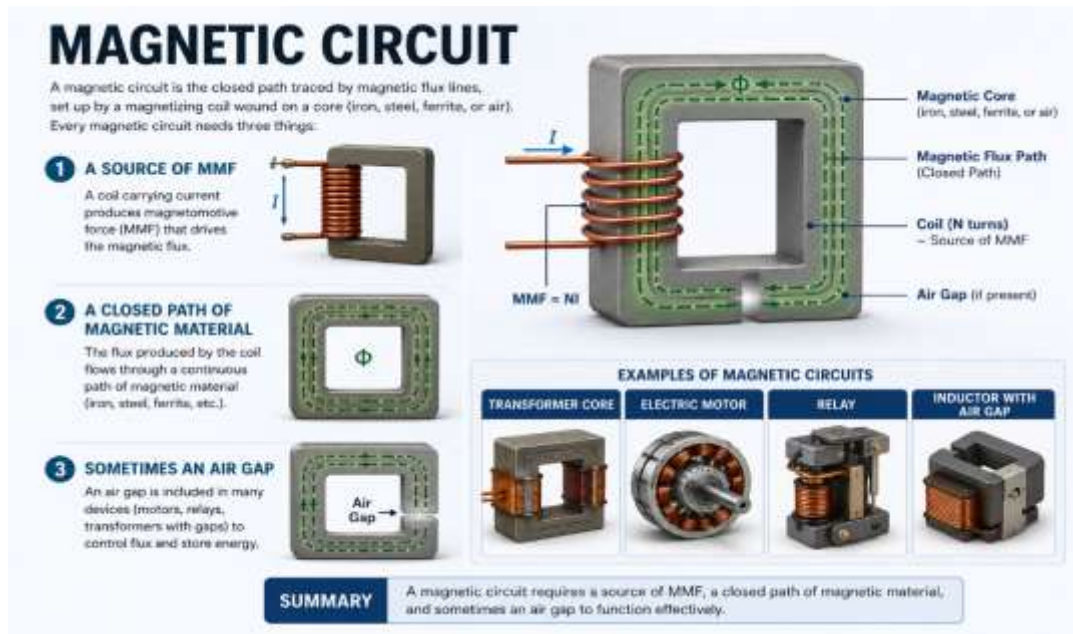


Basics of Magnetic Circuits

Introduction to Magnetic Circuits

A **magnetic circuit** is the closed path traced by magnetic flux lines, set up by a magnetizing coil wound on a core (iron, steel, ferrite, or air). Every magnetic circuit needs three things:

1. A source of MMF (a coil carrying current)
2. A closed path of magnetic material for flux to flow through
3. Sometimes an air gap (as in motors, relays, transformers with gaps)

