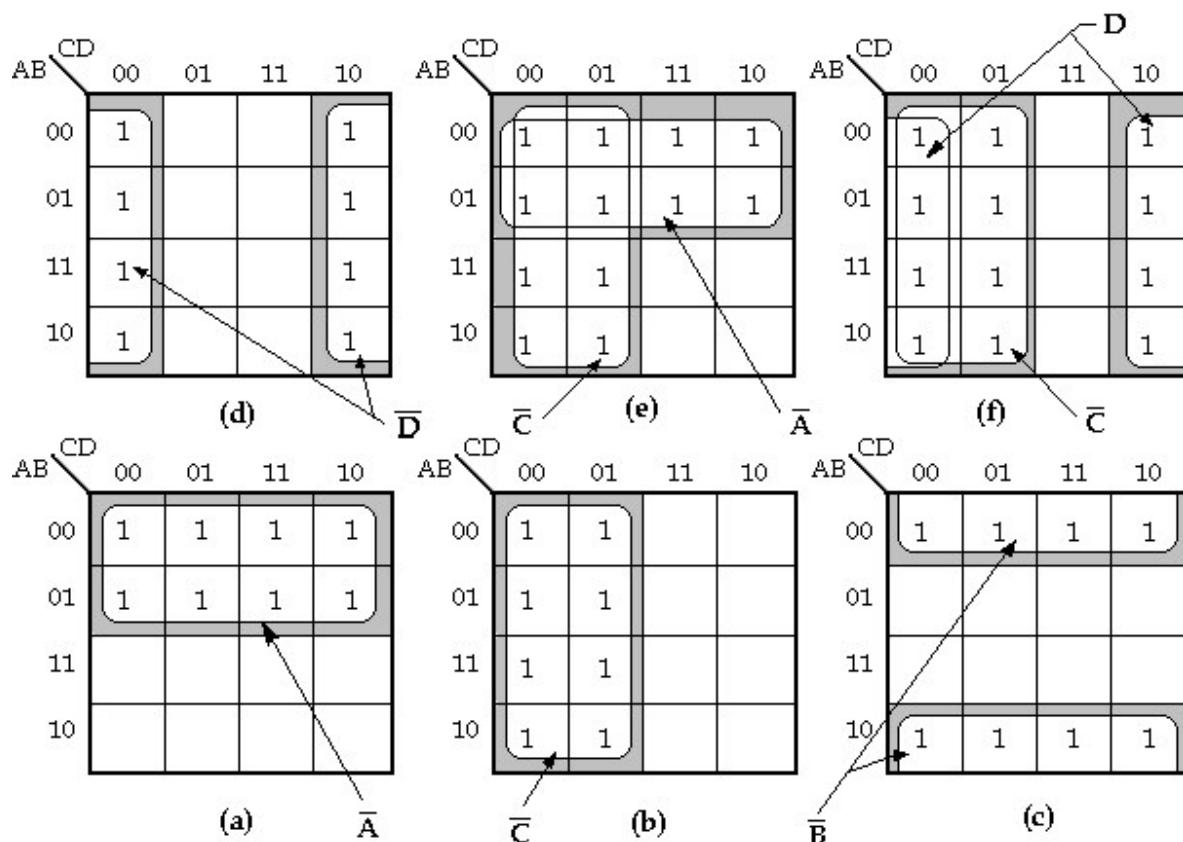
Simplification of Sum of Products Expressions: (Minimal Sums)

The generalized procedure to simplify Boolean expressions as follows:

1. Plot the K-map and place 1's in those cells corresponding to the 1's in the sum of product expression. Place 0's in the other cells.

2. Check the K-map for adjacent 1's and encircle those 1's which are not adjacent to any other 1's. These are called **isolated 1's**.

3. Check for those 1's which are adjacent to only one other 1 and encircle such **pairs**.

4. Check for **quads** and **octets** of adjacent 1's even if it contains some 1's that have already been encircled. While doing this make sure that there are minimum number of groups.

5. Combine any pairs necessary to include any 1's that have not yet been grouped. 6.Form the simplified expression by summing product terms of all the groups.

Three- Variable Map:

1. Simplify the Boolean expression,

F(x, y, z) = ∑m (3, 4, 6, 7).

Soln:
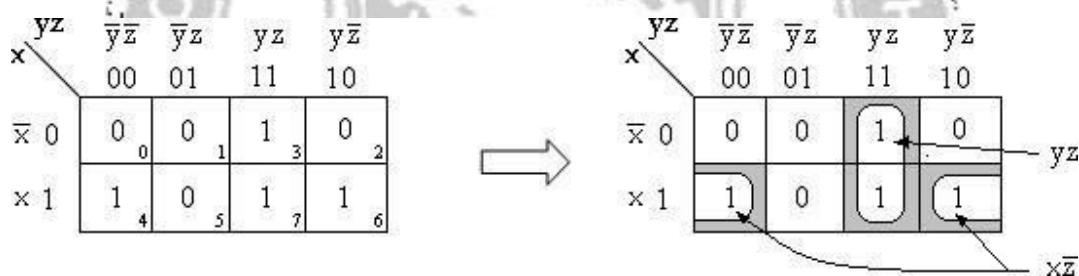
F = yz+ xz'

2. **F(x, y, z) = ∑m (0, 2, 4, 5, 6).**

Soln:
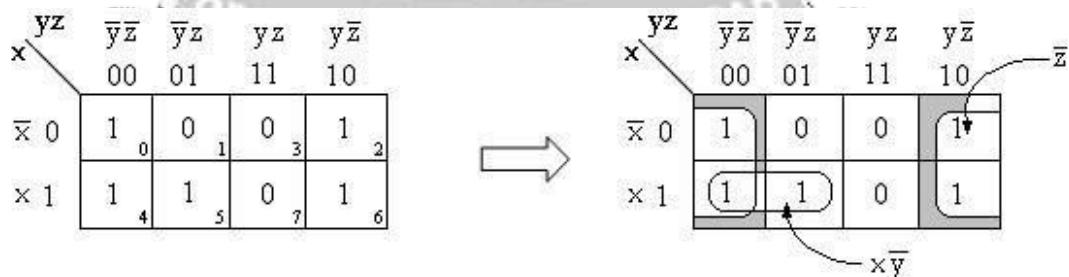
F = z'+ xy'

3.F = A'C + A'B + AB'C + BC Soln:

