## INTRODUCTION TO DSP

# Discrete Signals and Systems

**Discrete signals :** These signals take only finite amplitude levels and they are present at discrete time intervals. The signal which is discrete in time and amplitude is called *digital signal*.

**Examples :** PCM signal, digital image signal, digital speech etc.

**Discrete systems :** These systems process discrete or digital signals. These systems are made up of digital flip - flops, shift registers, counters, ALU, shifters etc.

## CLASSIFICATION OF SIGNALS

**Discrete time signal:**

A signal that is defined for discrete instants of time is known as discrete time signal. Discrete time signals are continuous in amplitude and discrete in time. It is also obtained by sampling a continuous time signal. It is denoted by x(n) and shown in Figure 1.2.2

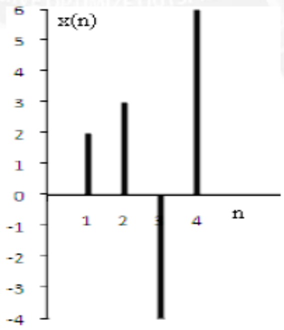



**Figure 1.2.2 Discrete time signal**

## CONTINUOUS TIME AND DISCRETE TIME SIGNAL

### Continuous time signal:

A signal that is defined for every instants of time is known as continuous time signal. Continuous time signals are continuous in amplitude and continuous in time. It is denoted by x(t) and shown in Figure 1.2.1

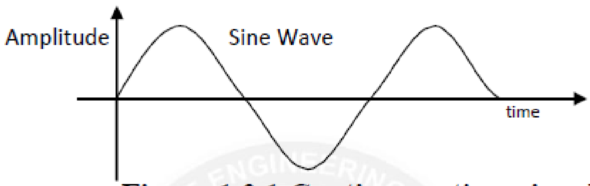



**Figure 1.2.1 Continuous time signal**

## EVEN (SYMMETRIC) AND ODD (ANTI-SYMMETRIC) SIGNAL

### Continuous domain:

**Even signal:**

A signal that exhibits symmetry with respect to t=0 is called even signal Even signal satisfies the condition $x(t) = x(-t)$

**Odd signal:**

A signal that exhibits anti-symmetry with respect to t=0 is called odd signal Odd signal satisfies the condition $x(t) = -x(-t)$

Even part $\boldsymbol{x_e(t)}$ and Odd part $\boldsymbol{x_0(t)}$ of continuous time signal $\boldsymbol{x(t)}$:

Even component of $x(t)$ *is* $x_e(t) = \frac{1}{2}[x(t) + x(-t)]$

Odd component of $x(t)$ *is* $x_o(t) = \frac{1}{2}[x(t) - x(-t)]$

**Discrete domain:**

**Even signal:**

A signal that exhibits symmetry with respect to n=0 is called even signal

Even signal satisfies the condition $x(n) = x(-n)$.

**Odd signal:**

A signal that exhibits anti-symmetry with respect to n=0 is called odd signal

Odd signal satisfies the condition $x(n) = -x(-n)$.

Even part $\boldsymbol{x_e(n)}$ and Odd part $\boldsymbol{x_0(n)}$ of discrete time signal $\boldsymbol{x(n)}$:

Even component of $x(n)$ is $x_e(n) = \frac{1}{2}[x(n) + x(-n)]$

Odd component of $x(n)$ is $x_o(n) = \frac{1}{2}[x(n) - x(-n)]$

## PERIODIC AND APERIODIC SIGNAL

**Periodic signal:**

A signal is said to periodic if it repeats again and again over a certain period of time.

**Aperiodic signal:**

A signal that does not repeat at a definite interval of time is called aperiodic signal.

