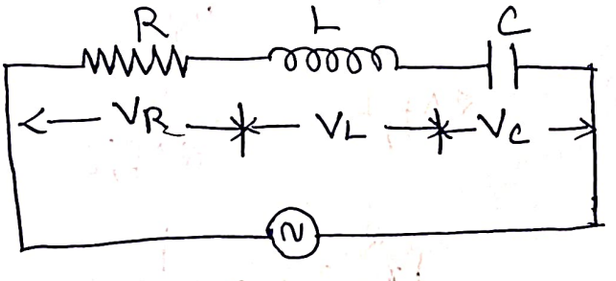
Resonance :-

The state where the inductive reactance X_L equals the capacitive reactance X_C, causing the impedance of the circuit to be minimum (equal to the reactance R) and the current to be at its maximum. This specific frequency called the resonant frequency makes the circuit behave as a purely resistive ckt.

Resonance in series RLC circuit :-



Apply KVL to the above RLC circuit

$$V = V_R + V_L + V_C \quad \text{——} \quad (1)$$

where V_R = voltage drop across Resistor

V_L = vtg drop across Inductor

V_C = vtg drop across capacitor.

$$V_R = IR \;,\; V_L = jIX_L \;,\; V_C = -jIX_C \quad \text{——} \quad (2)$$

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Sub ② in ① ⇒

$$V = IR + jIX_L - jIX_C$$

$$= I(R + jX_L - jX_C)$$

$$\frac{V}{I} = R + j(X_L - X_C)$$

$$|Z| = \sqrt{R^2 + (X_L - X_C)^2}$$

At resonant frequency $X_L - X_C = 0$

$$\therefore X_L = X_C$$

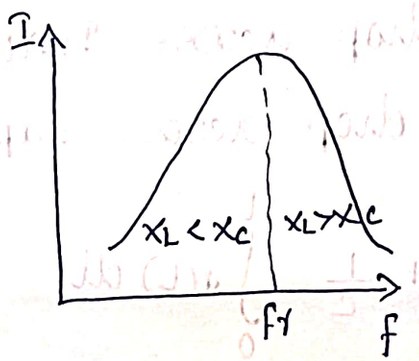
$$\omega_r L = \frac{1}{\omega_r C}$$

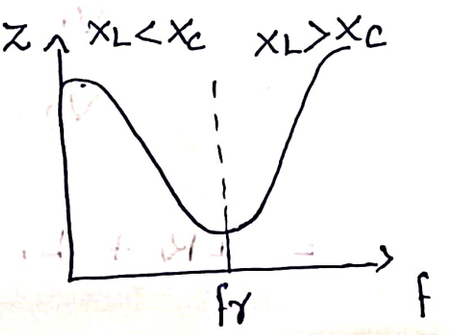
$$2\pi f_r L = \frac{1}{2\pi f_r C}$$

$$f_r^2 = \frac{1}{2\pi L \, 2\pi C}$$

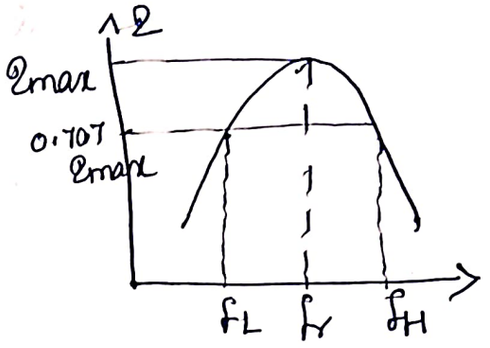
$$\boxed{f_r = \frac{1}{2\pi\sqrt{LC}}}$$