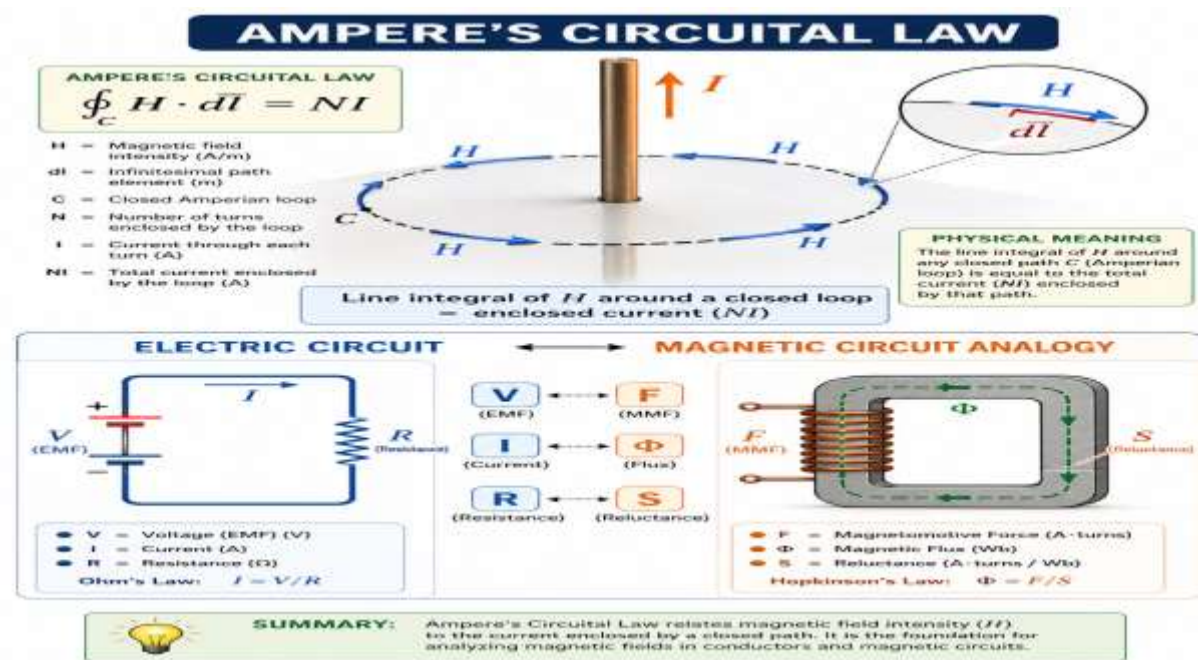


Governing law — Ampere's Circuital Law:

$$\oint \mathbf{H} \cdot d\mathbf{l} = NI$$

This states that the line integral of magnetic field intensity (H) around a closed path equals the total current (MMF) enclosed. It is the magnetic equivalent of Kirchhoff's voltage law, and it is where the

formula $F = NI$ actually comes from.



Electric–Magnetic Analogy (expanded)

Electric circuit	Magnetic circuit
EMF, V (volts)	MMF, F (ampere-turns)
Current, I (amperes)	Flux, Φ (weber)
Resistance, R (ohms)	Reluctance, S (AT/Wb)
Conductance, $1/R$	Permeance, $1/S$
Resistivity, ρ	Reluctivity, $1/\mu$
Conductivity, σ	Permeability, μ
Current density, $J = I/A$	Flux density, $B = \Phi/A$
Ohm's law: $V = IR$	$F = \Phi S$
KVL: $\sum V = 0$ in a loop	$\sum F = \sum \Phi S$ in a loop
KCL: $\sum I = 0$ at a node	$\sum \Phi = 0$ at a junction

Important difference to remember: Unlike current in a wire (where almost no energy is lost in an ideal conductor), maintaining flux in a magnetic circuit *does* require continuous energy input, because there is always some hysteresis and eddy current loss in real magnetic materials. The analogy is mathematical, not physical.

Magnetic Flux (Φ)

Definition: Total number of magnetic lines of force passing through a given cross-sectional area.

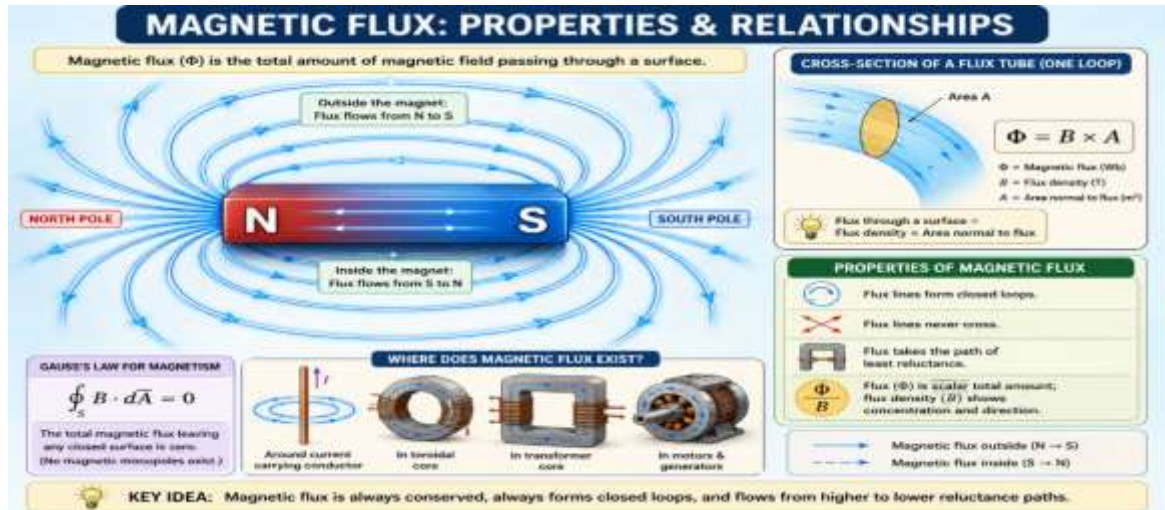
- **Symbol:** Φ **Unit:** Weber (Wb) = Volt-second (V·s)

Two ways to calculate flux:

1. From flux density: $\Phi = B \times A$
2. From the magnetic Ohm's law: $\Phi = F / S = NI / S$

Properties of magnetic flux

- Flux lines always form closed loops (they never start or end — this is Gauss's law for magnetism, $\oint \mathbf{B} \cdot d\mathbf{A} = 0$).
- Flux lines never cross each other.
- Flux takes the path of least reluctance (just as current takes the path of least resistance).
- Flux is a scalar quantity representing total "amount," while flux density is a vector-like quantity representing "concentration and direction."

