ROHININ COLLEGE OF ENGINEERING AND TECHNOLOGY
Approved by AICTE & Affliated to anna university
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DEPARTMENT OF MECHANICAL ENGINEERING

NAME OF THE SUBJECT: ENGINEERING MECHANICS

SUBJECT CODE : ME3351

REGULATION 2021

UNIT III: DISTRIBUTED FORCES

Moment of Inertia [polar]

State parallel Axis theorem

It state that, if the moment of inertia of a plane area about an axis through its centroid be denoted by IG the moment of inertia of the area about an axis AB parallel to the first and at a distance ‘h’ from the centroid is given by

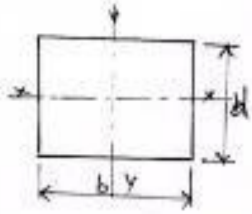
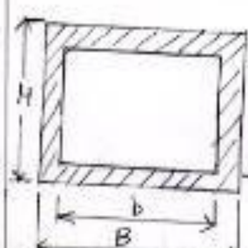
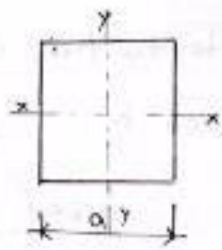
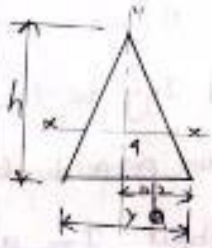
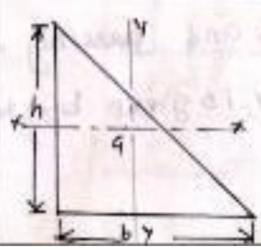
$$I_{AB} = IG + Ah^2$$

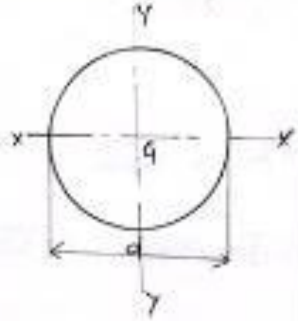
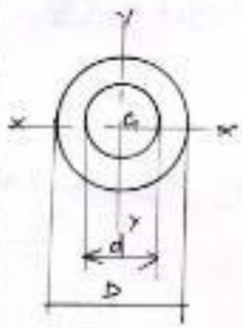
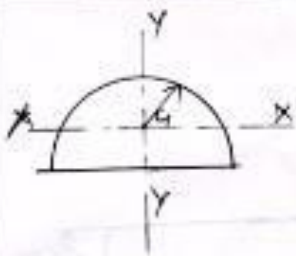
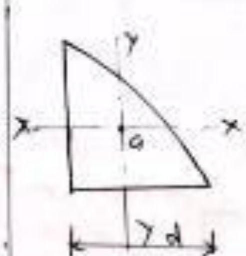
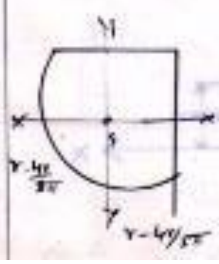
State perpendicular axis Theorem:

It states that ‘If I_{xx} and I_{yy} be the moment of inertia of a plane section about two perpendicular axes meeting at ‘o’ the moment of inertia about I_{zz} about the axis z-z perpendicular to the plane and passing through the intersection of X-X and Y-Y is given by the relation

$$I_{zz} = I_{xx} + I_{yy}$$

Moment of inertia.

Sl.No	Name	Figure	G ($\bar{x}$ and $\bar{y}$) about Area A $\bar{x}$ from left $\bar{y}$ from base	I_{xx}, I_{yy} self centroidal Axis
1	Rectangle		$\bar{x} = b/2$ $\bar{y} = d/2$ $A = b \times d$	$I_{xx} = \frac{bd^3}{12}$ $I_{yy} = \frac{db^3}{12}$ $I_{zz} = I_{xx} + I_{yy}$
2	Hollow Rectangle		$\bar{x} = B/2$ $\bar{y} = H/2$ $A = [Bb - Hh]$	$I_{xx} = \frac{1}{12}[BH^3 - bh^3]$ $I_{yy} = \frac{1}{12}[HB^3 - hb^3]$
3	Square		$\bar{x} = a/2$ $\bar{y} = a/2$ $A = a^2$	$I_{xx} = \frac{a^4}{12}$ $I_{yy} = \frac{a^4}{12}$
4	Triangle		$\bar{x} = b/2$ $\bar{y} = h/3$ $A = \frac{1}{2}bh$	$I_{xx} = \frac{bh^3}{36}$ $I_{yy} = \frac{hb^3}{48}$
5	Right angle Triangle		$\bar{x} = b/3$ $\bar{y} = h/3$ $A = \frac{1}{2}bh$	$I_{xx} = \frac{bh^3}{36}$ $I_{yy} = \frac{hb^3}{36}$ $I_{xx} + I_{yy} = I_{zz}$