**Continuous domain:**

A Continuous time signal is said to periodic if it satisfies the condition

$$x(t) = x(t + T) \text{ where } T \text{ is fundamental time period}$$

If the above condition is not satisfied, then the signal is said to be aperiodic

Fundamental time period

$$T = 2\pi/\Omega$$

where Ω is fundamental angular frequency in rad/sec

**Discrete domain:**

A Discrete time signal is said to periodic if it satisfies the condition

$$x(n) = x(n + N) \ \ where\ N\ is\ fundamental\ time\ period$$

If the above condition is not satisfied, then the signal is said to be aperiodic

Fundamental time period

$$N = 2\pi m/\omega$$

where ω is fundamental angular frequency in rad/sec,

$m$ is smallest positive integer that makes N as positive integer.

**ENERGY AND POWER SIGNAL**

**Energy signal:**

The signal which has finite energy and zero average power is called energy signal. The non-periodic signals like exponential signals will have constant energy and so non periodic signals are energy signals.

i.e., For energy signal, $0 < E < \infty \ \ and \ \ P = 0$

For Continuous time signals,

$$Energy\ E = \lim_{T\to\infty} \int_{-T}^{T} |x(t)|^2 dt$$

For Discrete time signals,

$$Energy\ E = \lim_{N\to\infty} \sum_{n=-N}^{N} |x(n)|^2$$

**Power signal:**

The signal which has finite average power and infinite energy is called power signal. The periodic signals like sinusoidal complex exponential signals will have constant power and so periodic signals are power signals.

i.e., For power signal, $0 < P < \infty \quad and \quad E = \infty$

For Continuous time signals,

$$Average\ power\ P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 dt$$

For Discrete time signals,

$$Average\ power\ P = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

## DETERMINISTIC AND RANDOM SIGNALS

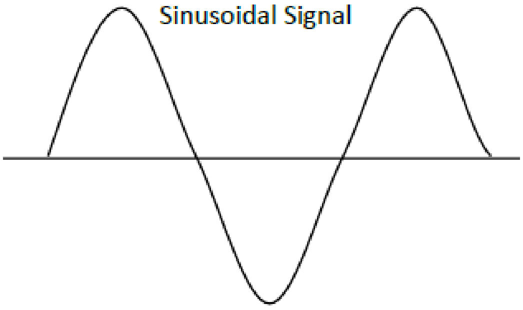
**Deterministic signal:**

A signal is said to be deterministic if there is no uncertainty over the signal at any instant of time i.e., its instantaneous value can be predicted. It can be represented by mathematical equation.

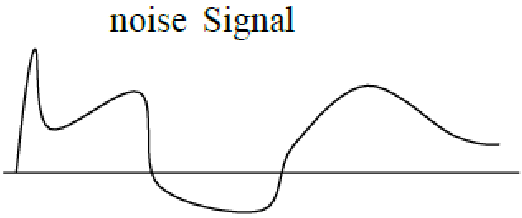
Example: sinusoidal signal

**Random signal (Non-Deterministic signal):**

A signal is said to be random if there is uncertainty over the signal at any instant of time i.e., its instantaneous value cannot be predicted. It cannot be represented by mathematical equation.




Deterministic signal




Random signal

## CAUSAL AND NON-CAUSAL SIGNAL

**Continuous domain:**

**Causal signal:**

A signal is said to be causal if it is defined for $t \geq 0$.

$$i.e., \quad x(t) = 0 \quad for\ t < 0$$

Non-causal signal:

A signal is said to be non-causal, if it is defined for $t < 0$ or for both $t < 0$ and $t \geq 0$

$$i.e., \quad x(t) \neq 0 \quad for\ t < 0$$

When a non-causal signal is defined only for $t<0$, it is called as anti-causal signal

**Discrete domain:**

**Causal signal:**

A signal is said to be causal, if it is defined for $n \geq 0$.

$$i.e., \quad x(n) = 0 \quad for\ n < 0$$

Non-causal signal:

A signal is said to be non-causal, if it is defined, for $n < 0$ or for both $n < 0$ $and\ n \geq 0$

$$i.e., \quad x(n) \neq 0 \quad for\ n < 0$$

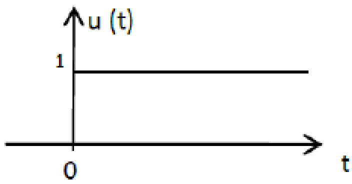
When a non-causal signal is defined only for $n<0$, it is called as anti-causal signal.