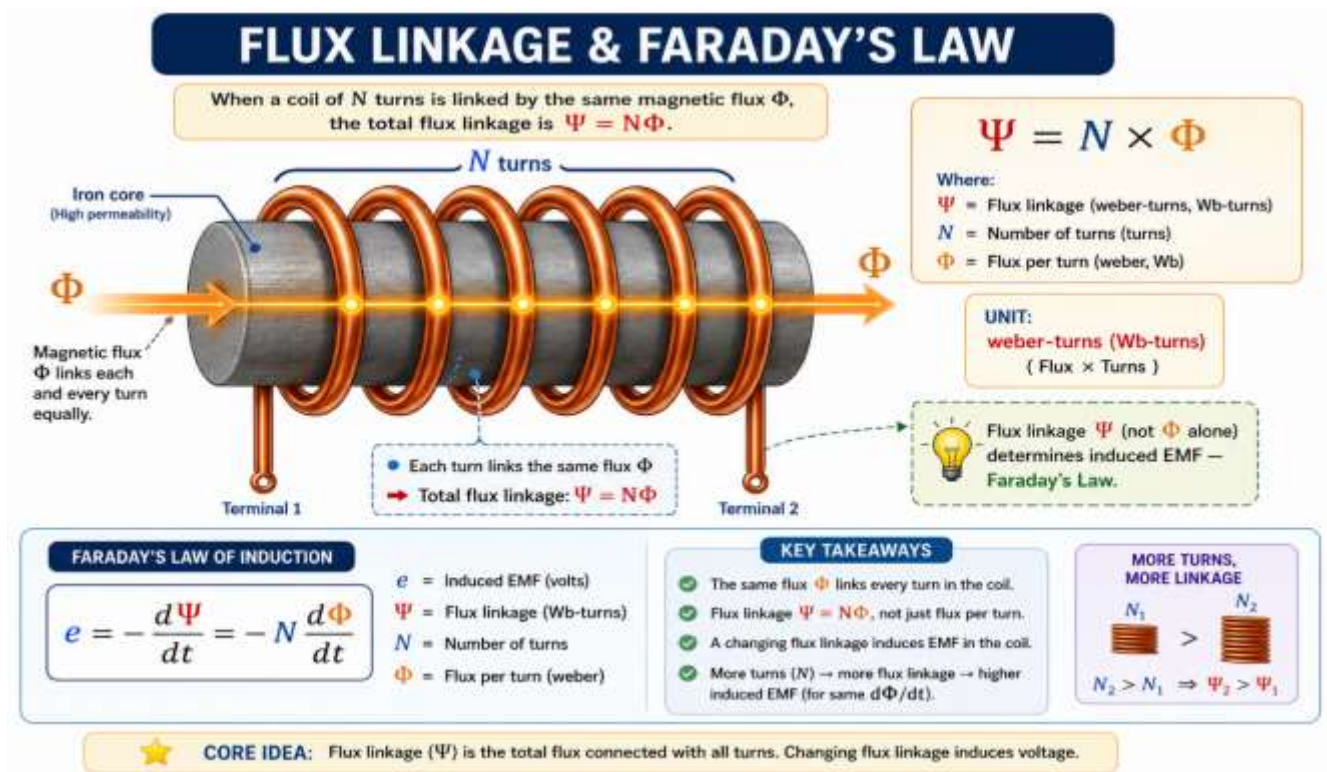


Flux linkage ($N\Phi$)

When a coil has N turns and each turn links the same flux Φ , the **total flux linkage** is:

$$\Psi = N \times \Phi \text{ (unit: weber-turns, Wb-turns)}$$

This quantity, not Φ alone, is what determines induced EMF (Faraday's law uses flux *linkage*, especially important once you study self and mutual inductance below).



Flux Density (B)

Definition: Flux per unit cross-sectional area, measured perpendicular to the direction of flux flow.

