EINSTEIN'S COEFFICIENTS A & B

Einstein calculated the probability of transitions by assuming, the atomic system is in equilibrium with electromagnetic radiation.

Consider an assembly of atoms at an absolute temperature T in which the atoms are in different energy states.

If N_0 is the number of atoms per unit volume in the ground state, then the number of atoms per unit volume in the exited state is given by Maxwell-Boltzmann law as

$$N = N_0\, e^{(-E/KT)} \quad \text{-------------------- (1)}$$

Where K → Boltzmann constant

If N_1 and N_2 are the number of atoms per unit volume in the energy states E1 and E_2, then from eqn (1) we can write

$$N_1 = N_0\, e^{(-E_1/KT)} \quad \text{-------------------- (2)}$$

$$N_2 = N_0\, e^{(-E_2/KT)} \quad \text{-------------------- (3)}$$

$$\frac{N_1}{N_2} = \frac{N_0 e^{\frac{-E_1}{KT}}}{N_0 e^{\frac{-E_2}{KT}}}$$

$$\frac{N_1}{N_2} = \exp\left(\frac{E_2 - E_1}{KT}\right)$$

$$\frac{N1}{N2} = e^{\left(\frac{h\gamma}{KT}\right)} \quad \text{-------------------- (4)}$$

When this assembly of atoms is exposed to light radiations of energy hυ, the transition will take place in the following process.

PROCESS1: STIMULATED ABSORPTION

The atoms in the ground state E_1 raised to the excited energy state E_2 by absorbing a photon of energy hυ, provided the photon energy is equal to the energy difference [$E_2 - E_1$] = hυ. This process is called stimulated absorption and it is an upward transition.

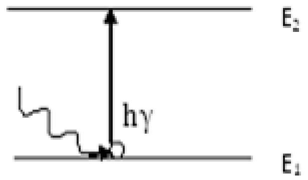

Figure 16

The number of transitions N_{ab} occurring per unit volume is

$N_{ab} = N_1 B_{12}$ Q -------------------- (5)

Where B_{12} → probability of transition from E_1 to E_2.

Q → Absorbed energy

PROCESS2: SPONTANEOUS EMISSION

The atoms in the excited state E2 returns to the ground state by emitting a photon of energy hν without the action of any external agency. Such process is called spontaneous emission and this process is a downward transition.

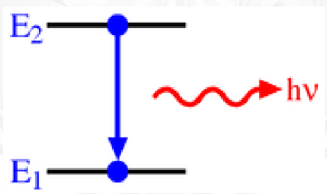

Figure 17

The number of transitions N_{sp} occurring per unit volume is

$N_{sp} = N_2 A_{21}$

Where B_{12} → probability of transition from E_2 to E_1.

PROCESS 3: STIMULATED EMISSION:

The interaction between the atom and the photon of the excited state will bring the atom to the ground state. This process is called stimulated emission and is downward transition.

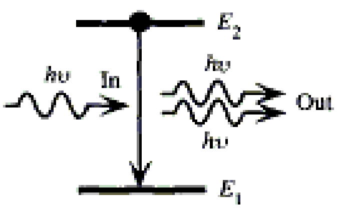

Figure 18

The number of transitions N_{sp} occurring per unit volume is

$$N_{sp} = N_2 B_{21}\ Q$$

Where B_{12} is the probability of transition from E_2 to E_1.

Q→Emitted energy

The Co- efficient A21, B_{21} andB_{12} are known as Einstein's co-efficient.

Under equilibrium, Upward transition = Downward transition

$$N_1 B_{12}\ Q = N_2 A_{21} + N_2 B_{21}\ Q$$

$$N_2 A_{21} = N_1 B_{12}\ Q - N_2 B_{21}\ Q$$

$$= [\ N_1 B_{12} - N_2 B_{21}]\ Q$$

$$Q = \frac{N_2 A_{21}}{N_1 B_{12} - N_2 B_{21}} \qquad \text{--------------------- (6)}$$

Taking $N_2 B_{21}$ outside from the denominator

$$Q = \frac{N_2 A_{21}}{N_2 B_{21}\left(\frac{N_1 B_{12}}{N_2 B_{21}} - 1\right)}$$

$$Q = \left[\frac{A_{21}}{B_{21}}\right] \frac{1}{\left(\frac{N_1 B_{12}}{N_2 B_{21}} - 1\right)}$$

$$Q = \left[\frac{A_{21}}{B_{21}}\right] \frac{1}{\left(\frac{B_{12}}{B_{21}} e^{\left(\frac{h\gamma}{KT}\right)} - 1\right)} \qquad \text{--------------------- (7)}$$

The above equation must satisfy Planck's radiation law

$$Q = \frac{8\pi h\vartheta^3}{c^3} \frac{1}{\left(e^{\left(\frac{h\gamma}{KT}\right)} - 1\right)} \qquad \text{--------------------- (8)}$$

By equating (7) and (8) we get,

$$\frac{8\pi h\vartheta^3}{c^3} \frac{1}{\left(e^{\left(\frac{h\gamma}{KT}\right)} - 1\right)} = \left[\frac{A_{21}}{B_{21}}\right] \frac{1}{\left(\frac{B_{12}}{B_{21}} e^{\left(\frac{h\gamma}{KT}\right)} - 1\right)}$$

$$\frac{A_{21}}{B_{21}} = \frac{8\pi h\vartheta^3}{c^3}$$ ---------------------(9)

$$\frac{B_{12}}{B_{21}} e^{\left(\frac{h\gamma}{KT}\right)} = e^{\left(\frac{h\gamma}{KT}\right)}$$

$B_{12} = B_{21}$ under thermal equilibrium. --------------------(10)

A_{21}, B_{21} are called Einstein's co-efficient for spontaneous emission and stimulated emission probability per time.