## BASIC (ELEMENTARY OR STANDARD) CONTINUOUS TIME SIGNALS

### Step signal

Unit Step signal is defined as

$$u(t) = 1 \quad for\ t \geq 0$$
$$= 0 \quad for\ t < 0$$

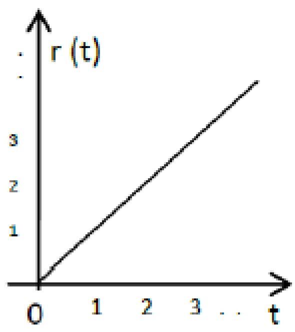


Unit step signal

### Ramp signal

Unit ramp signal is defined as

$$r(t) = t \quad for\ t \geq 0$$
$$= 0 \quad for\ t < 0$$

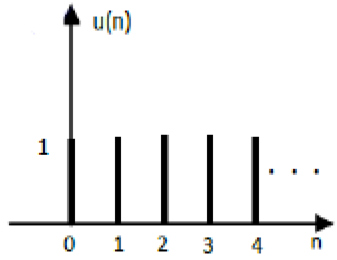


Unit ramp signal

## Step signal

Unit Step signal is defined as

$$u(n) = 1 \;\; for\ n \geq 0$$
$$= 0 \; for\ n < 0$$

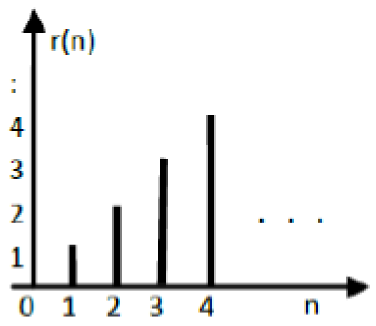



Unit step signal

## Unit Ramp signal

Unit Ramp signal is defined as

$$r(n) = n \;\; for\ n \geq 0$$
$$= 0 \quad for\ n < 0$$




Unit Ramp signal

# Basic Elements of DSP

- Fig. 1.1.1 shows the block diagram of basic elements of DSP. The analog input signal can be any one of the signal generated in nature such as sound, video, temperature, pressure, flow, sesmic signals, biomedical signals etc.

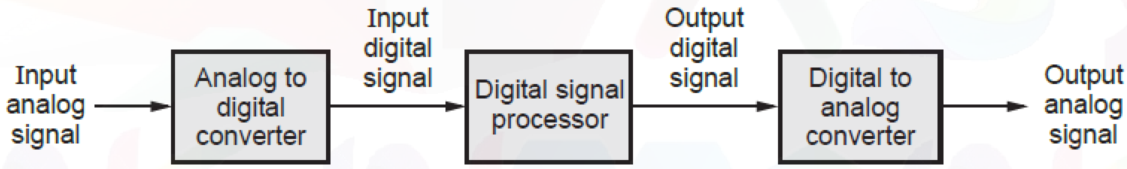



**Fig. 1.1.1 Basic elements of DSP**

- **Analog to digital converter :** Analog input signal is converted to digital form. It determines the sampling rate and quantization error in digitizing operation.
- **Digital signal processor :** The DSP processor receives digital signal from A/D converter and performs the operations such as amplification, attenuation, filtering, spectral analysis, feature extraction, correlation, transformation etc. operations on digital data.
- The DSP processor consists of ALU, shifter, serial ports, interrupts, address generators, clocks, counters, etc. for its functioning.
- The DSP processors have special architectural features due to which DSP operations are implemented fast on it compared to other general purpose microprocessors.
- The DSP processors include most of the peripherals such as memories, counters, timers, clocks and A/D - D/A converters on - chip. Thus they provide single chip solution.
- **Digital to analog converter :** The processed digital signals are provided back in their analog form by D/A converters. For example, sound, image and video signals are required in analog form. D/A conversion again depends upon speed of sampling.