- **Symbol:** B **Unit:** Tesla (T) = Wb/m² $B = \Phi / A = \mu_0 \mu_r H$

The B-H Curve (magnetization curve)

For ferromagnetic materials (iron, steel), B does not increase linearly with H forever:

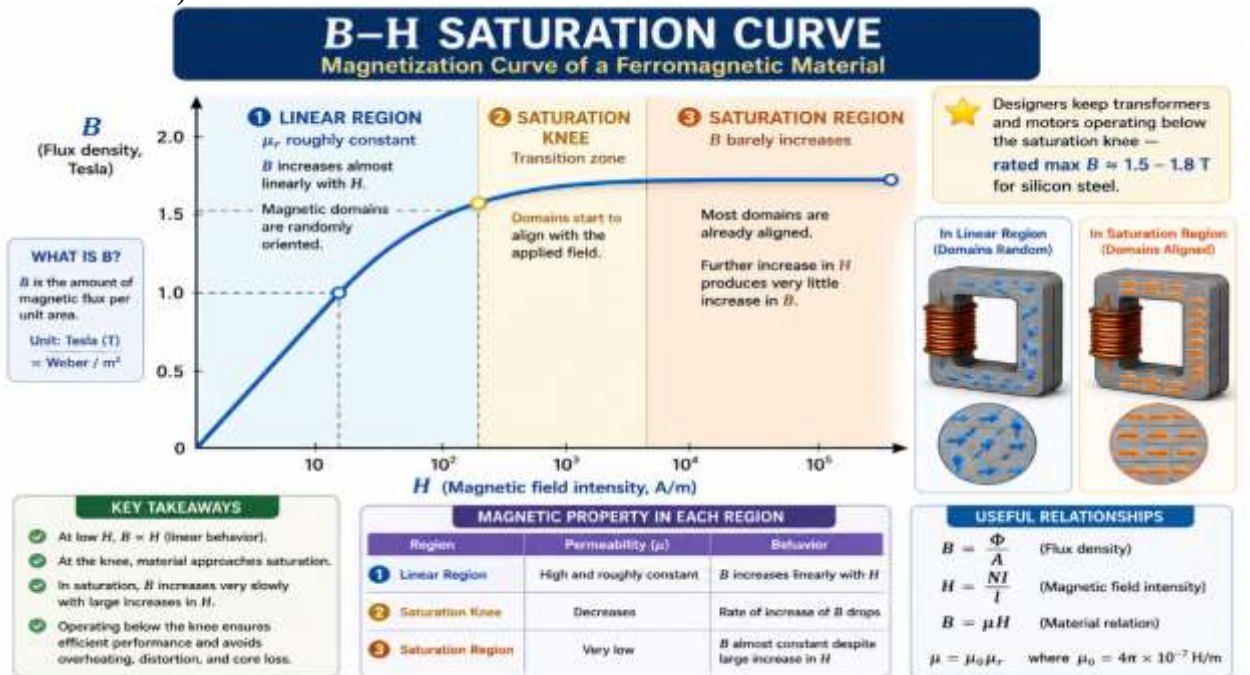
- At low H, B rises almost linearly (μ_r is roughly constant).
- At high H, the material **saturates** — B increases very little even for a large increase in H, because nearly all magnetic domains are already aligned.
- This is why μ_r is not truly constant — it changes with operating point, and is highest in the linear/unsaturated region.

Practical implication: Designers try to keep transformers and motors operating below the saturation "knee" of the B-H curve, otherwise a small increase in current gives almost no extra flux, and losses shoot up.

Why flux density matters more than flux alone

Two devices could have the same total flux Φ , but if one has a smaller cross-section, its B will be much higher — closer to saturation and with higher core losses. So B (not just Φ) is the parameter engineers actually design against, using the material's rated maximum flux density (e.g., ~1.5–1.8 T

for silicon steel).



Magneto motive Force (MMF) and Reluctance

(As clarified earlier: "reluctance" is the concept that belongs here in place of "magnetic resonance," which is unrelated to circuit theory.)

MMF

$$F = N I \text{ (ampere-turns, AT)}$$

MMF is what "pushes" flux around the magnetic circuit, exactly as EMF pushes current around an electric circuit.

