MAGNETIC FIELD INTENSITY DUE TO AN INFINITE WIRE CARRYING A CURRENT

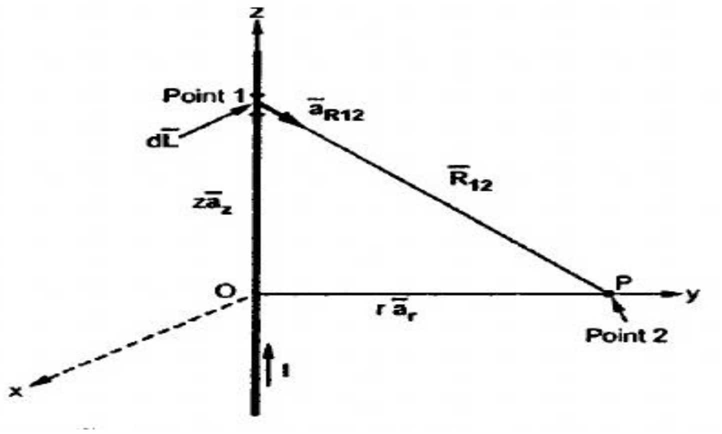


Fig5: H due to infinitely long straight conductor carrying current I

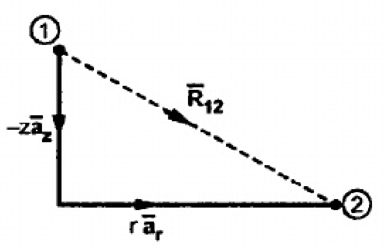
Consider a infinitely long straight conductor is placed along z axis.
The current passing through the conductor is a direct current of I Amp.
The field intensity H at a point P is also to be calculated which is at a distance r from the z-axis as shown in above fig.
Consider small differential element at point 1 along z- axis at a distance z from origin. I $d\bar{l}$ = Idz$\overline{az}$.

The distance vector joining point 1 &point 2 is $\overline{R_{12}}$ are
$\overline{R_{12}}$ = -z $\overline{az}$ + r $\overline{ar}$

$$a\overline{R_{12}} = \frac{\overline{R_{12}}}{|R_{12}|} = \frac{r\,\overline{ar} - z\overline{az}}{\sqrt{r^2+z^2}}$$

$$d\bar{l} \times \overline{R_{12}} = \begin{vmatrix} \overline{ar} & \overline{a\emptyset} & \overline{az} \\ 0 & 0 & dz \\ r & 0 & -z \end{vmatrix}$$

$= \overline{ar}$ (0-0) – (- r dz)$\overline{a\emptyset}$ + $\overline{az}$ (0-0)
= r dz$\overline{a\emptyset}$



R_{12} is neglected for convenience
I $d\bar{l}$ x aR_{12}= $\frac{Ir\,dz\,\overline{a\emptyset}}{\sqrt{r^2+z^2}}$
According to Biot-savart law
H =$\frac{I\,dl\,X\,\overline{aR}_{12}}{4\pi R_{12}^2} = \frac{Ir\,dz\,\overline{a\emptyset}}{4\pi\sqrt{r^2+z^2}(r^2+z^2)^2} = \frac{Ir\,dz\,\overline{a\emptyset}}{4\pi(r^2+z^2)^{\frac{3}{2}}}$
Thus the total field intensity $\bar{H}$ can be obtained by integrating dH over the entire length of the conductor.$\bar{H}$= $\int_{-\infty}^{\infty} d\bar{H} = \int_{-\infty}^{\infty} \frac{Ir\,dz\,a\theta}{4\pi(r^2+z^2)}$
Put z = r tan θ, $z^2 = r^2 tan^2\theta$
dz = r $sec^2\theta$, z=$-\infty$, $\theta = -\frac{\pi}{2}$
Z = $+\infty$, $\theta = \frac{\pi}{2}$

$$\bar{H} = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{Ir\,rsec^2\theta}{4\pi(r^2 + r^2tan^2\theta)^{\frac{3}{2}}}\, d\theta\, a\emptyset$$

1+$tan^2\theta = sec^2\theta$

$\bar{H} = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \frac{Ir^2 sec^2\theta}{4\pi r^3 sec^3\theta} \mathrm{d}\theta \; a\emptyset \;\; = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \frac{I}{4\pi r \; \sec\theta} d\theta \; a\emptyset \;\; = \frac{I}{4\pi r} \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} cos\theta \; d\theta a\emptyset$

$= \frac{I}{4\pi r} [\sin\theta]_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \; a\emptyset$

$= \frac{I}{4\pi r} \left[sin\frac{\pi}{2} - sin\left(\frac{-\pi}{2}\right)\right] \; a\emptyset$

$\bar{H} = \frac{I}{4\pi r} \; 2 \; a\emptyset \quad = \frac{I}{2\pi r} \; a\emptyset \quad A/m$

$\bar{B} = \mu H \quad = \frac{\mu I}{2\pi r} \; a\emptyset$ wb/m^2.

