3.5 MINIMUM SPANNING TREE - KRUSKAL'S ALGORITHM

A second greedy strategy is repeatedly to select the edges in order of smallest weight and accept an edge if it does not cause a cycle.

Steps:

1. Initially, there are |V| single-node trees. Adding an edge merges two trees into one.
2. When the algorithm terminates, there is only one tree, and this is the minimum spanning tree.
3. The algorithm terminates when enough edges are accepted.

The strategy

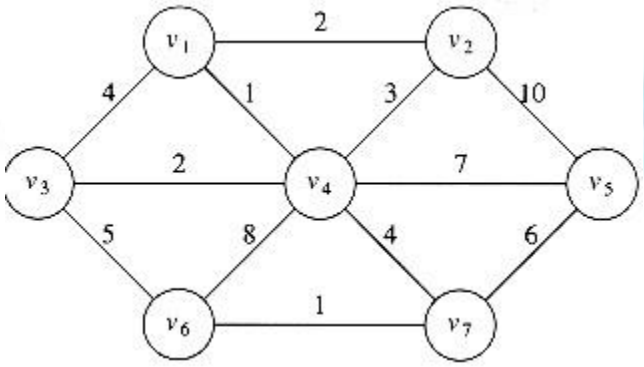
i. The edges are built into a minheap structure and each vertex is considered as a sigle node tree.

ii. The deletemin operation is used to find the minimum cost edge (u,v).

iii. The vertices u and v are searched in the spanning tree set S and if the returned sets are not same then (u,v) is added to the set s with the constraint that adding (u,v) will not create a cycle in spanning tree set S.

iv. Repeat step (ii) and (iii) until a spanning tree is constructed with |V| - 1 edges.

Example



i. Initially all the vertices are single node trees.

ii. Select the smallest edge v1 to v4, both the nodes are different sets, it does not form cycle.

iii. Select the next smallest edge v6 to v7. These two vertices are different sets; it does not form a cycle, so it is included in the MST.

iv. Select the next smallest edge v1 to v2. These two vertices are different sets; it does not form a cycle, so it is included in the MST.

v. Select the next smallest edge v3 to v4. These two vertices are different sets; it does not form a cycle, so it is included in the MST.

vi. Select the next smallest edge v2 to v4 both v2 and v4 are same set, it forms cycle so v2 – v4 edge is rejected.

vii. Select the next smallest edge v1 to v3, it forms cycle so v1 – v3 edge is rejected.

viii. Select the next smallest edge v4 to v7, it does not form a cycle so it is included in the tree.

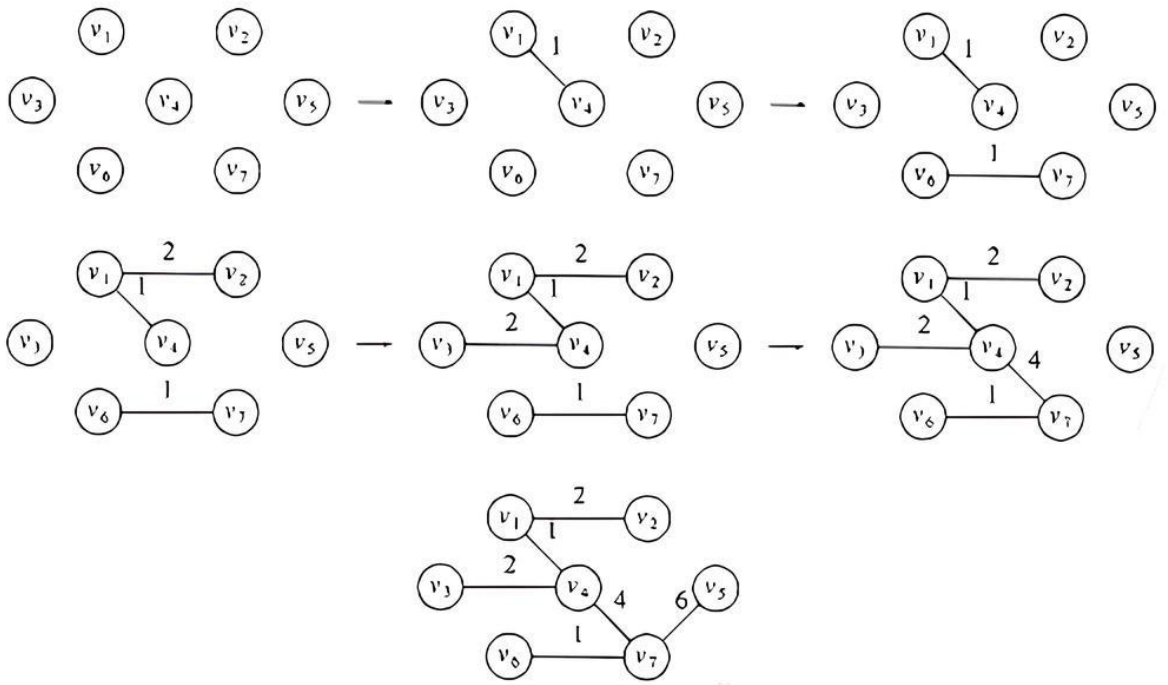
ix. Select the next smallest edge v3 to v6, it forms a cycle so v3 – v6 edge is rejected.

x. Select the next smallest edge v5 to v7, it does not form a cycle so it is included in the tree.

Edge	Weight	Action
(v1,v4)	1	Accepted
(v6,v7)	1	Accepted
(v1,v2)	2	Accepted
(v3,v4)	2	Accepted
(v2,v4)	3	Rejected
(v1,v3)	4	Rejected
(v4,v7)	4	Accepted
(v3,v6)	5	Rejected
(v5,v7)	6	Accepted
(*v*3,*v*6)	5	Rejected
(*v*5,*v*7)	6	Accepted

Figure: Action of Kruskal's algorithm on G

All the nodes are included. The cost of minimum spanning tree = 16 (2 + 1+ 2 + 4 + 1 + 6).



Routine for kruskals algorithm

```
void kruskal( graph G )
{
    int EdgesAccepted;
    DisjSet S;
    PriorityQueue H;
    vertex u, v;
    SetType uset, vset;
    Edge e;
    Initialize( S ); // form a single node tree
    ReadGraphIntoHeapArray( G, H );
    BuildHeap( H );
    EdgesAccepted = 0;
    while( EdgesAccepted < NumVertex-1 )
    {
        e = DeleteMin( H ); // Selection of minimum edge
```

```
        uset = Find( u, S );
        vset = Find( v, S );
        if( uset != vset )
        {
          /* accept the edge */
          EdgesAccepted++;
          SetUnion( S, uset, vset);
        }
        }
}
```

- The appropriate data structure is the union/find algorithm
- The worst-case running time of this algorithm is O(|E| log |E|), which is dominated by the heap operations. Notice that since |E| = O(|V|2), this running time is actually O(|E| log |V|).