Reluctance

$$S = l / (\mu_0 \mu_r A)$$

- Directly proportional to path length l
- Inversely proportional to cross-sectional area A
- Inversely proportional to permeability ($\mu_0 \mu_r$)

Series magnetic circuits

When flux flows through sections of different length, area, or material one after another (like a ring core with an air gap), total reluctance adds up just like series resistors:

$$S_{\text{total}} = S_1 + S_2 + S_3 + \dots$$

$$F = \Phi(S_1 + S_2 + S_3 + \dots) = \Phi S_1 + \Phi S_2 + \Phi S_3 + \dots$$

Each term ΦS_n is like a "magnetic voltage drop" across that section — this is why the method is often called the **MMF drop method**, directly parallel to voltage drops in a series electric circuit.

Air gaps in series circuits: Even a very short air gap contributes a huge reluctance because $\mu_r(\text{air}) = 1$, while $\mu_r(\text{iron})$ can be 1000–10,000. A 1 mm air gap can have reluctance comparable to a metre of iron path. This is why motors and transformers are designed to minimize unwanted air gaps.

Parallel magnetic circuits

When the flux path splits into two or more parallel branches (like a shell-type transformer core), permeances add:

$$1/S_{\text{total}} = 1/S_1 + 1/S_2 + \dots$$

This mirrors parallel resistors, and total flux from the source splits among branches inversely proportional to their reluctance (just as current splits inversely proportional to resistance).

Worked example — series magnetic circuit with air gap

An iron ring, mean length 0.5 m, cross-section $4 \times 10^{-4} \text{ m}^2$, $\mu_r = 1200$, has an air gap of 1 mm cut into it. Coil has 800 turns carrying 3 A. Find total reluctance and flux.

Solution:

Iron path length = $0.5 - 0.001 = 0.499 \text{ m}$ (air gap length subtracted from iron length)

Reluctance of iron: $S_{\text{iron}} = l_{\text{iron}} / (\mu_0 \mu_r A) = 0.499 / (4\pi \times 10^{-7} \times 1200 \times 4 \times 10^{-4}) \approx 8.27 \times 10^5 \text{ AT/Wb}$

Reluctance of air gap ($\mu_r = 1$ for air): $S_{\text{air}} = l_{\text{air}} / (\mu_0 A) = 0.001 / (4\pi \times 10^{-7} \times 4 \times 10^{-4}) \approx 1.99 \times 10^6 \text{ AT/Wb}$

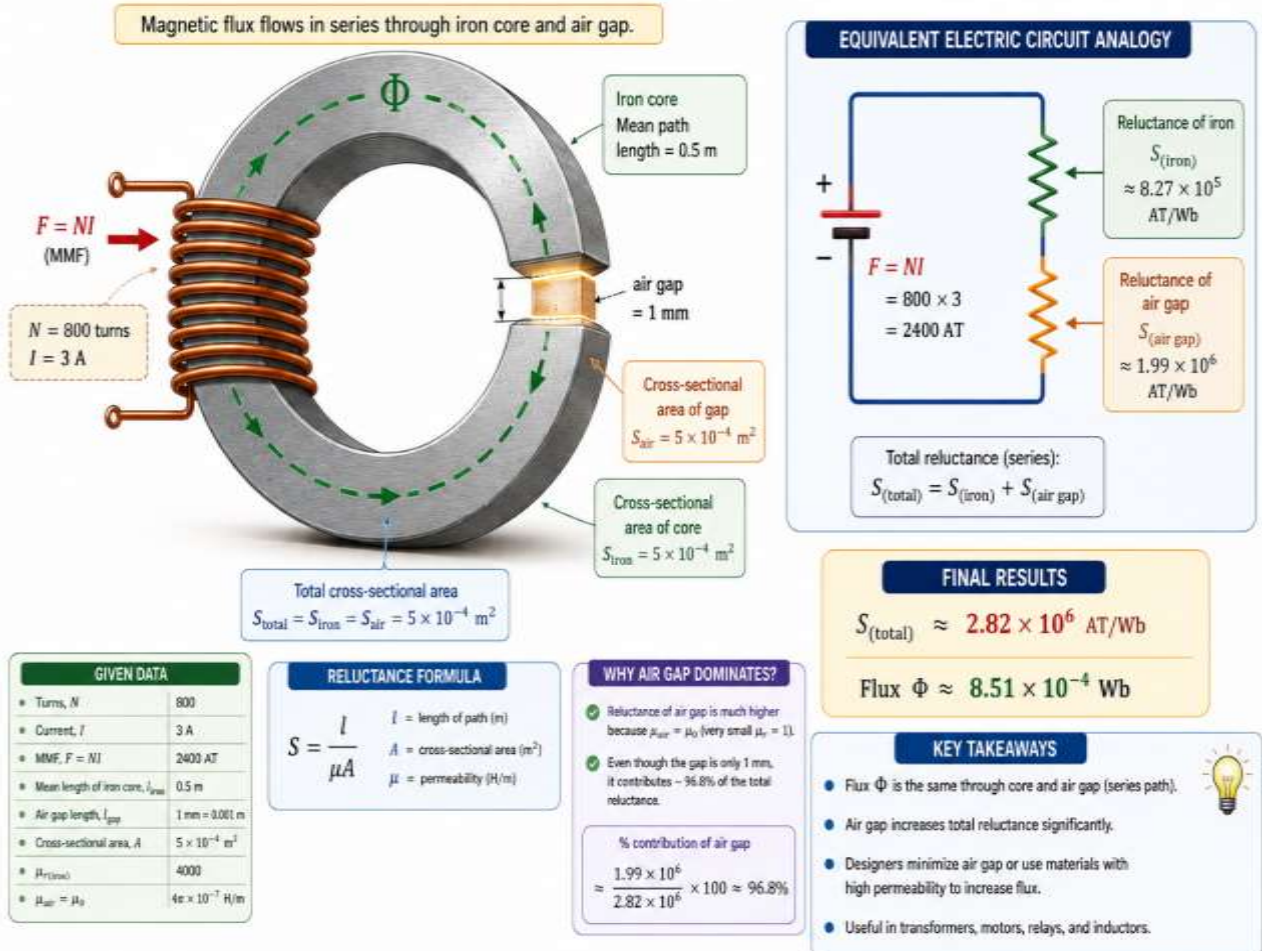
Total reluctance: $S_{\text{total}} = S_{\text{iron}} + S_{\text{air}} \approx 8.27 \times 10^5 + 1.99 \times 10^6 \approx 2.82 \times 10^6 \text{ AT/Wb}$