$= A'C\ (B+ B') + A'B\ (C+ C') + AB'C + BC\ (A+ A')$

$= \underline{A'BC}+ A'B'C + \underline{A'BC} + A'BC' + AB'C + ABC + \underline{A'BC}$

$= A'BC+ A'B'C + A'BC' + AB'C + ABC$

$= m_3+ m_1+ m_2+ m_5+ m_7$

$= \sum m\ (1, 2, 3, 5, 7)$
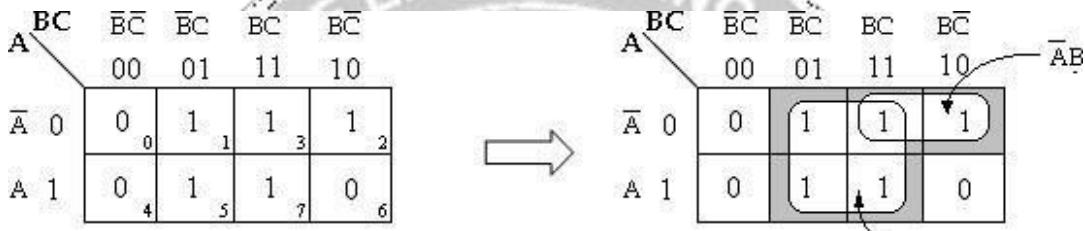
F = C + A'B

4. AB'C + A'B'C + A'BC + AB'C' + A'B'C' <u>Soln:</u>

$= m5 + m1 + m3 + m4 + m0$

$= \sum m\ (0, 1, 3, 4, 5)$
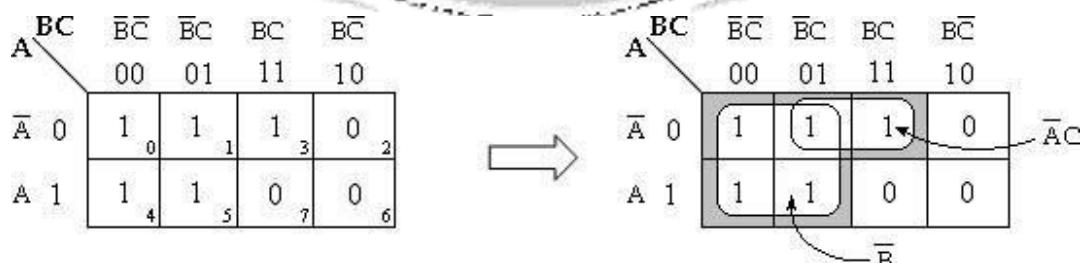
F = A'C + B'

Four - Variable Map:

1. Simplify the Boolean expression,

   Y = A'BC'D' + A'BC'D + ABC'D' + ABC'D + AB'C'D + A'B'CD' <u>Soln:</u>
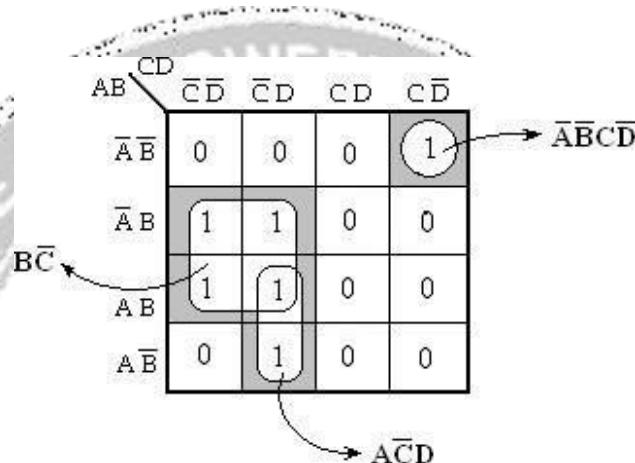
Therefore,   Y= A'B'CD'+ AC'D+ BC'

2. **F (w, x, y, z) = ∑ m(0, 1, 2, 4, 5, 6, 8, 9, 12, 13, 14)** <u>Soln:</u>
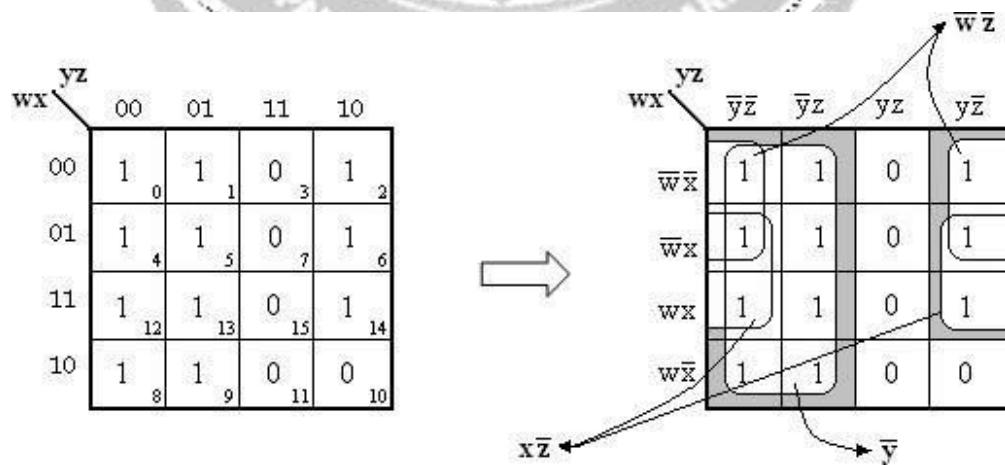
Therefore,

   F= y'+ w'z'+ xz'

3. **F= A'B'C'+ B'CD'+ A'BCD'+ AB'C'**

   = A'B'C' (D+ D') + B'CD' (A+ A') + A'BCD'+ AB'C' (D+ D')

   = A'B'C'D+ A'B'C'D'+ AB'CD'+ A'B'CD'+ A'BCD'+ AB'C'D+ AB'C'D'

