Electric potential

The electric field intensity E is calculated from the electric scalar potential.

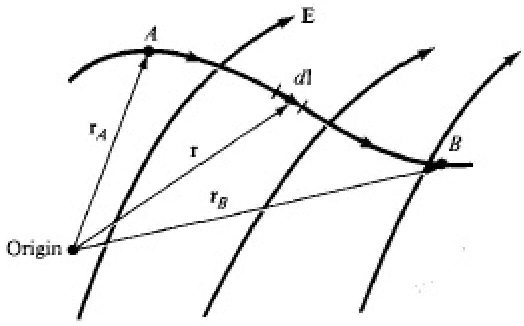

Fig 29. Displacement of point charge Q is an electrostatic field E

In fig 29 a point charge Q is move from A to point B in an electric field E. From Coulomb's law : The force on Q is$F = Q.E$ the work done to displacing the charge by dl is

$dw = -F\, dl = -QE\, dl$……… (1)

$'-' \rightarrow$ work is being done by an external agent.

The potential energy required to move the charge Q from A to B is

$W = -Q \int_A^B E\, dl$ ………… (2)

Work done by an external source for moving a charge a Q from A to B in an electric field E is potential difference. The potential difference between A & B is V_{AB}

$V_{AB} = \frac{W}{Q} = -\int_A^B E\, dl$……….(3)

[work done by a unit charge]

Note :

- In V_{AB} $A \rightarrow$ Initial point, B $\rightarrow$ final point
- V_{AB} is negative, there is a loss in potential energy, V_{AB} is positive, there is a gain in potential energy.
- V_{AB} is independent of path taken

➤ V_{AB} is measured in joules per coulomb (or) volts (v)

In Fig 3 the electric field E due to a point charge Q located at the origin.

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}.a_r \quad \ldots\ldots\ldots\ldots \quad (4)$$

Sub (4) in (3) r_B

$$(3) \Longrightarrow V_{AB} = -\int_{r_A}^{r_B} \frac{Q}{4\pi\varepsilon_0 r^2}.a_r.dr\ \ a_r$$

$$= \frac{-Q}{4\pi\varepsilon_0}\int_{r_A}^{r_B} r^{-2} a_r\, dr$$

$$= \frac{-Q}{4\pi\varepsilon_0}\left[\frac{r^{-1}}{-1}\right]_{r_A}^{r_B}$$

$$= \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r_B} - \frac{1}{r_A}\right] \quad \ldots\ldots\ldots\ldots \quad (5)$$

$$V_{AB} = V_B - V_A \quad \ldots\ldots\ldots\ldots \quad (6)$$

$V_B \& V_A \rightarrow$ Absolute potentials at B & A respectively

To choose infinity as reference ie

$V_A = 0 \qquad\qquad r_A \rightarrow \propto$

Sub $r_B \rightarrow r$ in eqn (5)

$$V_{AB} = V = \frac{Q}{4\pi\varepsilon_0 r} \quad \ldots\ldots\ldots\ldots \quad (7)$$

In eqn (5) & (6) E points in the radial direction. Any contribution from a displacement in the θ & φ direction is

$$E.dl = E\ Cos\ \alpha\ dl = E.dr \quad \ldots\ldots\ldots \quad (8)$$

$\alpha \rightarrow$ angle between E and dl

Conservative :

Vectors whose line integral does not depend on the path of integration are called conservative. (E is conservative)

The potential at any point is the potential difference between that point and chosen point (reference point) at which the potential is zero

$$V = -\int_{\propto}^{r} E.dl \quad \ldots\ldots\ldots\ldots \quad (9)$$

If the point charge Q is not located at the origin but at a point whose position vector is r'. the potential at $V_{(x,y,z)}(or)\ V(r)$ at r becomes

$$V(r) = \frac{Q}{4\pi\varepsilon_0|r-r'|} \quad \ldots\ldots\ldots \quad (10)$$

The electric field due to 'N' point charges located at points with position vectors $r_1, r_2 \ldots. r_n$. The potential at $'r'$ is obtained by superposition principle.