Notice: the 1 mm air gap contributes more than double the reluctance of the entire 0.499 m iron path — this shows how dominant air gaps are.

MMF: $F = NI = 800 \times 3 = 2400 \text{ AT}$

Flux: $\Phi = F / S_{\text{total}} = 2400 / (2.82 \times 10^6) \approx 8.51 \times 10^{-4} \text{ Wb}$

MAGNETIC CIRCUIT WITH AIR GAP - SERIES RELUCTANCE



Self-Inductance (L)

Definition: The property of a coil by which it opposes any change in its own current by inducing a self-EMF, according to Faraday's and Lenz's laws.

Derivation from Faraday's Law

Faraday's law: induced EMF = $-(\text{rate of change of flux linkage})$

$$e = -(d\Psi/dt) = -N(d\Phi/dt)$$

Since $\Phi \propto I$ in a linear magnetic circuit ($\Phi = NI/S$), we can write $\Psi = LI$, so:

$$e = -L(dI/dt)$$

Comparing the two expressions for e gives the defining relation:

$$L = \Psi/I = N\Phi/I$$

Formula from circuit geometry

Substituting $\Phi = NI/S$ into $L = N\Phi/I$:

$$L = N^2/S = (N^2 \mu_0 \mu_r A) / l$$

Key insight: L depends on N^2 , not N . Doubling the turns quadruples the inductance — this is one of the most tested relationships in exams.

Energy stored in an inductor

Energy is stored in the magnetic field itself, not "used up":

$$W = (1/2) L I^2 \text{ (joules)}$$

This is the magnetic analogy of kinetic energy ($\frac{1}{2}mv^2$) — L behaves like "magnetic mass" and I like "magnetic velocity."

Factors affecting self-inductance

1. Number of turns N ($\propto N^2$)
2. Cross-sectional area A of the core ($\propto A$)
3. Length of the magnetic path l ($\propto 1/l$)
4. Permeability of the core material μ_r ($\propto \mu_r$) — this is why inductors use iron/ferrite cores instead of air cores when high inductance is needed in a small size.

SELF-INDUCTANCE (L) OF A COIL

A changing current in a coil creates a changing magnetic field, which induces an EMF in the same coil that opposes the change.

I (increasing current)

induced EMF e — opposes the change (Lenz's law)

N turns

A

μ_r (core permeability)

l

KEY FORMULAS

Inductance of a long solenoid (air core or magnetic core):

$$L = \frac{N^2 \mu_0 \mu_r A}{l}$$

L = inductance (henry, H)
 N = number of turns (turns)
 μ_0 = permeability of free space = $4\pi \times 10^{-7}$ H/m
 μ_r = relative permeability of core (dimensionless)
 A = cross-sectional area of core (m^2)
 l = magnetic path length (m)

Energy stored in the magnetic field:

$$W = \frac{1}{2} L I^2 \text{ (joules)}$$

where I = current (A)

Geometry factors that determine inductance: number of turns N , cross-sectional area A , magnetic path length l , and permeability μ_r .

