

## UNIT III

### STATISTICAL PROCESS CONTROL

Meaning and significance of statistical process control (SPC) – construction of control charts for variables and attributes. Process capability – Meaning, significance and measurement – Six sigma - concepts of process capability. Reliability concepts – Definitions, reliability in series and parallel, product life characteristics curve. Total productive maintenance (TMP), Tero Technology.

### PROCESS CAPABILITY

#### Process

The process is a set of interrelated activities that convert the inputs such as equipment, materials, people, methods, and environment into desirable output.

The quality of the process is measured by the output of product quality characteristics. If product characteristics/specifications conform to engineering/customer requirements, then the process is considered stable/capable and performing well as per set standards.

A process is considered stable if there are only Common Causes of Variation present in the process.

#### Process Capability Meaning and Definition

Minimum spread of process variation present in a manufacturing process.

Process capability means how likely a product is going to meet the design specification.

It is the 6 Sigma range of process **Inherent variation**.

In SPC, Control limits are one of the most basic requirements of a **process capability** study, So it is very important to ensure the condition of the 4M input while calculating trial control limits.

**Process Capability** = +/- 3 Sigma = Total spread of 6 Sigma

#### Need for process capability (PC)

- For meeting customer requirements/specifications.
- To measure and control the spread of the process.
- To provide more realistic tolerances for product dimensions.
- Improve the process performance capability

#### Factors influencing PC

- Condition of machine and equipment.
- Operation type and operational conditions.
- Type of Raw Materials.
- The Skill of Operators and Inspectors.
- Measurement method.
- Gauge/Instruments condition.

## Tools for PC estimation

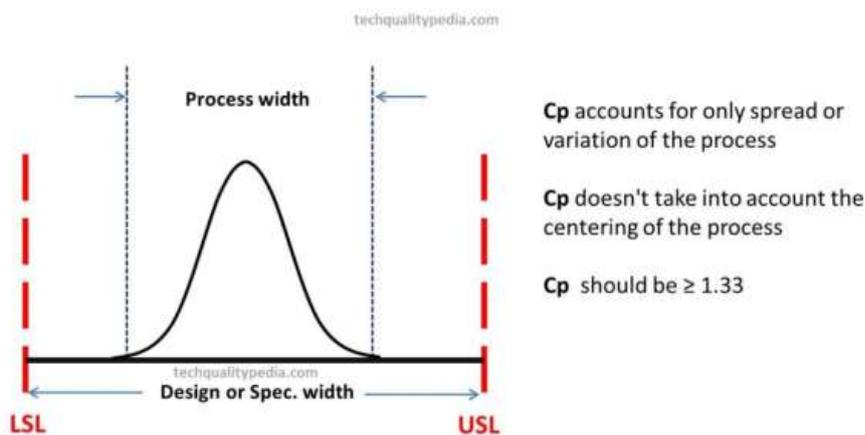
- Histogram.
- Control charts.
- Analysis of Variances.
- Run chart.

## Process Capability Indices: Cp and Cpk

### Cp Process Capability

**Process capability – Cp:** It is defined as the tolerance width divided by the total spread of the process.

$$Cp = \frac{\text{Design or Specification Width}}{\text{Process Width}} = \frac{USL - LSL}{6 \text{ Sigma}}$$



