CONDUCTORS IN STATIC ELECTRIC FIELD

Introduction:

Electric field can exist in free space and also in material media. Based on electrical properties material can be classified as conductors and nonconductors. Nonconducting materials are usually known as insulators or dielectrics. Conduction ,electric current and polarization are the electrical properties of Conductors Susceptibility, permittivity, linearity, isotropy,homogeneity,dielectric strength and relaxation time are the properties of dielectric.

Properties of Materials:

Materials are classified in terms of their conductivity σ, in mhos per meter (℧/m) Or Siemens per meter (S/m), conductors and nonconductors or technically as metals and insulators (dielectrics) .The conductivity of a material depends on temperature and frequency. A material with high conductivity is referred to as conductors (metals, $\sigma \gg 1$) where as one with low conductivity is referred to as in insulators (dielectric, $\sigma \ll 1$).A material whose conductivity is lies between metals and insulator is called semiconductor .Example germanium and silicon . Examples of metal copper and aluminium. Example of insulator rubber and glass.

The conductivity of metal increases with decrease in temperature . At temperature near absolute zero ($T= 0^o$ K) some conductors exhibit infinite conductivity called superconductors example lead and aluminium . The conductivity of lead at 4^o K is of the order of 10^{20} mhos/m.

In metal large number of electrons are available for conduction current. Dielectric have few electrons for conduction current.

Conductors:

A conductor has abundance of charge that is free to move. In an isolated conductor when an external electric field E_e is applied ,the positive free charges are pushed along the same direction as the applied field, while the negative charge move in opposite direction. Free charge accumulate on the surface of the conductor and form induced surface charge .The induced surface charges setup an internal induced field E_i , which cancels the external field E_e.

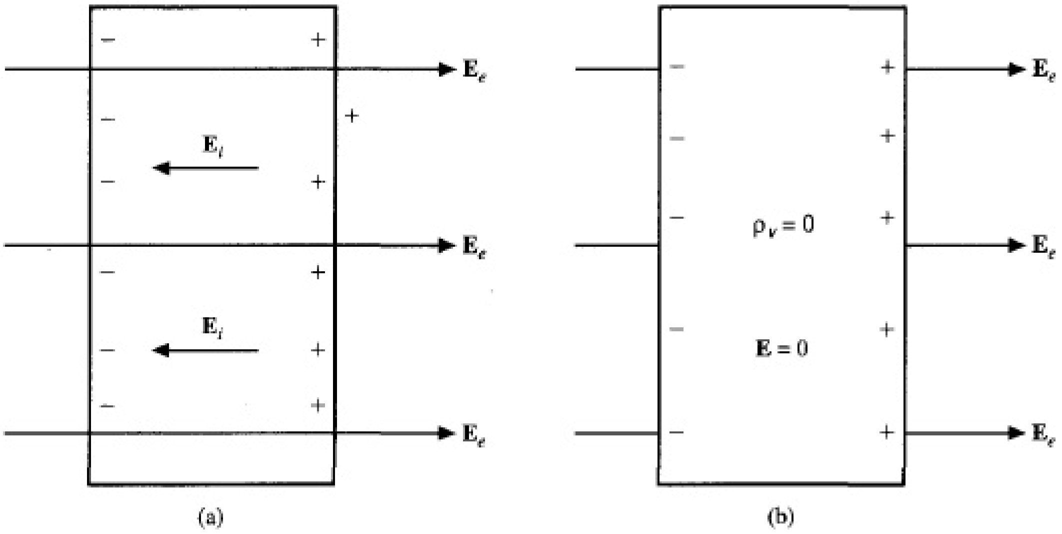
A perfect conductor cannot contain an electrostatic field within it. A conductor is called equipotential body, that is the potential is same everywhere in the conductor. $E = -\nabla V = 0.$

Consider ohm's law $J = \sigma E$. J → current density. In a perfect conductor $\sigma \to \infty$, the electric field inside the conductor must vanish .In other words , E →0 because $\sigma \to \infty$ in a perfect conductor. If some charges are introduced in the interior of a conductor , the charges will move to the surface and redistribute themselves quickly in a manner that the field inside the conductor vanishes. According to Gauss's law , if E = 0, the charge density ρ_v must be zero. A perfect conductor cannot contain an electrostatic field within it .

Under Static Conditions

E=0 , $\rho_v = 0$, $V_{ab} = 0$ inside a conductor

Consider a conductor whose ends are maintained at a potential difference V. In this case $E \neq 0$ inside the conductor ,There is no static equilibrium in the conductor since the conductor is not isolated but wired to source of electromotive force, which force the free charges to move and prevents the eventual establishment of electrostatic equilibrium. An electric field inside the conductor cause the flow of current , the flow of electrons encounter a damping force called resistance . If a conductor has uniform cross section S and of length l . The direction of the electric field E produced is same as the direction of the flow of positive charges or current I. This direction is opposite to the direction of flow of electrons.



(a) (b)

Fig1:(a) An isolated conductor under the influence of an applied field. (b) a conductor has zero electric field under static conditions.

The electric field applied is uniform and is given by

$$E = \frac{V}{l} \qquad \text{--- (1)}$$

Current density $J = \frac{I}{S}$ ---(2)

Substitute equation (1) and (2) in ohm's law

$$J = \sigma E = \frac{I}{S} = \sigma \frac{V}{l} \qquad \text{---(3)}$$

Hence $R = \frac{V}{I} = \frac{l}{\sigma S}$ --(4)

$\rho_c = \frac{1}{\sigma}$ is the resistivity of the material

$$R = \frac{\rho_c l}{S} \qquad \text{---(5)}$$

Equation (5) is used to find the resistance of any conductor of uniform cross section.

The resistance of a conductor of nonuniform cross section is

$$R = \frac{V}{I} = \frac{\int E \, .dl}{\int \sigma E \, .ds} \qquad \text{---(6)}$$

$V = - \int E.dl$ represent the drop in above equation because

$\int E.dl < 0 \; if \; I > 0$

Power in watts is defined as the rate of change of energy W (in joules) or force times velocity.

$$\int \rho_V dv \, E \, .u = \int E \, .\rho_V \, u \, dv$$

$$P = \int E.J \, dv \qquad \text{--(7)}$$

Equation (7) is also known as Joule's law .

The power density(watts/m^3) is given by

$$w_p = \frac{dp}{dv} = E.J = \sigma |E|^2 \qquad \text{---(8)}$$

For conductor with uniform cross section dv= ds dl

Eqn (7) becomes $P = \int_L E\,dl \int_S J\,ds = VI$

$P = I^2 R$ Common form of joules law in electric circuit theory.

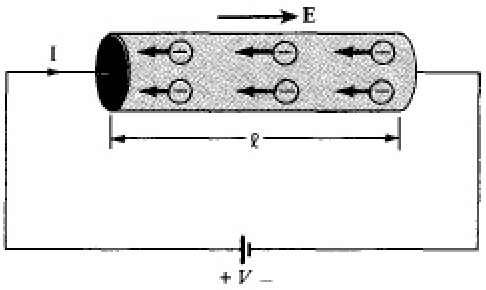


Fig 2: A conductor of uniform cross section under an applied E field.