★ PHYSICAL MEANING: Inductance is the ability of a coil to store energy in its magnetic field and oppose changes in current.

💡 $L \propto N^2$ — doubling the turns quadruples the inductance.

PARAMETERS & THEIR EFFECTS	VOLTAGE-CURRENT RELATIONSHIP	TYPICAL CORE MATERIALS (μ_r)	EFFECT OF TURNS (N^2 RELATIONSHIP)
N (turns): $L \propto N^2$ More turns → much higher inductance. A (area): $L \propto A$ Larger core area → higher inductance. μ_r (permeability): $L \propto \mu_r$ Higher permeability core → higher inductance. l (length): $L \propto \frac{1}{l}$ Longer magnetic path → lower inductance.	Induced back-EMF (opposes change in current): $e = -L \frac{dI}{dt}$ e = induced EMF (V) L = inductance (H) dI/dt = rate of change of current (A/s) The negative sign [−] shows that e opposes the change in current (Lenz's law).	Air (no core) $\mu_r \approx 1$ Ferrite $\mu_r \approx 10^2 - 10^3$ Silicon steel $\mu_r \approx 10^3 - 10^4$ Permalloy $\mu_r \approx 10^5 - 10^6$ ★ Higher μ_r → higher L, but cores can saturate.	If all other factors stay the same: $N_2 = 2N_1$ $\downarrow$ $L_2 = 4L_1$ Number of Turns (N)

✅ KEY IDEA: A coil with inductance resists changes in current. The stronger the magnetic field (higher N , A , μ_r ; shorter l), the greater the inductance and the more energy it can store.

🔧 Used in inductors, chokes, transformers, motors, filters, and many power circuits.

Mutual Inductance (M)

Definition: The property by which a change in current in one coil induces an EMF in a neighboring coil, due to the common flux linking both.

Derivation

If coil 1 (N_1 turns, current I_1) produces flux Φ_1 , and a fraction of it (Φ_{12}) links coil 2 (N_2 turns):

$$M = N_2 \Phi_{12} / I_1$$

Induced EMF in coil 2:

$$e_2 = -M (dI_1/dt)$$

By reciprocity (a fundamental result from field theory), the mutual inductance is the same regardless of which coil is "driving":

$$M_{12} = M_{21} = M$$

Formula from circuit geometry

For two coils sharing the same magnetic core (same reluctance path):

$$M = (N_1 N_2) / S = (N_1 N_2 \mu_0 \mu_r A) / l$$

Dot convention (polarity of mutual EMF)

In circuit diagrams, coils are marked with a dot at one terminal of each winding. If both currents enter (or both leave) the dotted terminals, the mutually induced EMFs *add* to the self-induced EMFs; if one enters and the other leaves the dotted terminal, they *subtract*. This convention avoids having to redraw the actual winding direction every time — essential in transformer and coupled-circuit analysis.