   = m_1+ m_0+ m_{10}+ m_2+ m_6+ m_9+ m_8

   = **∑ m (0, 1, 2, 6, 8, 9, 10)**
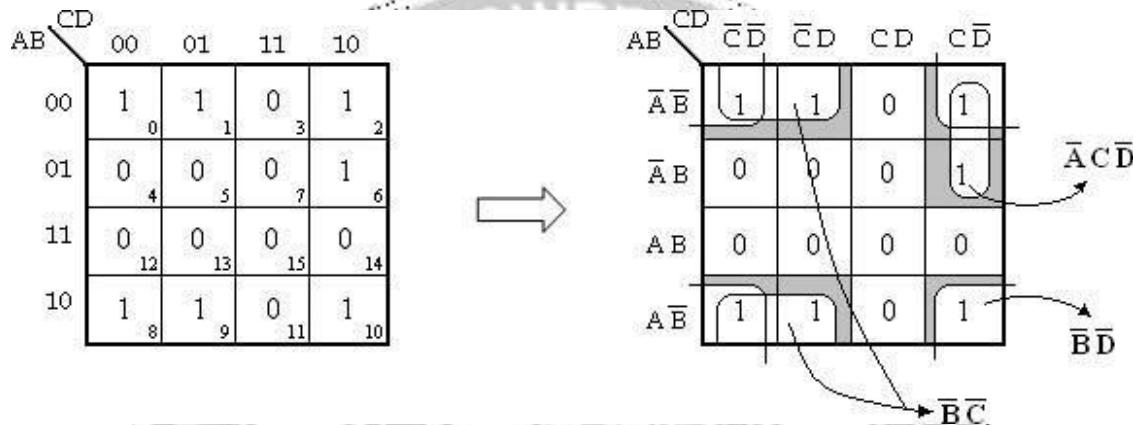
Therefore, **F= B'D'+ B'C'+**

   A'CD'.

4. **Y= ABCD+ AB'C'D'+ AB'C+ AB**

   = ABCD+ AB'C'D'+ AB'C (D+D')+ AB (C+C') (D+D')

   = ABCD+ AB'C'D'+ AB'CD+ AB'CD'+ (ABC+ABC') (D+ D')

   = <u>ABCD</u>+ AB'C'D'+ AB'CD+ AB'CD'+ <u>ABCD</u>+ ABCD'+ ABC'D+ ABC'D'

   = ABCD+ AB'C'D'+ AB'CD+ AB'CD'+ ABCD'+ ABC'D+ ABC'D'

   = m15+ m8+ m11+ m10+ m14+ m13+ m12

   = **∑ m (8, 10, 11, 12, 13, 14, 15)**
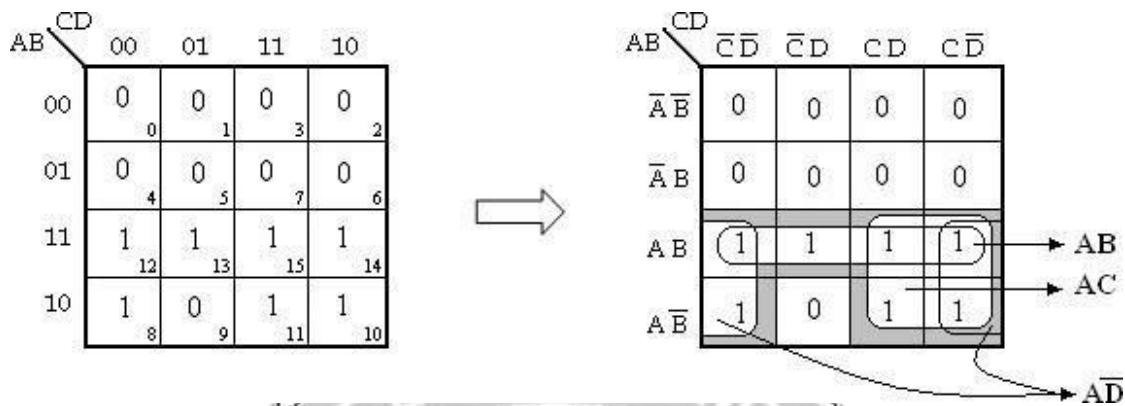
Therefore, **Y= AB+ AC+AD'.**

5. **Y (A, B, C, D)= ∑ m (7, 9, 10, 11, 12, 13, 14, 15)**
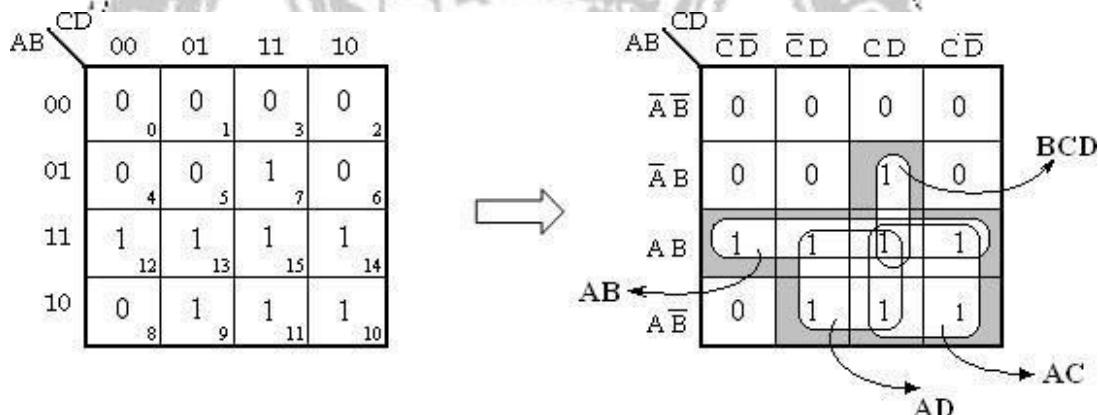
Therefore,

   Y= AB+ AC+ AD+BCD.