### Calculation of Process Capability (Cp) :

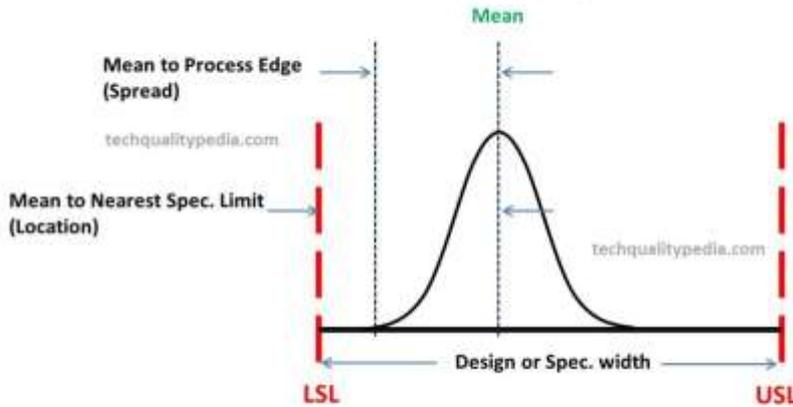
$$Cp = \frac{\text{Design Tolerance}}{6\sigma} = \frac{USL - LSL}{6\sigma}$$

USL = Upper Specification Limit, LSL = Lower Specification Limit

Cp indicates the spread of variation present in a process.

### Cpk Process Capability Index | Cpk in Quality

$$Cpk = \frac{\text{Distance from the Mean to Nearest Specification Limit}}{\text{Distance from Mean to Process Edge}}$$



Cpk accounts for both spread and location of process or centering of the process

Cpk should be  $\geq 1.33$

**Cpk Process Capability Index:** Cpk is the capability index that accounts for both the spread and location of the process or the centering of the process.

**Cpk in quality** is a measure of the process performance capability.

It measures how close your manufacturing process is to the center of the design specification limits.

### Cpk Formula | Calculation

Cpk in the manufacturing process should be greater or equal to 1.33, and it is defined as the minimum of Cpu and Cpl.

#### Calculation of Process Capability Index (Cpk) :

Cpk = Minimum of Cpk<sub>U</sub> and Cpk<sub>L</sub>

$$Cpk_U = \frac{USL - \bar{X}}{3\sigma} \quad \text{and} \quad Cpk_L = \frac{\bar{X} - LSL}{3\sigma}$$

### Cp and Cpk Difference

<i>Cp</i>	<i>Cpk</i>
<b>Cp</b> is the Process Capability and it is defined as the tolerance width divided by the process width(6Sigma).	<b>Cpk</b> is the Process Capability Index and measures how close your manufacturing process is to the center of the design specification limits.
<b>Cp</b> doesn't take into account the centering of the process. Cp accounts for only spread or variation of the process	<b>Cpk</b> take into accounts for both spread and centering of the process
Formula: <b>Cp</b> = Design Tolerance/6Sigma = (USL-LSL)/6Sigma	Formula: <b>Cpk</b> is the minimum value of Cpu or Cpl

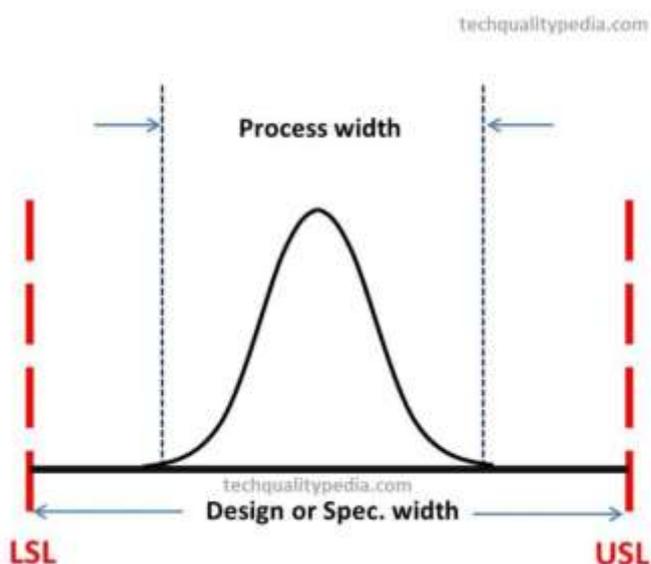
<p>Where  USL- Upper Specification Limit,  LSL-Lower Specification Limit  Sigma(<math>\sigma</math>) – Short-term Standard Deviation</p>	<p>denoted as <b>Cpk = Min (Cpu, Cpl)</b></p> <p>Cpu = (USL – X Double Bar)/(3Sigma)  Cpl = (X Double bar – LSL)/(3Sigma)</p> <p>Where  Cpu – Upper Capability Index,  Cpl – Lower Capability Index  Sigma(<math>\sigma</math>) – Short-term Standard Deviation</p>
<p><b>Cp</b> should not looked into isolation, it is the only used to find out whether the process is capable or not i.e. Cp <math>\geq</math>1.33</p>	<p><b>Cpk</b> should always be looked into, it is used to find out whether the process is capable to meet the customer requirements or not i.e. Cpk <math>\geq</math>1.33</p>