# Advantages and Disadvantages of Digital Signal Processing

**Advantages of DSP over analog signal processing :**

i) DSP systems are highly **flexible**. They can be reconfigured for some other operation by changing software program.

ii) DSP systems do not suffer from component tolerances. Hence they are highly **accurate**.

iii) Digital storage do not suffer from **noise and distortion**.

iv) **Mathematical operations** can be easily implemented in digital domain.

v) DSP systems are **cheaper and compact** in size.

vi) Performance of DSP is **repeatable** and hence **reliable**.

vii) DSP systems are **adaptive** and **universally compatible**.

**Disadvantages of DSP :**

i) Difficult to process **high bandwidth analog** signals.

ii) DSP systems are **expensive** for small applications.

iii) DSP systems cannot be implemented **without power supply**. But analog circuits can be implemented with passive components.

# Applications of DSP

**i) Voice and speech :** Speech recognition, voice mail, speech coding/decoding, speech synthesis, speaker identification etc.

**ii) Telecommunications :** Cellular phones, video conferencing, packet switching, echo cancellation, digital EPABXs, multiplexing, adaptive equalizers, data encryption etc.

**iii) Consumer applications :** Digital audio/video/television, music systems, Synthesizers, toys etc.

**iv) DSP for graphics and imaging :** 3-D and 2-D visualization, animation, pattern recognition, image coding, image transmission, image enhancement, satellite imaging, robot vision.

**v) DSP for military and defense :** Radar processing, sonar processing, Navigation, Missile guidance, RF modems, Secure communication.

**vi) Biomedical engineering :** X - ray storage, enhancement, ultrasound equipment, CT scanning equipments, MRI, ECG analysis, EEG brain mappes hearing aids, patient monitoring systems.

vii) **Automotive applications :** Vibration analysis, battery operated cars, voice commands, engine control, antiskid brakes, cellular phones, adaptive ride control

viii) **Control applications :** Servo control, robot control, laser print control, disk control, engine control, motor control.

ix) Other areas of applications include : Industrial applications, instrumentation.

## SAMPLING THEOREM

It is one of useful theorem that applies to digital communication systems.

Sampling theorem states that "A band limited signal $\boldsymbol{x(t)}$ with $\boldsymbol{X}(\omega) = \mathbf{0}$ for $|m| \geq \omega\boldsymbol{m}$ can be represented into and uniquely determined from its samples $\boldsymbol{x(nT)}$ if the sampling frequency $\boldsymbol{fs} \geq \mathbf{2}\boldsymbol{fm}$, where $\boldsymbol{fm}$ is the frequency component present in it".

(i.e) for signal recovery, the sampling frequency must be at least twice the highest frequency present in the signal.

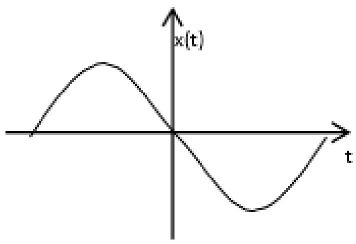



Fig 4.1

$\delta_T(t)$ is impulse train

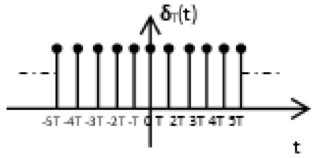



Fig 4.2

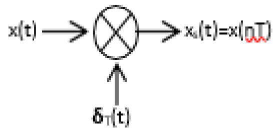



Fig 4.3

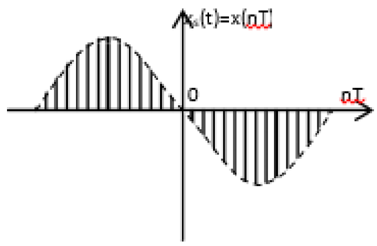



Fig 4.4

Analog signal $x(t)$ is input signal as shown in Fig 4.1, $\delta_T(t)$ is the train of impulse shown in Fig 4.2. Sampled signal $x_s(t)$ is the product of signal $x(t)$ and impulse train $\delta_T(t)$ as shown in Fig 4.2