6. Y= A'B'C'D+ A'BC'D+ A'BCD+ A'BCD'+ ABC'D+ ABCD+ AB'CD

   = m1+ m5+ m7+ m6+ m13+ m15+ m11

   = **∑ m (1, 5, 6, 7, 11, 13, 15)**
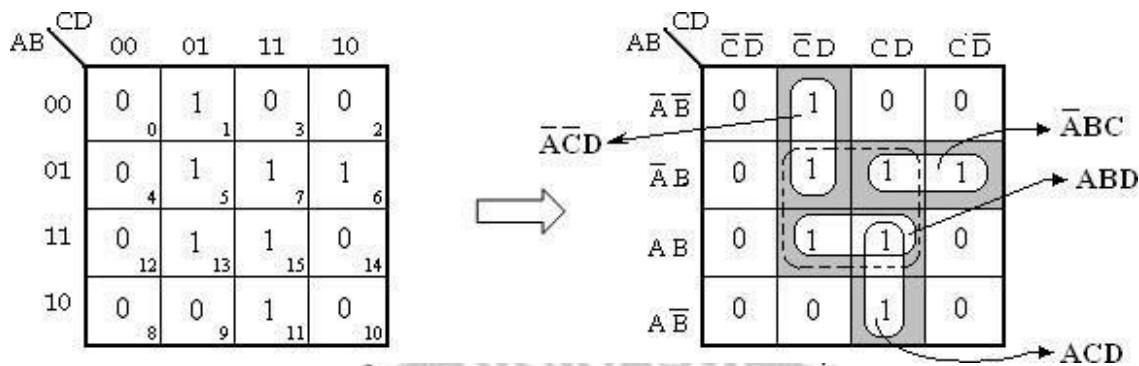
In the above K-map, the cells 5, 7, 13 and 15 can be grouped to form a quad as indicated by the dotted lines. In order to group the remaining 1's, four pairs have to be formed. However, all the four 1's covered by the quad are also covered by the pairs. So, the quad in the above k-map is redundant.

Therefore, the simplified expression will be,

Y = A'C'D+ A'BC+ ABD+ ACD.

7. **Y= ∑ m (1, 5, 10, 11, 12, 13, 15)**
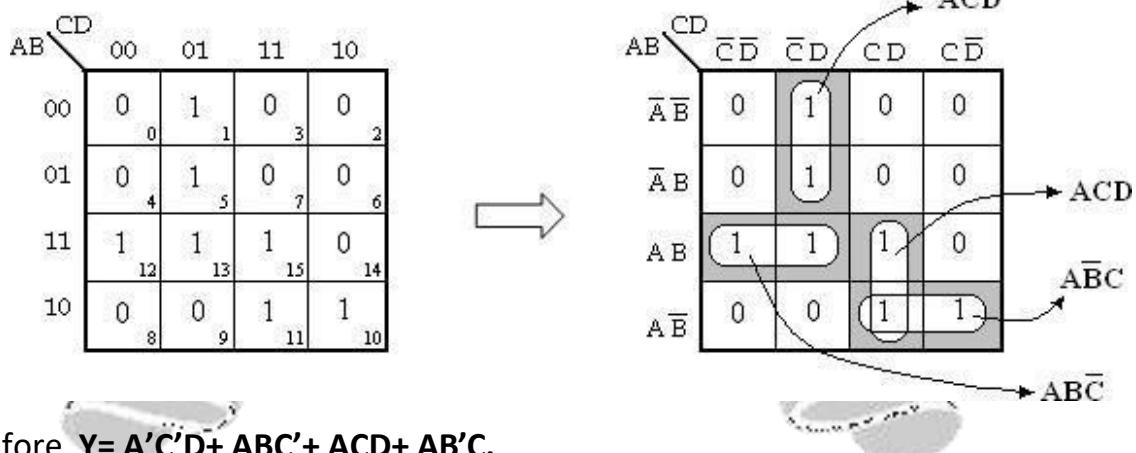
Therefore, **Y= A'C'D+ ABC'+ ACD+ AB'C.**

8. **Y= A'B'CD'+ ABCD'+ AB'CD'+ AB'CD+ AB'C'D'+ ABC'D'+ A'B'CD+ A'B'C'D'**
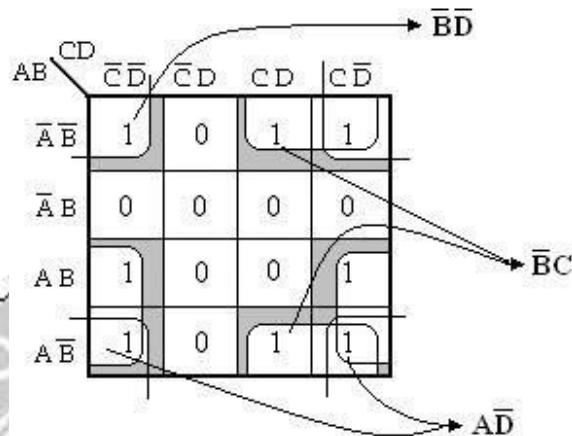
Therefore, **Y= AD'+ B'C+ B'D'**

9. **F (A, B, C, D) = ∑ m (0, 1, 4, 8, 9, 10)**
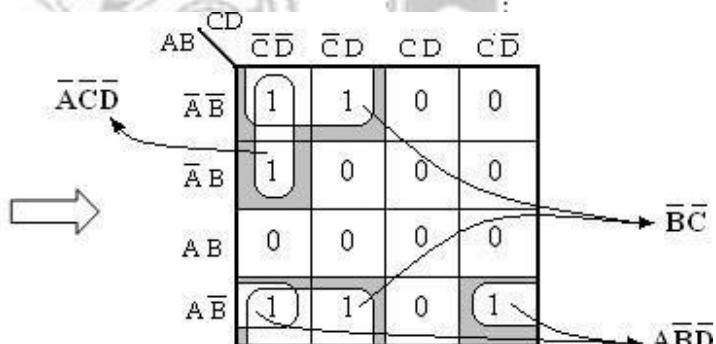
Therefore, **F= A'C'D'+ AB'D'+ B'C'.**

Simplification of Sum of Products Expressions: (Minimal Sums)

1. **Y= (A+ B+ C') (A+ B'+ C') (A'+ B'+ C') (A'+ B+ C) (A+ B+ C)**

$= M_1. M_3. M_7. M_4. M_0$

$= \prod M (0, 1, 3, 4, 7)$

$= \sum m (2, 5, 6)$

Y' = B'C'+ A'C+ BC.

Y= Y'' = (B'C'+ A'C+ BC)'

= (B'C')'. (A'C)'. (BC)'

= (B''+ C''). (A''+C'). (B'+ C') **Y =**

(B+ C). (A+C'). (B'+ C')

2. **Y= (A'+ B'+ C+ D) (A'+ B'+ C'+ D) (A'+ B'+ C'+ D') (A'+ B+ C+ D) (A+ B'+ C'+ D) (A+**

B'+ C'+ D') (A+ B+ C+ D) (A'+ B'+ C+ D')

= M12. M14. M15. M8. M6. M7. M0.