## Process Capability Index | Cp and Cpk

### Process Capability

**Process capability – Cp:** This is defined as the tolerance width divided by the total spread of the process (6 Sigma).

$$Cp = \frac{\text{Design or Specification Width}}{\text{Process Width}} = \frac{USL - LSL}{6 \text{ Sigma}}$$



**Cp** accounts for only spread or variation of the process

**Cp** doesn't take into account the centering of the process

**Cp** should be  $\geq$  1.33

Cp indicates the spread of variation present in a process.

Cp should not looked into isolation, it is only used to find out whether the process is capable or not i.e. Cp  $\geq$ 1.33

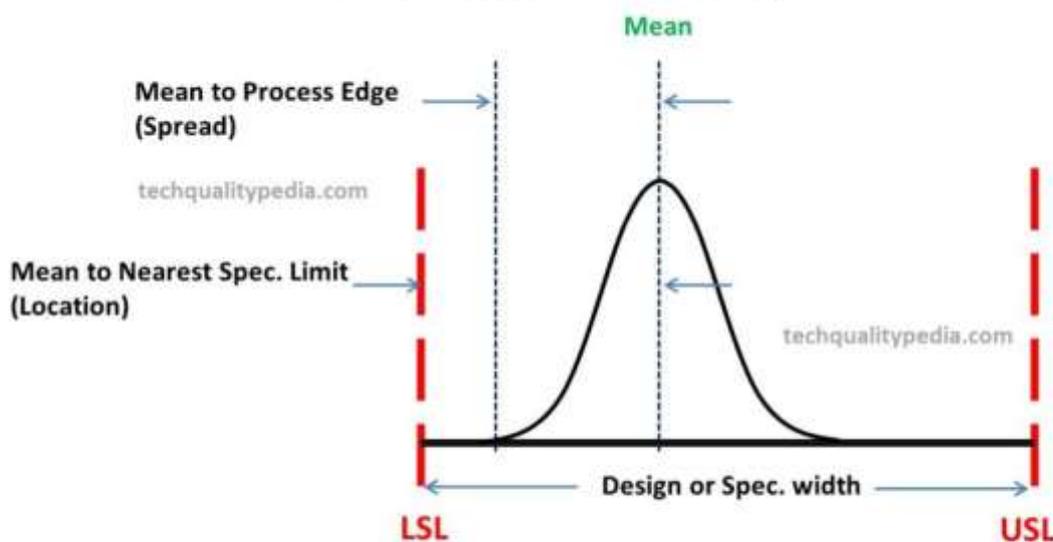
## Calculation of Process Capability (Cp) :

$$Cp = \frac{\text{Design Tolerance}}{6\sigma} = \frac{USL - LSL}{6\sigma}$$

**USL** = Upper Specification Limit, **LSL** = Lower Specification Limit

**Process Capability Index – Cpk:** This is the capability index that accounts for the centering of the process and is defined as the minimum of Cpk upper and Cpk lower.

$$Cpk = \frac{\text{Distance from the Mean to Nearest Specification Limit}}{\text{Distance from Mean to Process Edge}}$$



**Cpk** accounts for both spread and location of process or centering of the process

**Cpk** should be  $\geq 1.33$

Cpk is a measure of process performance capability.