$$\therefore x_s(t) = x(t).\,\delta_T(t)$$

$$we\ know\ \delta_T(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT) = \frac{1}{T} \sum_{n=-\infty}^{\infty} e^{jn\omega_s t}$$

$$\therefore x_s(t) = x(t).\frac{1}{T} \sum_{n=-\infty}^{\infty} e^{jn\omega_s t}$$

Applying Fourier transform on both sides

$$X_s(\omega) = \frac{1}{T} \sum_{n=-\infty}^{\infty} F[x(t)e^{jn\omega_s t}]$$

$$X_s(\omega) = \frac{1}{T} \sum_{n=-\infty}^{\infty} X(\omega - n\omega_s)$$

$$where\ \omega_s = 2\pi f_s = \frac{2\pi}{T}$$

$$\therefore X_s(\omega) = \frac{1}{T} \sum_{n=-\infty}^{\infty} X(\omega - \frac{2\pi n}{T})$$

$$(or)$$

$$X_s(f) = f_s \sum_{n=-\infty}^{\infty} X(f - nf_s) \qquad where\ f_s = \frac{1}{T}$$

Where $X(\omega)$ or $X(f)$ is Spectrum of input signal.

Where $X_s(\omega)$ or $X_s(f)$ is Specturm of sampled signal.

Spectrum of continuous time signal x(t) with maximum frequency $\omega m$ is shown in Fig 4.5

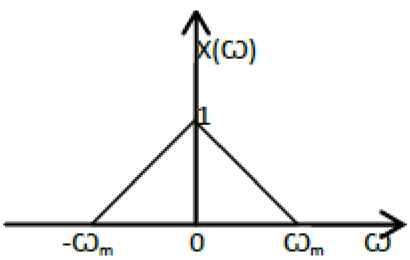



Fig 4.5 Spectrum of x(t)

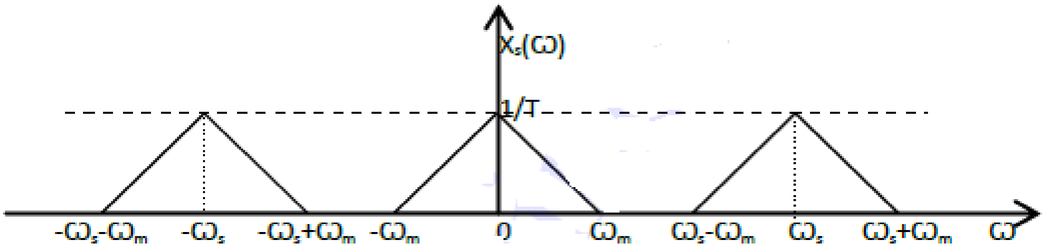



Fig 4.6 Spectrum of $xs\ t$ when $\boldsymbol{\omega s} - \boldsymbol{\omega m} > \boldsymbol{\omega m}$

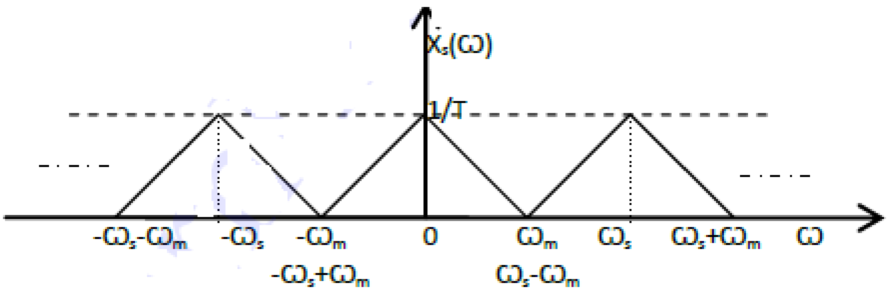



Fig 4.7 Spectrum of $xs\ t$ when $\boldsymbol{\omega s} - \boldsymbol{\omega m} = \boldsymbol{\omega m}$