M13 = ∏M (0, 6, 7, 8, 12, 13, 14, 15)

Y' = B'C'D'+ AB+ BC

Y= Y'' = (B'C'D'+ AB+ BC)'

= (B'C'D')'. (AB)'. (BC)'

$$= (B''+ C''+D''). (A'+B'). (B'+ C')$$

$$= (B+ C+ D). (A'+ B'). (B'+ C')$$

Therefore, **Y= (B+ C+ D). (A'+ B'). (B'+ C')**

3. **F(A, B, C, D)= ∏M (0, 2, 3, 8, 9, 12, 13, 14, 15)**
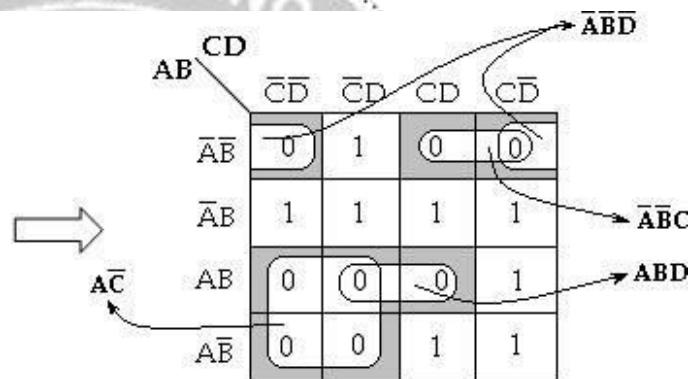
Y' = A'B'D'+ A'B'C+ ABD+ AC'

Y= Y'' = (A'B'D'+ A'B'C+ ABD+ AC')'

$$= (A'B'D')'. (A'B'C)'. (ABD)'. (AC')'$$

$$= (A''+ B''+ D''). (A''+ B''+C'). (A'+ B'+ D'). (A'+ C'')$$

$$= (A+ B+ D). (A+ B+ C'). (A'+ B'+ D'). (A'+ C)$$

Therefore, **Y= (A+ B+ D). (A+ B+ C'). (A'+ B'+ D'). (A'+ C)**

4. **F(A, B, C, D)= ∑m (0, 1, 2, 5, 8, 9, 10)**

$$= ∏M (3, 4, 6, 7, 11, 12, 13, 14, 15)$$

Y' = BD'+ CD+ AB

Y= Y" = (BD'+ CD+ AB)'

    = (BD')'. (CD)'. (AB)'

    = (B'+ D"). (C'+ D'). (A'+ B')

    = (B'+ D). (C'+ D'). (A'+ B')

Therefore, **Y= (B'+ D). (C'+ D'). (A'+ B')**

Don't care Conditions:

        A don't care minterm is a combination of variables whose logical value is not specified. When choosing adjacent squares to simplify the function in a map, the don't care minterms may be assumed to be either 0 or 1. When simplifying the function, we can choose to include each don't care minterm with either the 1's or the 0's, depending on which combination gives the simplest expression.

1. F (x, y, z) = ∑m (0, 1, 2, 4, 5)+ ∑d (3, 6, 7)

    F (x, y, z) = 1

2. F (w, x, y, z) = ∑m (1, 3, 7, 11, 15)+ ∑d (0, 2, 5)
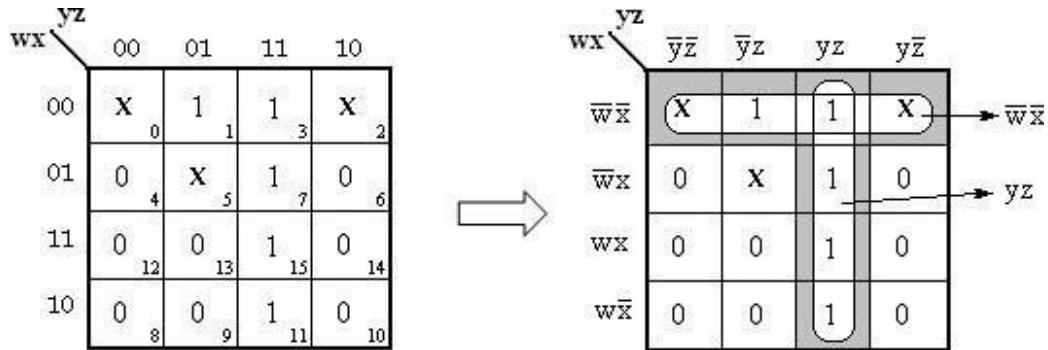
F (w, x, y, z) = w'x'+ yz

3. F (w, x, y, z) = ∑m (0, 7, 8, 9, 10, 12)+ ∑d (2, 5, 13)
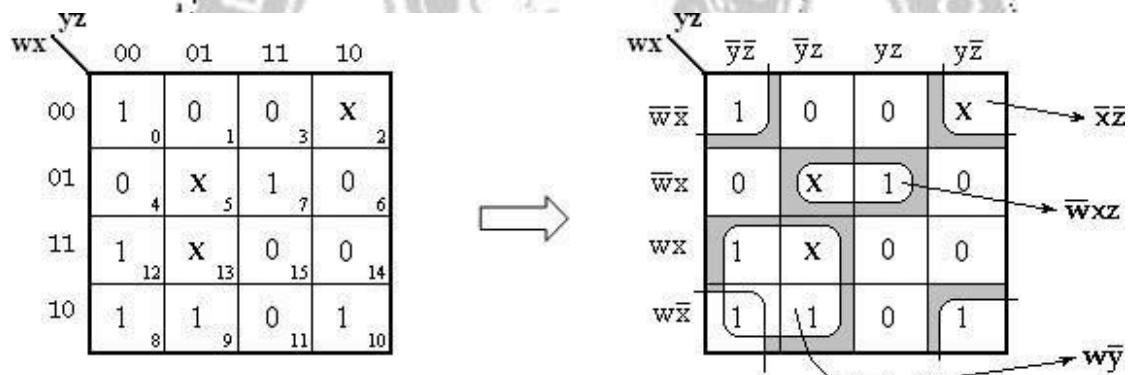
F (w, x, y, z) = w'xz+ wy'+ x'z'.