Cpk indicates the shifting or closeness of the process average from the target or mean value.

Cpk should always be looked into, it is used to find out whether the process is capable of meeting the customer's requirements or not i.e.  $Cpk \geq 1.33$

**Cpk Process Capability Index**

### Calculation of Process Capability Index (Cpk) :

**Cpk** = Minimum of  $Cpk_U$  and  $Cpk_L$

$$Cpk_U = \frac{USL - \bar{X}}{3\sigma} \quad \text{and} \quad Cpk_L = \frac{\bar{X} - LSL}{3\sigma}$$

### Process Standard Deviation ( $\sigma$ ) :

$$\sigma = R/d_2$$

$d_2$  is the constant depending upon the subgroup size (n)

Subgroup Size (n)	$d_2$
2	1.128
3	1.693
4	2.059
5	2.326
6	2.534
7	2.704
8	2.847
9	2.970

### Example :

Specification :  $30 \pm 4.0$ , Standard Deviation( $\sigma$ ) = 1  
USL is 34 and LSL is 26

**When  $\bar{X} = 30$ ,  $Cpk = Cp = 1.33$**

$$Cp = \text{Design Tolerance} / 6\sigma = 8 / 6 * 1 = 1.33$$

$$Cpk = \text{Min of } \left[ \frac{34 - 30}{3 * 1}, \frac{30 - 26}{3 * 1} \right] = 1.33$$

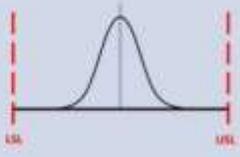
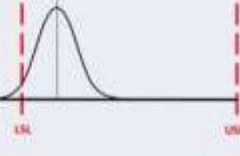
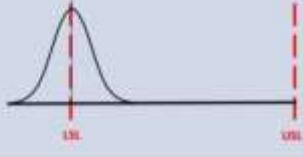
**When  $\bar{X} = 32$ ,  $Cpk = 0.67$**

$$Cp = \text{Design Tolerance} / 6\sigma = 8 / 6 * 1 = 1.33$$

$$Cpk = \text{Min of } \left[ \frac{34 - 32}{3 * 1}, \frac{32 - 26}{3 * 1} \right] = 0.67$$

Interpretations of Cp and Cpk

## Cp Vs Cpk Interpretation

<p><math>C_p = 1.33</math> <math>C_{pk} = 1.33</math></p>		<p>Process is capable of meeting the customer spec. requirement. Process is perfectly centered</p>
<p><math>C_p = 1.33</math> <math>C_{pk} = 0.97</math></p>		<p>Process is not capable of meeting the customer spec. requirement. Process is not centered, and produce defects. Re-adjustment reqd.</p>
<p><math>C_p = 1.33</math> <math>C_{pk} = 0</math></p>		<p>Process is not capable of meeting the customer spec. requirement. Process is not centered, shifted outside the spec. limit, and result in high defects. Re-adjustment reqd.</p>

**When the  $C_p$  value is greater than 1 i.e.  $C_p$  value  $>1$ :** The process spread is less and all products fall within the specification limit. Here the process is said to be *quite capable* of meeting the specification limit.

**When the  $C_p$  value is equal to 1 i.e.  $C_p$  value  $=1$ :** The process spread is a little wide but running within the designed specification limit. Here the process is said to be *just capable* of meeting the specification limit.

**When the  $C_p$  value is less than 1 i.e.  $C_p$  value  $<1$ :** The process spread is large and most of the products fall outside the specification limit. Here the process is said to be *incapable*.

**When the  $C_p$  value is equal to  $C_{pk}$  i.e.  $C_p=C_{pk}$ :** The Process means is said to be at centre.

**When the  $C_{pk}$  value is less than 1 i.e.  $C_{pk}$  value  $<1$ :** Indicates that the mean of the process is shifted from the target and defects will be produced.