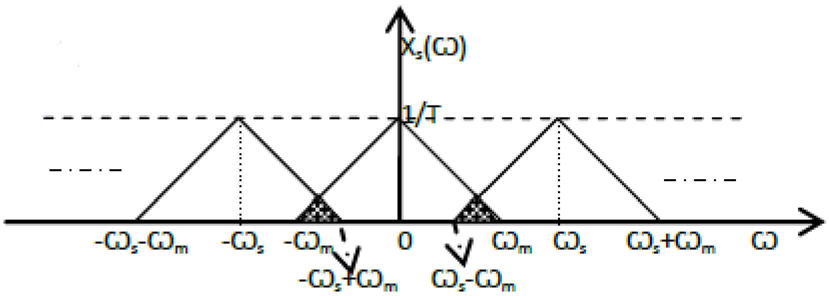



Fig 4.8 Spectrum of $x_s(t)$ when $\boldsymbol{\omega_s} - \boldsymbol{\omega_m} < \boldsymbol{\omega_m}$

For $\boldsymbol{\omega s} > 2\boldsymbol{\omega m}$

The spectral replicates have a larger separation between them known as guard band which makes process of filtering much easier and effective. Even a non-ideal filter which does not have a sharp cut off can also be used.

For $\boldsymbol{\omega s} = \boldsymbol{2\omega m}$

There is no separation between the spectral replicates so no guard band exists and $X(\omega)$ can be obtained from $Xs(\omega)$ by using only an ideal low pass filter (LPF) with sharp cutoff.

For $\boldsymbol{\omega s} < 2\boldsymbol{\omega m}$

The low frequency component in $Xs\ \omega$ overlap on high frequency components of $X\ \omega$ so that there is presence of distortion and $X\ \omega$ cannot be recovered from $Xs\ \omega$ by using any filter. This distortion is called aliasing.

So we can conclude that the frequency spectrum of $Xs(\omega)$ is not overlapped for $\boldsymbol{\omega s} - \boldsymbol{\omega m} \geq \boldsymbol{\omega m}$, therefore the Original signal can be recovered from the sampled signal.

For $\boldsymbol{\omega s} - \boldsymbol{\omega m} < \boldsymbol{\omega m}$, the frequency spectrum will overlap and hence the original signal cannot be recovered from the sampled signal.

∴ For signal recovery,

$$\omega_s - \omega_m \geq \omega_m (i.e) \quad \boldsymbol{\omega_s} \geq \boldsymbol{2\omega_m}$$

$$(or)$$

$$\boldsymbol{f_s} \geq \boldsymbol{2f_m}$$

i.e., Aliasing can be avoided if $fs \geq 2fm$

**Aliasing effect (or) fold over effect**

It is defined as the phenomenon in which a high frequency component in the frequency spectrum of signal takes identity of a lower frequency component in the spectrum of the sampled signal.

When $fs < 2fm$ , (i.e) when signal is under sampled, the individual terms in equation

$$X_s(\omega) = \frac{1}{T}\sum_{n=-\infty}^{\infty} x(\omega - n\omega_s)$$

get overlap. This process of spectral overlap is called frequency folding effect.

**Occurrence of aliasing**

Aliasing Occurs if

i) The signal is not band-Limited to a finite range.

ii) The sampling rate is too low.

**To Avoid Aliasing**

i) $x(t)$ should be strictly band limited.

It can be ensured by using anti-aliasing filter before the sampler.

ii) $f_S$ should be greater than $2f_m$ .

