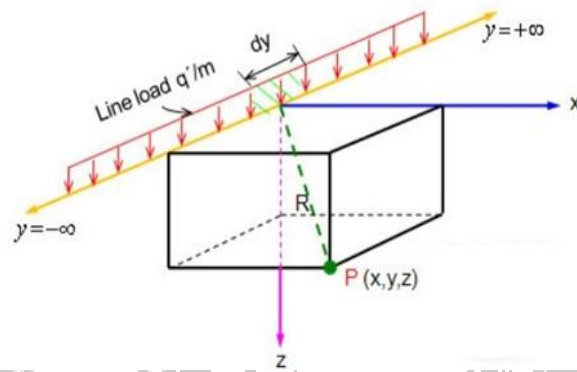


3) Stress due to line load:

The vertical stress in a soil mass due to a vertical line load can be obtained using Boussinesq solution. Let the vertical line load be of intensity q per unit length, along the y axis, acting on the surface of a semi infinite soil mass



Let us consider the load acting on a small length dy . The load can be taken as a point load of $q' dy$ using Boussinesq solution the vertical stress at P is given by

$$\sigma_z = \frac{3Q}{2\pi z^2} \left[\frac{(Z)^5}{(r^2 + z^2)^{5/2}} \right]$$

$$\Delta\sigma_z = \frac{3q'dy}{2\pi} \left[\frac{(Z)^3}{(r^2 + z^2)^{5/2}} \right] \text{ --- (1)}$$

The vertical stress at P due to line load extending from $-\infty$ to $+\infty$ is obtained by integration,

$$\sigma_z = \frac{3q'z^3}{2\pi} \int_{-\infty}^{+\infty} \frac{dy}{(r^2 + z^2)^{5/2}}$$

We know that $r^2 = x^2 + y^2$

$$\sigma_z = \frac{3q'z^3}{2\pi} \int_{-\infty}^{+\infty} \frac{dy}{(x^2 + y^2 + z^2)^{5/2}} \text{ --- (2)}$$

Substitute $x^2 + y^2 = u^2$ in eqn (2)

$$\sigma_z = \frac{3q'z^3}{2\pi} \int_{-\infty}^{+\infty} \frac{dy}{(u^2 + z^2)^{5/2}} \text{ --- (3)}$$

Let $y = u \tan \theta$ $dy = u \sec^2 \theta \cdot d\theta$

When $y = -\infty$, $\tan \theta = -\infty$, $\theta = -\pi/2$

$y = +\infty$, $\tan \theta = \infty$, $\theta = \pi/2$

Eqn (3) can be written as,

$$\sigma_z = \frac{3q'z^3}{2\pi} \int_{-\pi/2}^{\pi/2} \frac{dy}{(u^2 + u^2 \tan^2 \theta)^{5/2}}$$

$$\sigma_z = \frac{3q'z^3 \cdot 2}{2\pi} \int_0^{\pi/2} \frac{dy}{(u^2 + u^2 \tan^2 \theta)^{5/2}}$$

$$\sigma_z = \frac{3q'z^3}{\pi} \int_0^{\pi/2} \frac{dy}{u^2(1 + \tan^2 \theta)^{5/2}}$$

Substitute $dy = u \cdot \sec^2 \theta \cdot d\theta$ & $[1 + \tan^2 \theta = \sec^2 \theta]$

$$\sigma_z = \frac{3q'z^3}{\pi} \int_0^{\pi/2} \frac{u \cdot \sec^2 \theta \cdot d\theta}{u^2(\sec^2 \theta)^{5/2}}$$

$$\sigma_z = \frac{3q'z^3}{\pi u^4} \int_0^{\pi/2} \cos^3 \theta \cdot d\theta$$

$$\sigma_z = \frac{3q'z^3}{\pi u^4} \int_0^{\pi/2} \cos^2 \theta \cdot \cos \theta \cdot d\theta$$

$$\sigma_z = \frac{3q'z^3}{\pi u^4} \int_0^{\pi/2} (1 - \sin^2 \theta) \cdot \cos \theta \cdot d\theta \text{ --- (4)}$$

Let $\sin \theta = t$

$\cos \theta \cdot d\theta = dt$

When $\theta = 0$, $\sin \theta = 0$

$\theta = \pi/2$, $\sin \theta = 1$

Eqn (4) becomes

$$\sigma_z = \frac{3q'z^3}{\pi u^4} \int_0^1 (1 - t^2) \cdot dt$$

$$= \frac{3q'z^3}{\pi u^4} \left(t - \frac{t^3}{3} \right)_0^1$$

$$= \frac{3q'z^3}{\pi u^4} \left(1 - \frac{1}{3} \right)$$

$$= \frac{3q'z^3}{\pi u^4} \cdot \frac{2}{3}$$

$$= \frac{2q'z^3}{\pi((u^2)^2)}$$

$$\sigma_z = \frac{2q'z^3}{\pi((x^2 + z^2)^2)}$$

$$\sigma_z = \frac{2q'}{\pi z \left(1 + \left(\frac{x}{z} \right)^2 \right)^2}$$

$$\sigma_z = \frac{2q'}{\pi z} \cdot \left[\frac{1}{1 + \left(\frac{x}{z} \right)^2} \right]^2 \text{-----(5)}$$

$$\sigma_z = (I_B) \frac{q'}{z}$$

$$I_B = \frac{2}{\pi} \cdot \left[\frac{1}{1 + \left(\frac{x}{z} \right)^2} \right]^2$$

I_B = Boussinesq influence factor for line load

When the point P lies vertically below the line load $x=0$

$$\sigma_z = \frac{2q'}{\pi z}$$

Problems

1) A line load of 100 kN/m run extends to a long distance. Determine the intensity of vertical stress at a point 2 m below the surface for the following two cases.

i) Directly under the line load

ii) At a distance of 2 m perpendicular to the line load. Use Boussinesq theory

Solution:

$$q' = 100 \text{ kN/m}$$

case i) $z=2\text{m}, x=0$

$$\sigma_z = \frac{2q'}{\pi z} \cdot \left[\frac{1}{1 + \left(\frac{x}{z}\right)^2} \right]^2$$

$$\sigma_z = \frac{2 \times 100}{\pi \times 2} \cdot \left[\frac{1}{1 + \left(\frac{0}{2}\right)^2} \right]^2$$

$$\sigma_z = 31.83 \text{ kN/m}^2$$

case ii) $z=2\text{m}, x=2$

$$\sigma_z = \frac{2 \times 100}{\pi \times 2} \cdot \left[\frac{1}{1 + \left(\frac{2}{2}\right)^2} \right]^2$$

$$\sigma_z = 7.96 \text{ kN/m}^2$$