**When the  $C_{pk}$  value is greater than 1 i.e.  $C_{pk}$  value  $>1$ :** The center or mean of the process may be shifted from the target but still the process is capable of meeting design specifications.

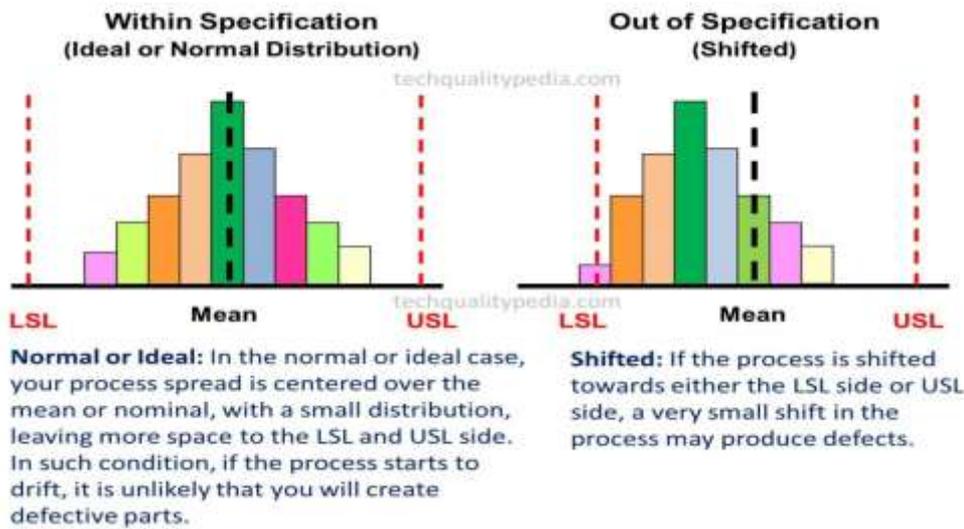
To achieve Six Sigma quality in the organization, we must reduce the variation in the process to achieve the desired value of  $C_p$ .

### Interpretations of Process Capability Histograms

The histograms show various frequency distributions or process spreads in the manufacturing process, and some common cases are described here along with the graph:

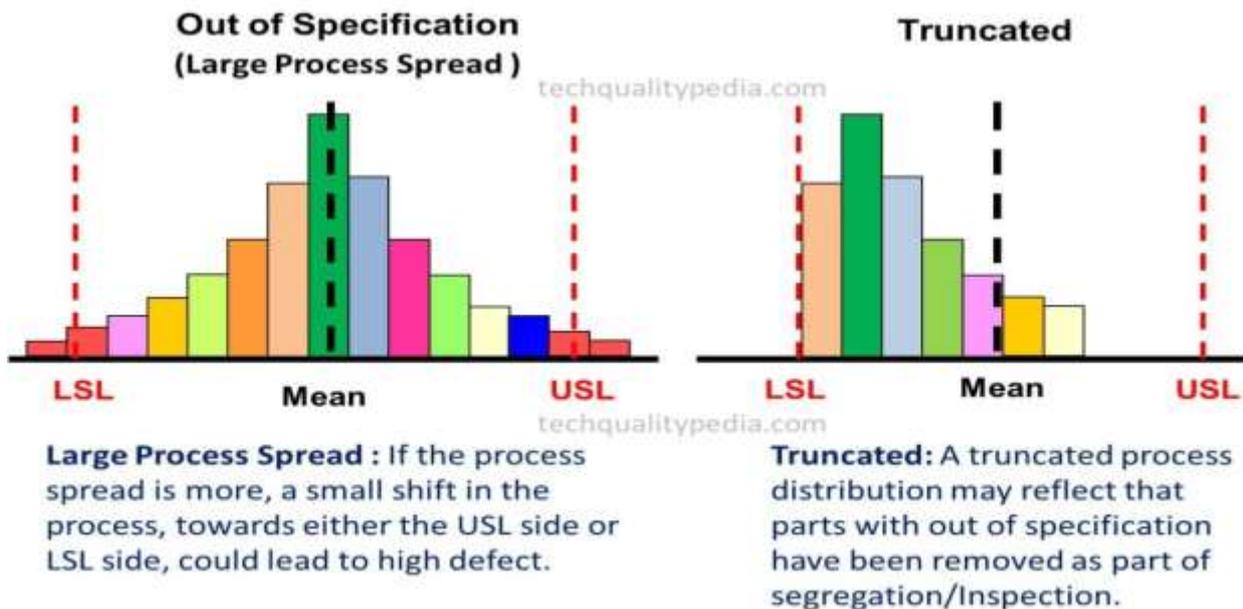
**Normal or Ideal:** In the normal or ideal case, your process spread is centered over the mean or nominal, with a small distribution, leaving more space to the LSL and USL side. In such conditions, if the process starts to drift, it is unlikely that you will create defective parts.