Magnetic field intensity due to finite wire carrying a current I

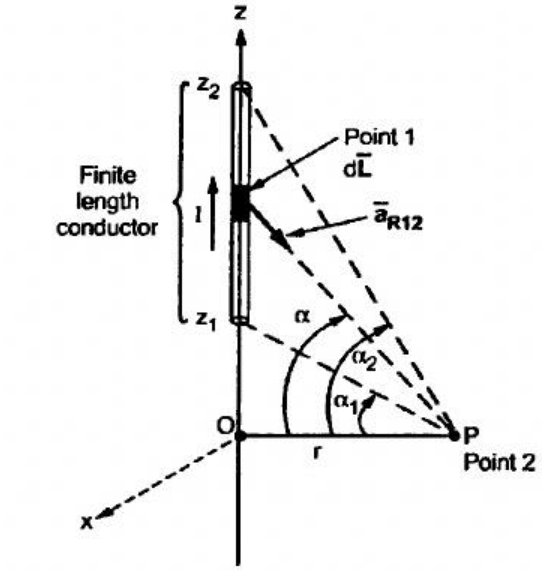


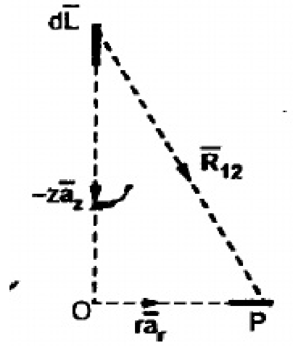
Fig 6: Magnetic field H due to finite conductor carrying current I

Consider a conductor of finite length placed along z-axis as shown in figure. It carries a direct current I. The perpendicular distance of point P from z axis is 'r'.

The conductor is placed such that its one end is at $z=z_1$ while other at $z=z_2$. Consider a differential element dl along z-axis at a distance z from origin.

$d\bar{l} = dz\overline{az}$.

The unit vector in the direction joining differential element to point P Is $a\overline{R_{12}}$ and can be expressed as,



$a\overline{R_{12}} = \frac{R_{12}}{|R_{12}|} \;=\; \frac{-z\,\overline{az}+r\overline{ar}}{\sqrt{(-z)^2+r^2}} \;=\; \frac{r\,\overline{ar}-z\overline{az}}{\sqrt{r^2+z^2}}$

$d\bar{l} \times a\overline{R_{12}} = r\,dz\,\overline{a\emptyset}$

This is some as obtained in the infinite long conductors.

$$d\bar{l} \times \overline{R_{12}} = \begin{vmatrix} \overline{ar} & \overline{a\emptyset} & \overline{az} \\ 0 & 0 & dz \\ r & 0 & -z \end{vmatrix}$$

$= \overline{ar}$ (0-0) – (- r dz)$\overline{a\emptyset}$ + $\overline{az}$ (0-0)

= r dz$\overline{a\emptyset}$

I $d\bar{l} \times a\overline{R_{12}} = \frac{Ir\ dz\ a\emptyset}{\sqrt{r^2+z^2}}$

According to Biot-savart law dH at point P is,

$$dH = \frac{I\ d\bar{l} \times aR_{12}}{4\pi R_{12}^2} = \frac{Ir\ dz\ a\emptyset}{4\pi\sqrt{r^2+z^2}\left(\sqrt{r^2+z^2}\right)^2} = \frac{Ir\ dz\ a\emptyset}{4\pi(r^2+z^2)^{\frac{3}{2}}}$$

The total H at P due to conductor of finite length can be obtained by integrating dH over z = z_1 to z =z_2

$$H=\int_{z_1}^{z_2} d\bar{H} = \int_{z_1}^{z_2} \frac{Ir\ dz\ \overline{a\emptyset}}{4\pi(r^2+z^2)^{\frac{3}{2}}}$$

Put z= r$\tan\theta$, $z^2 = r^2 tan^2\theta$

dz = r $sec^2\theta$

For z = z_1 z_1= r $\tan\alpha_1$

For z = z_2 z_2= r $\tan\alpha_2$

$$\alpha_1 = \tan^{-1}\left({z_1}/{r}\right), \alpha_2 = \tan^{-1}\left({z_2}/{r}\right)$$

$$\bar{H} = \int_{\theta_1}^{\theta_2} \frac{Ir\ rsec^2\alpha\ d\alpha\ a\emptyset}{4\pi(r^2+r^2tan^2\alpha)^{\frac{3}{2}}} = \int_{\theta_1}^{\theta_2} \frac{I\,r^2sec^2\alpha\ d\alpha\ a\emptyset}{4\pi r^3[1+tan^2\alpha]^{\frac{3}{2}}}$$

$$= \int_{\theta_1}^{\theta_2} \frac{I\ d\alpha\ a\emptyset}{4\pi r \sec\alpha} = \int_{\theta_1}^{\theta_2} \frac{I\ \cos\theta\alpha d\emptyset\ a\emptyset}{4\pi r}$$

$\bar{H}\frac{I}{4\pi r}[\sin\alpha]_{\alpha}^{\alpha_2}$ a$\emptyset$ = $\frac{I}{4\pi r}[\sin\alpha_2 - \sin\alpha_1]\ a\emptyset$

B= $\mu\bar{H} = \frac{\mu I}{4\pi r}[\sin\alpha_2 - \sin\alpha_1]\overline{a\emptyset}\,wb/m^2$

Note: While using the result if segment carrying current I is not along z-axis then the direction of $\bar{H}$ not be in $\overline{a\emptyset}$.It depends on in which plane segment carrying current is placed. The magnitude of H is $\frac{1}{4\pi r}$ [sinα_2 – sinα_1] but direction is always normal to the plain containing the source and to be decided by right banded screw rule.