4. F (w, x, y, z) = ∑m (0, 1, 4, 8, 9, 10)+ ∑d (2, 11)

Soln:
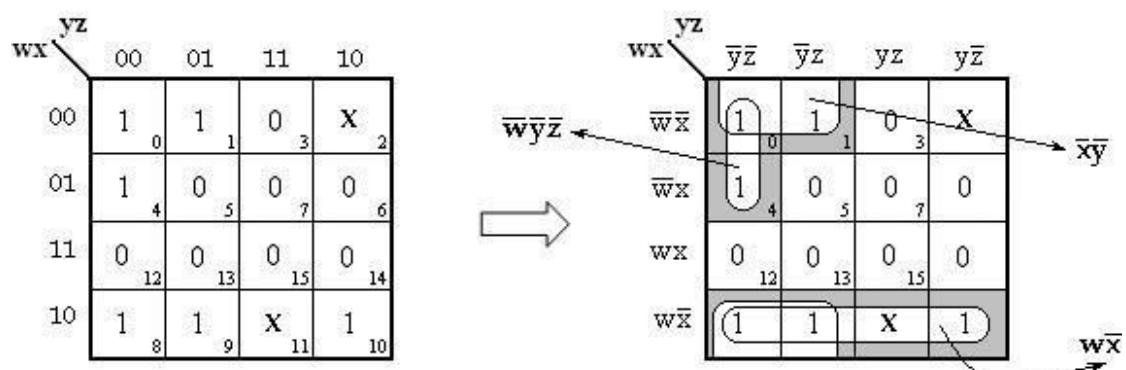
F (w, x, y, z) = wx'+ x'y'+ w'y'z'.

5. **F(A, B, C, D) = ∑m (0, 6, 8, 13, 14)+ ∑d (2, 4, 10)** <u>**Soln:**</u>
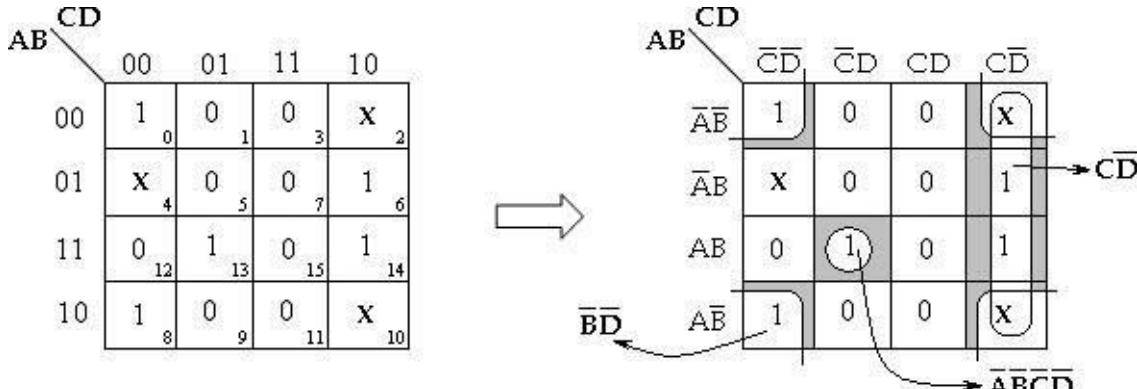
F(A, B, C, D) = CD'+ B'D'+ A'B'C'D'.

<u>**Five- Variable Maps:**</u>

A 5- variable K- map requires 2^5= 32 cells, but adjacent cells are difficult to identify on a single 32-cell map. Therefore, two 16 cell K-maps are used.

If the variables are A, B, C, D and E, two identical 16- cell maps containing B, C, D and E can be constructed. One map is used for A and other for A'.

In order to identify the adjacent grouping in the 5- variable map, we must imagine the two maps superimposed on one another ie., every cell in one map is adjacent to the corresponding cell in the other map, because only one variable changes between such corresponding cells.
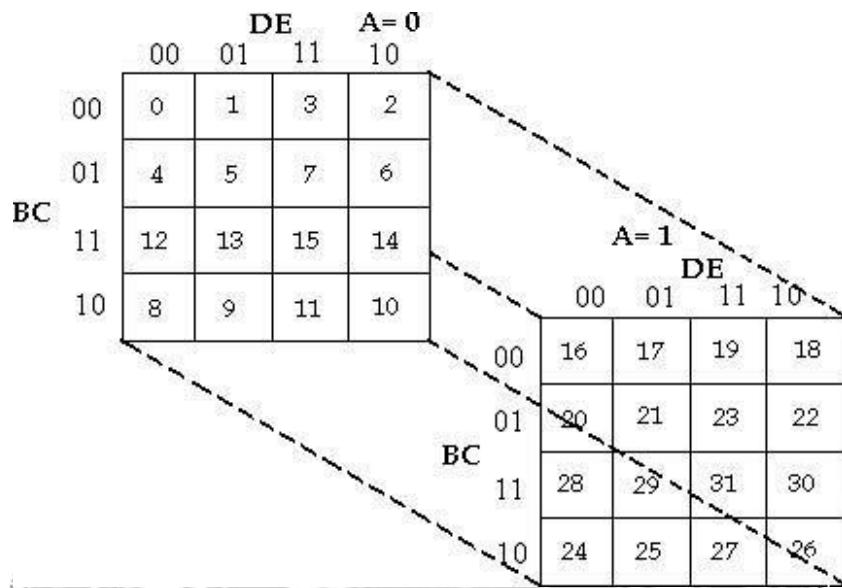
Five- Variable Karnaugh map (Layer Structure)

Thus, every row on one map is adjacent to the corresponding row (the one occupying the same position) on the other map, as are corresponding columns. Also, the rightmost and leftmost columns within each 16- cell map are adjacent, just as they are in any 16- cell map, as are the top and bottom rows.
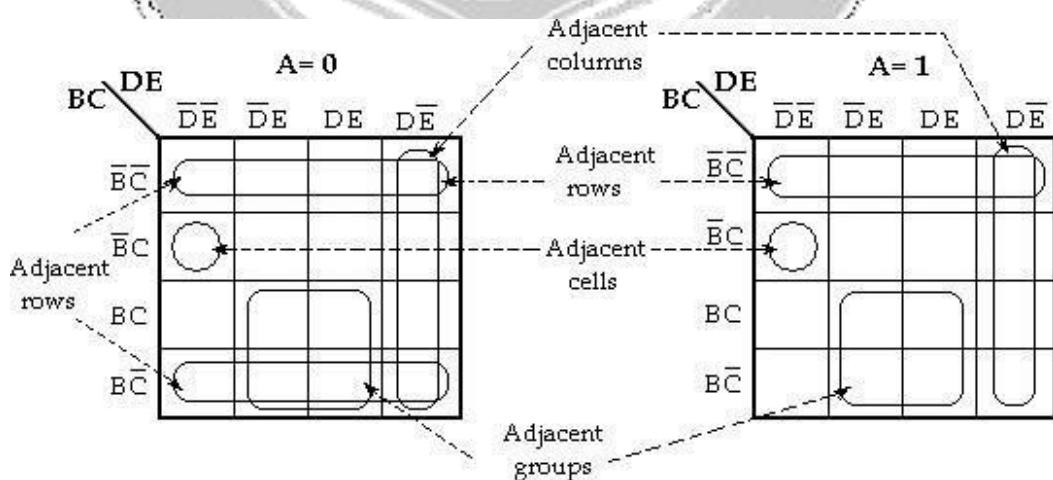

Typical subcubes on a five-variable map

However, the rightmost column of the map is not adjacent to the leftmost column of the other map.