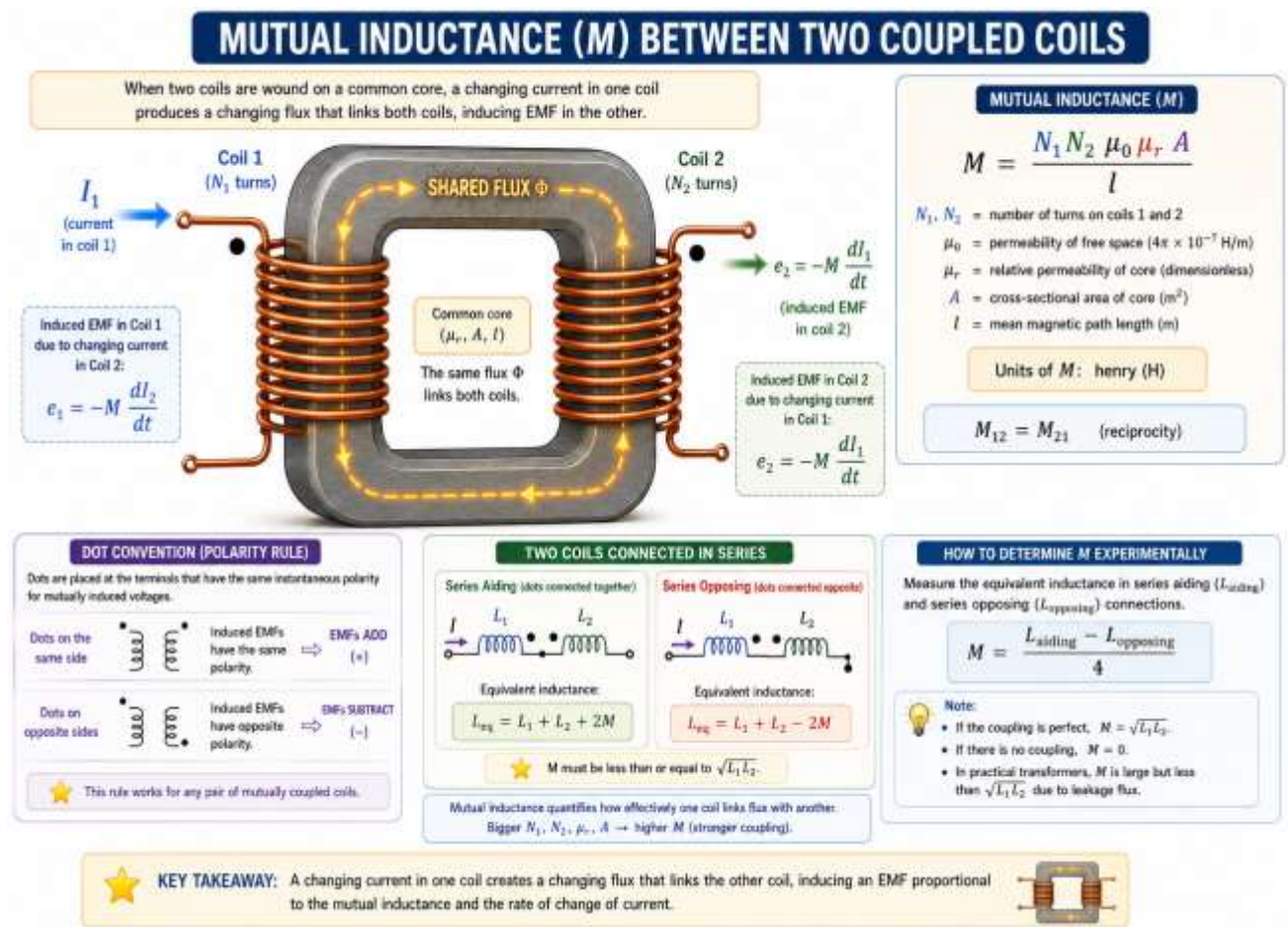
Series-connected coupled coils

Two coils L_1 and L_2 with mutual inductance M can be connected in series in two ways:

- **Series aiding** (fluxes add): $L_{\text{total}} = L_1 + L_2 + 2M$
- **Series opposing** (fluxes oppose): $L_{\text{total}} = L_1 + L_2 - 2M$

A quick way to find M experimentally: measure L_{total} in both configurations and use

$$M = (L_{\text{aiding}} - L_{\text{opposing}}) / 4$$



Coefficient of Coupling (k)

Definition: The fraction of flux produced by one coil that successfully links the other coil.

$$M = k \sqrt{L_1 L_2} \Rightarrow k = M / \sqrt{L_1 L_2}$$

Why k is never exactly 1 in practice

Some flux produced by coil 1 always "leaks" through the air or the gaps in the core instead of linking coil 2 — this is called **leakage flux**. Leakage flux:

- Does not contribute to energy transfer between coils
- Is represented in transformer models as a small "leakage inductance" in series with each winding
- Is minimized by winding both coils tightly on the same core, interleaving windings, and using high-permeability core material

Typical k values

Type of coupling	Typical k
Air-core coils, far apart	0.01 – 0.5

Type of coupling	Typical k
Air-core coils, close together	0.5 – 0.8
Iron-core transformer (well designed)	0.95 – 0.99
Ideal transformer (theoretical)	1.0

Why k matters in real devices

- **Power transformers** are designed for k as close to 1 as possible, to minimize leakage reactance and maximize efficiency of power transfer.
- **RF/tuned circuits** (radio tuning coils) deliberately use lower, adjustable k to control bandwidth and selectivity.
- **Wireless charging coils** operate with relatively low k (loosely coupled), which is why they are less efficient than a wired transformer connection and are sensitive to coil alignment/distance.