$$V(r) = \frac{Q_1}{4\pi\varepsilon_0|r-r_1|} + \frac{Q_2}{4\pi\varepsilon_0|r-r_2|} + \cdots + \frac{Q_n}{4\pi\varepsilon_0|r-r_n|}$$

$$V(r) = \frac{1}{4\pi\varepsilon_0}\sum_{k=1}^{n}\frac{Q_k}{|r-r_k|} \text{ (point charges)} \quad \ldots\ldots\ldots\ldots\ldots \ (11)$$

Continuous charge distributions

Line charge (potential at r)

$$Q = \rho_L\, dl$$

$$V(r) = \frac{1}{4\pi\varepsilon_0}\int_L \frac{\rho_L(r')}{|r-r'|}\, dl' \quad \ldots\ldots\ldots \quad (12)$$

Surface charge $Q = \rho_s\, ds$

$$V(r) = \frac{1}{4\pi\varepsilon_0}\int_S \frac{\rho_s(r')}{|r-r'|}\, ds' \ \ldots\ldots\ldots.. \ (13)$$

Volume charge :

$$Q = \rho_v\, dv$$

$$V(r) = \frac{1}{4\pi\varepsilon_0}\int_v \frac{\rho_v(r')}{|r-r'|}\, dv' \quad \ldots\ldots\ldots \quad (14)$$

From eqn (7) to (14). The reference point has been choosen at infinity. If any other point is choosen as reference, equation. (10) $\Longrightarrow$

$$V = \frac{Q}{4\pi\varepsilon_0 r} + C \text{ ....} \qquad (15)$$

$C \rightarrow$ Constant, ie determined at the chosen point of reference

2. The potential v can be determined in two ways :

- Charge distribution
- Electric field 'E'

If the charge distribution is know we use one of the eqn (10) to (15)

IF 'E' is known

$$V = -\int E.dl + C \qquad \text{.........} (16)$$

$$V_{AB} = V_B - V_A = -\int_A^B E.dl = \frac{W}{Q} \qquad \text{.........} (17)$$

Relationship between E and V. Maxwell's equation

The potential difference between points A and B is independent of the path taken. (From previous section)

$V_{BA} = -V_{AB}$

ie $V_{AB} + V_{BA} = \oint E.dl = 0$

$$\oint_L E.dl = 0 \text{.........} \qquad (1)$$

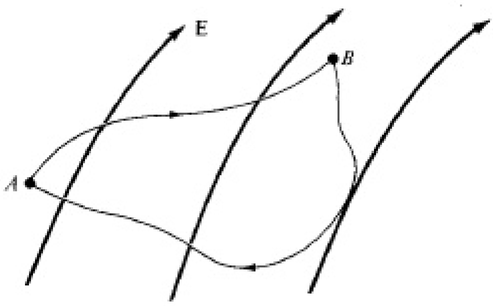

Fig30 : conservative nature of an electrostatic field

ie, the line integral of 'E' along a closed path is zero. No net work is done in moving a charge along a closed path in an electrostatic field.

Stokes theorem

$$\oint E\ dl = \oint_s\ (\nabla \text{x}\ E)\ ds = 0$$

$$\nabla \text{x}\ E = 0 \ \ldots\ldots\ldots\ (2)$$

Any vector field satisfied eqn (1) or (2) is said to be conservative or irrotational.

(1) → Maxwell's second equation in integral form

(2) → Maxwell's second equation in point form (or) Differential form

The potential $V = -\int E\ dl$

$$dV = -E.dl = -E_x\ dx - E_y\ dy - E_z\ dz\ \ldots\ldots\ldots \qquad (3)$$

A total charge in $V(x, y, z)$ is the sum of partial charges with respect to x, y, z variables

$$dV = \frac{\partial v}{\partial x}\ dx + \frac{\partial v}{\partial y}\ dy + \frac{\partial v}{\partial z}\ dz\ \ldots\ldots\ldots\ldots\ (4)$$

Compare (3) & (4)

$$E_x = -\frac{\partial v}{\partial x} \qquad E_y = -\frac{\partial v}{\partial y} \qquad E_z = -\frac{\partial v}{\partial z}$$

Thus $E = -\nabla V$

ie, Electric field intensity is the gradient of v. $'-'$ sign indicates the direction of E is opposite to the direction in which 'v' increases.

$$\nabla \text{x} \nabla \text{V} = 0$$