1. Simplify the Boolean function

   F (A, B, C, D, E) = ∑m (0, 2, 4, 6, 9, 11, 13, 15, 17, 21, 25, 27, 29, 31)

Soln:
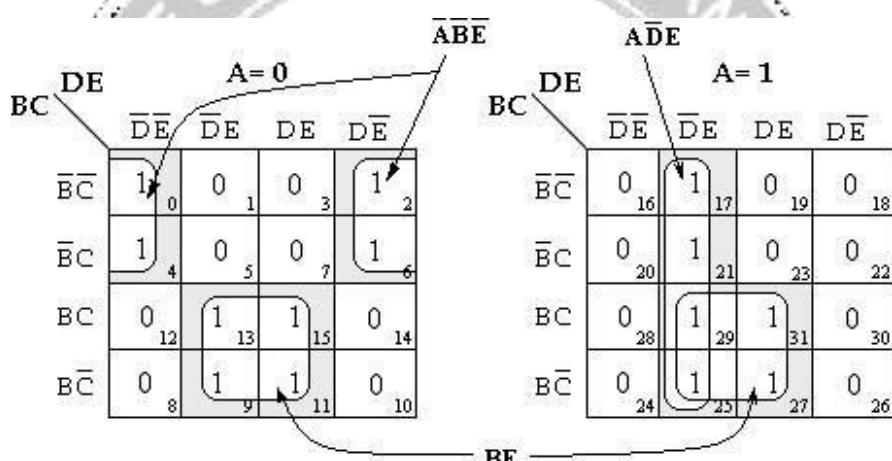
F (A, B, C, D, E) = A'B'E'+ BE+ AD'E

2. F (A, B, C, D, E) = ∑m (0, 5, 6, 8, 9, 10, 11, 16, 20, 24, 25, 26, 27, 29, 31)

Soln:
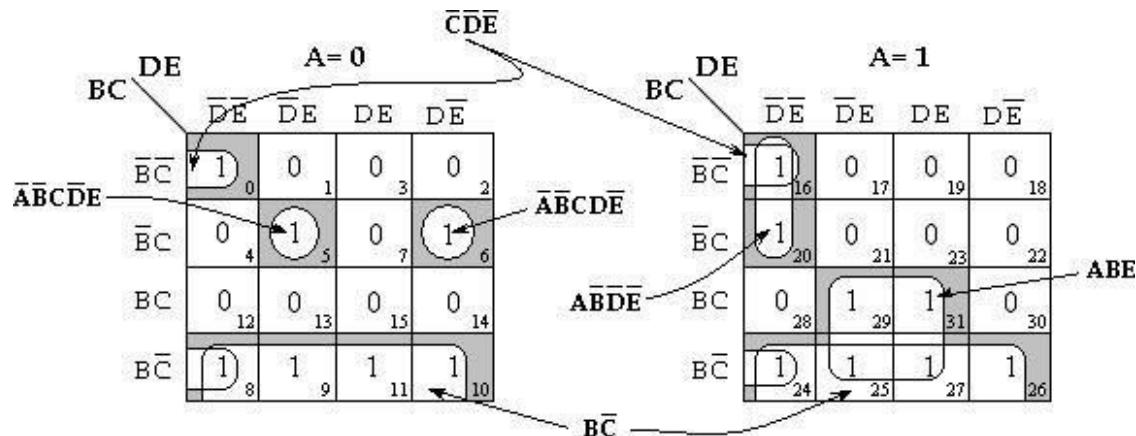
F (A, B, C, D, E) = C'D'E'+ A'B'CD'E+ A'B'CDE'+ AB'D'E'+ ABE+ BC'

3. F (A, B, C, D, E) = ∑m (1, 4, 8, 10, 11, 20, 22, 24, 25, 26)+∑d (0, 12, 16, 17)

Soln:
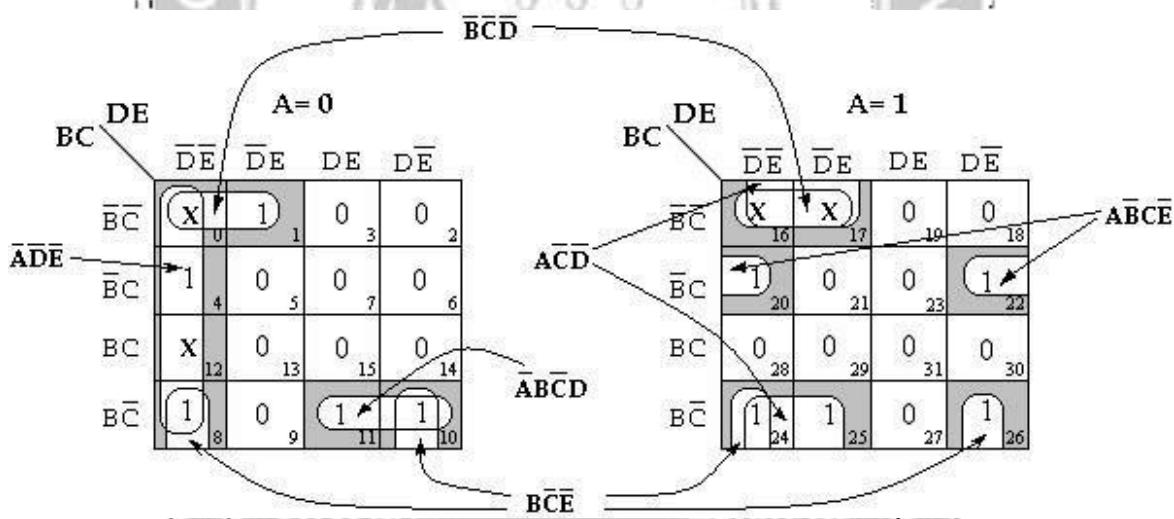
F (A, B, C, D, E) = B'C'D'+ A'D'E'+ BC'E'+ A'BC'D+ AC'D'+ AB'CE'