The current, impedance curve of RLC circuit are





At Half power frequency f_H and f_L



$$|Z| = \sqrt{2}\,R$$

$$\sqrt{R^2+(X_L-X_C)^2} = \sqrt{2}\,R$$

squaring on both sides,

$$R^2+(X_L-X_C)^2 = 2R^2$$

$$(X_L-X_C)^2 = 2R^2-R^2$$

$$(X_L-X_C)^2 = R^2$$

when $X_C > X_L$, the frequency is f_L

$$\therefore (X_C-X_L)^2 = R^2$$

$$X_C - X_L = R$$

$$\frac{1}{\omega_L C} - \omega_L L = R$$

$$\frac{1-\omega_L^2 LC}{\omega_L C} = R$$

$$1-\omega_L^2 LC = \omega_L RC$$

$$\Rightarrow \quad \omega^2 LC + \omega RC - 1 = 0$$

$$\omega_{L} = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$= \frac{-RC \pm \sqrt{R^2C^2+4LC}}{2LC}$$

$a = LC$
$b = RC$
$c = -1$

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$$\frac{-RC}{2LC} \pm \sqrt{\frac{(RC)^2 + 4LC}{4L^2C^2}}$$

$$= -\frac{R}{2L} + \sqrt{\frac{R^2C^2}{4L^2C^2} + \frac{4LC}{4L^2C^2}}$$

$$= \frac{-R}{2L} + \sqrt{\frac{R^2}{4L^2} + \frac{1}{LC}} = \frac{-R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$\therefore \frac{R}{2L} \ll \frac{1}{LC}.$$

$$\omega_L = \frac{-R}{2L} + \sqrt{\frac{1}{LC}}$$

$$2\pi f_L = \frac{-R}{2L} + \sqrt{\frac{1}{LC}}$$

$$f_L = \frac{1}{2\pi}\left[\frac{-R}{2L} + \sqrt{\frac{1}{LC}}\right]$$

$$f_L = \frac{-R}{4\pi L} + \frac{1}{2\pi}\sqrt{\frac{1}{LC}}$$

$$\boxed{f_L = \frac{-R}{4\pi L} + f_r}$$

At frequency f_H, $X_L > X_C$

$$X_L - X_C = R$$

$$\omega L - \frac{1}{\omega C} = R$$

$$\frac{\omega^2 LC - 1}{\omega C} = R$$

$$\omega^2 LC - 1 = \omega RC$$

$$\therefore \omega^2 LC - \omega RC - 1 = 0$$

where,
$a = LC$,
$b = -RC$
$c = -1$

$$\omega_H = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{RC \pm \sqrt{(RC)^2 + 4LC}}{2LC}$$

$$= \frac{RC}{2LC} \pm \sqrt{\frac{R^2C^2 + 4LC}{4L^2C^2}}$$

$$= \frac{R}{2L} \pm \sqrt{\frac{R^2C^2}{4L^2C^2} + \frac{4LC}{4L^2C^2}}$$

$$= \frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$\because \frac{R}{2L} \ll \frac{1}{LC}$

$$\omega_H = \frac{R}{2L} \pm \sqrt{\left(\frac{1}{LC}\right)}$$

$$\omega_H = \frac{R}{2L} + \frac{1}{\sqrt{LC}}$$

$$2\pi f_H = \frac{R}{2L} + \frac{1}{\sqrt{LC}}$$

$$f_H = \frac{R}{2\pi 2L} + \frac{1}{2\pi\sqrt{LC}}$$

$$f_H = \frac{R}{4\pi L} + \frac{1}{2\pi\sqrt{LC}} = \frac{R}{4\pi L} + f_r$$

$$BW = f_H - f_L$$

$$= \frac{R}{4\pi L} + f_r - \left[-\frac{R}{4\pi L} + f_r\right]$$

$$= \frac{2R}{2 \cdot 4\pi L}$$

$$BW = \frac{R}{2\pi L}$$

Quality Factor.

$$Q = \frac{\text{Resonant Frequency}}{\text{Bandwidth}}$$

$$= \frac{f_r}{f_H - f_L} = \frac{\frac{1}{2\pi\sqrt{LC}}}{\frac{R}{2\pi L}}$$

$$= \frac{1}{2\pi\sqrt{LC}} \propto \frac{2\pi L}{R}$$

$$Q.F = \frac{1}{R}\sqrt{\frac{L}{C}}$$