**Nyquist Rate**

It is the theoretical minimum sampling rate at which a signal can be sampled and still be reconstructed from its samples without any distortion

$$Nyquist\ rate\ fN = 2fm\ .\ Hz$$

**Data Reconstruction or Interpolation**

The process of obtaining analog signal $x(t)$ from the sampled signal $xs(t)$ is called data reconstruction or interpolation.

$$we\ know\ x_s(t) = x(t).\delta_T(t) = x(t)\sum_{n=-\infty}^{\infty}\delta(t-nT)$$

$$\delta(t-nT)\ exist\ \ only\ at\ t = nT$$

$$\therefore x_s(t) = x(nt)\sum_{n=-\infty}^{\infty}\delta(t-nT)$$

The reconstruction filter, which is assumed to be linear and time invariant, has unit impulse response h($t$).

The reconstruction filter, output $y(t)$ is given by convolution of $xs(t)$ and h($t$).

$$\therefore y(t) = x_s(t) * h(t) = \int_{-\infty}^{\infty} x(nT)\sum_{n=-\infty}^{\infty}\delta(\tau-nT).\ h(t-\tau)d\tau$$

$$= \sum_{n=-\infty}^{\infty} x(nT)\int_{-\infty}^{\infty}\delta(\tau-nT)\ h(t-\tau)d\tau$$

$$\delta(\tau-nT)\ exist\ only\ at\ \tau = nT$$

$$\delta(\tau-nT) = 1\ at\ \tau = nT$$

$$y(t) = \sum_{n=-\infty}^{\infty} x(nT)h(t-nT)$$

**Ideal Reconstruction filter**

The sampled signal $xs(t)$ is passed through an ideal LPF (Fig 4.9) with bandwidth greater than $fm$ and a pass band amplitude response of T, then the filter output is $x(t)$.

Transfer function of ideal reconstruction filter is

$$H(f) = T \;;\; |f| < 0.5f_s$$

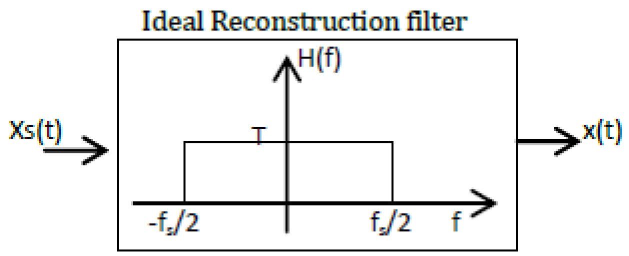



Fig 4.9

The impulse response of ideal reconstruction filter is

$$h(t) = \int_{\frac{-f_s}{2}}^{\frac{f_s}{2}} Te^{j\omega t}\, df$$

$$= \int_{-\frac{f_s}{2}}^{\frac{f_s}{2}} T\, e^{j2\pi ft}\, df = T\left[\frac{e^{j2\pi ft}}{j2\pi t}\right]_{-\frac{f_s}{2}}^{\frac{f_s}{2}} = \frac{T}{j2\pi t}\left[e^{j2\pi\frac{f_s}{2}t} - e^{-j2\pi\frac{f_s}{2}t}\right]$$

$$= \frac{1}{f_s\pi t}\left[\frac{e^{j2\pi\frac{f_s}{2}t} - e^{-j2\pi\frac{f_s}{2}t}}{2j}\right] = \frac{1}{\pi f_s t}\sin \pi f_s t = \sin c\; \pi f_s t$$

$$\therefore h(t-nT) = \sin c\; \pi f_s(t-nT) \quad \ldots\ldots\ldots\langle 1\rangle$$

$$y(t) = \sum_{n=-\infty}^{\infty} x(nT)h(t-nT)$$

Substitute equation 1 in above equation

$$\therefore y(t) = \sum_{n=-\infty}^{\infty} x(nT)\ \sin c\ \pi f_s(t - nT) = \sum_{n=-\infty}^{\infty} x(nT)\ \sin c\ \pi \left(\frac{t}{T} - n\right)$$

$$\left[\because f_s = \frac{1}{T}\right]$$