4. $F(A, B, C, D, E) = \sum m (0, 1, 2, 6, 7, 9, 12, 28, 29, 31)$

Soln:
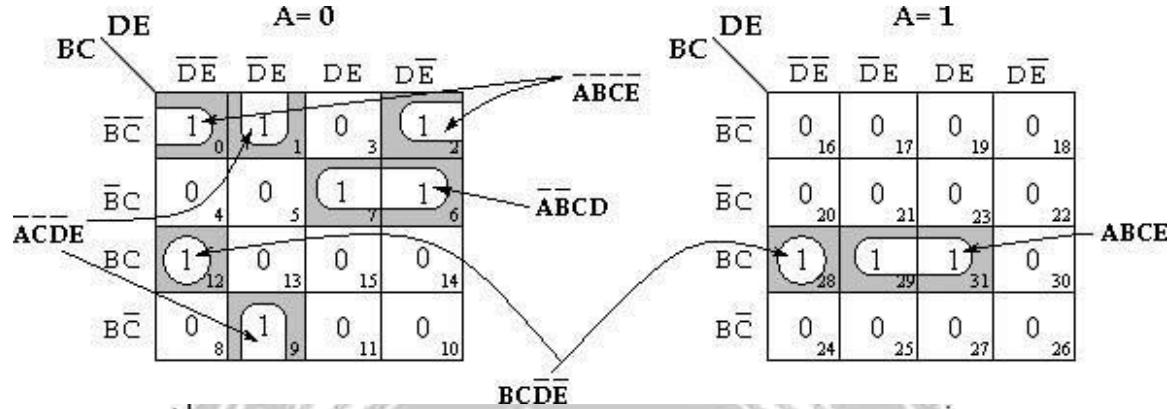
$F(A, B, C, D, E) = BCD'E' + ABCE + A'B'C'E' + A'C'D'E + A'B'CD$

5. $F(x_1, x_2, x_3, x_4, x_5) = \sum m (2, 3, 6, 7, 11, 12, 13, 14, 15, 23, 28, 29, 30, 31)$

Soln:
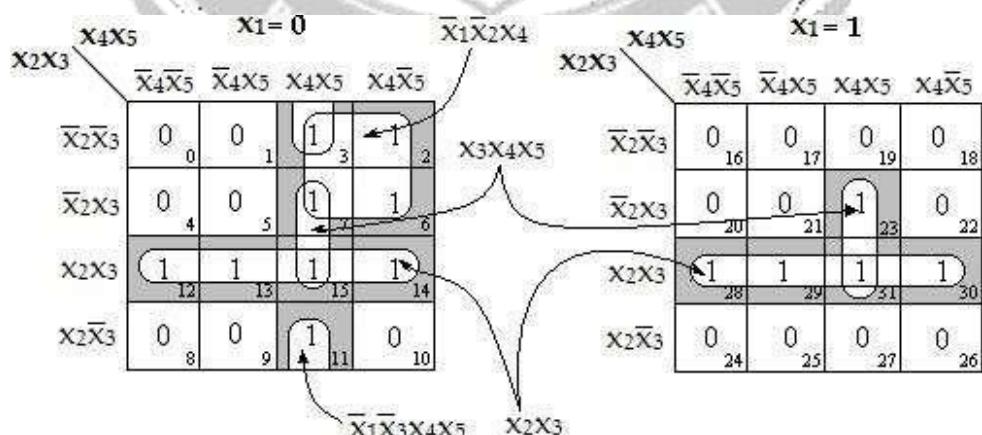
$F(x_1, x_2, x_3, x_4, x_5) = x_2 x_3 + x_3 x_4 x_5 + x_1' x_2' x_4 + x_1' x_3' x_4 x_5$

6. F (x_1, x_2, x_3, x_4, x_5) = ∑m (1, 2, 3, 6, 8, 9, 14, 17, 24, 25, 26, 27, 30, 31)+ ∑d (4, 5)

Soln:
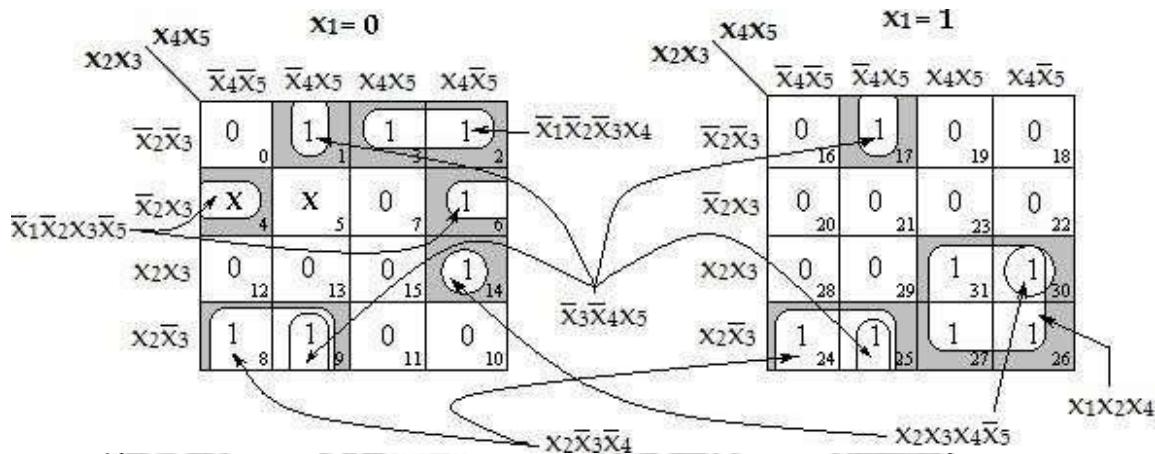
F (x_1, x_2, x_3, x_4, x_5) = $x_2x_3'x_4' + x_2x_3x_4x_5' + x_3'x_4'x_5 + x_1x_2x_4 + x_1'x_2'x_3x_5' + x_1'x_2'x_3'x_4$