**Shifted:** If the process is shifted towards either the LSL or USL sides, a very small shift may produce defects.



**Large Process Spread:** If the process spread is more, a small shift in the process, towards either the USL side or LSL side, could lead to high defects.

**Truncated:** A truncated process distribution may reflect that parts with out of specification have been removed from the data set as part of segregation/inspection.



### Cp and Cpk Difference

	Process Capability Cp	Process Capability Index Cpk
<b>Process Mean and Variability</b>	Assumes the process mean is centered within the specification limits. It measures potential capability but does not reflect actual performance if the process mean is off-center.	Accounts for shifts in the process mean and provides a more realistic assessment of process capability, considering the actual position of the process mean within the specification limits.

<b>When to Use</b>	Best used for evaluating the potential capability of a process under ideal conditions, where the process mean is perfectly centered.	Provides a practical view of process performance, considering real-world variations and shifts. It is used to assess how well the process performs in its actual state.
<b>Identifying Issues</b>	If Cp is low, it indicates that the process may not have the potential to meet specifications, assuming perfect centering.	A low Cpk value indicates actual performance issues, including process mean shifts or variability.
<b>Implementing Improvements</b>	Focuses on reducing variability and ensuring the process is centered within specification limits.	Address both centering issues and overall variability to improve process performance.
<b>Implications for Quality Control</b>	A high Cp value indicates that the process has the potential to meet specifications if centered properly. However, it does not guarantee that the process will consistently meet specifications if the mean is not centered.	A high Cpk value reflects actual process performance and centering. A lower Cpk value compared to Cp may suggest that the process mean is not centered or that there are significant variations affecting process performance.

### Cp Value | Normal Production Run | New Product Development Stage

Cp value should be  $\geq 1.33$  during the normal production process.

The recommended value of **Process Performance Capability-Pp** is  $\geq 1.67$  during the new product development (NPD) stage.

### When to Use Cp and Pp

**Pp** is used when evaluating the real-world performance of a process, accounting for actual variations and shifts. It is a measure of performance considering all sources of variation. Whereas **Cp** is used when assessing the potential of a process that is expected to be stable and centered. It is a measure of capability under ideal conditions.

### Cp and Cpk Process Capability Analysis

PROCESS CAPABILITY ANALYSIS		
Cp	Cpk	Remarks
✓	✓	<ul style="list-style-type: none"> <li>• Process is Capable.</li> <li>• Continue process</li> <li>• Process running closure or near to target.</li> </ul>
✓	✗	<ul style="list-style-type: none"> <li>• Process is potentially capable i.e. Process spread due to variation is less but not centered.</li> <li>• Bring Cpk closure to target by local actions.</li> </ul>
✗	✗	<ul style="list-style-type: none"> <li>• Process is not capable.</li> <li>• Management action required for process improvement.</li> </ul>