6	circle		$A = \pi r^2$ or $\frac{\pi d^2}{4}$ $\bar{x} = d/2$ $\bar{y} = d/2$	$I_{xx} = \frac{\pi d^4}{64}$ $I_{yy} = \frac{\pi d^4}{64}$ In r $I_{xx} = \frac{\pi r^4}{4}$ $I_{yy} = \frac{\pi r^4}{4}$ $I_{zz} = \pi/2\, r^4$
7	hollow circle		$\bar{x} = D/2$ $\bar{y} = D/2$ $A = \pi/4\,[D^2 - d^2]$	$I_{xx} = \pi/4\,[R^2 - r^2]$ $I_{yy} = \pi/4\,[R^4 - r^2]$ $I_{zz} = \pi/2\,[R^2 - r^2]$
8	semi circle 		$\bar{x} = r$ $\bar{y} = \frac{4r}{3\pi}$ $A = \frac{\pi r^2}{2}$	$I_{xx} = 0.11 r^4$ $I_{yy} = \frac{\pi r^4}{8} = \frac{\pi D^4}{128}$
9	Quadrant		$\bar{x} = \frac{4r}{3\pi}$ $\bar{y} = \frac{4r}{3\pi}$ $A = \frac{\pi r^2}{4}$	$I_{xx} = 0.055 r^4$ $I_{yy} = 0.055 r^4$
10	Quadrant		$\bar{x} = r - \frac{4r}{3\pi}$ $\bar{y} = r - \frac{4r}{3\pi}$	

Formula:

Moment of Inertia about the X axis

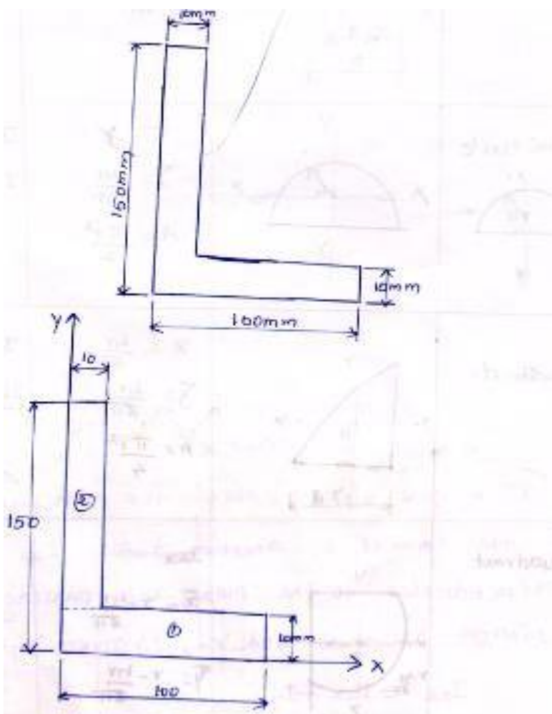
$$I = I_{xx1} + A_1[\bar{y} - y_1]^2 + I_{xx2} + A_2[\bar{y} - y_2]^2 + I_{xx3} + A_3[\bar{y} - y_3]^2 + \ldots\ldots\ldots\ldots..$$

Moment of Inertia about the Y axis

$$I_{yy} = I_{yy1} + A_1[\bar{x} - x_1]^2 + I_{yy2} + A_2[\bar{x} - x_2]^2 + I_{yy3} + A_3[\bar{x} - y_3]^2$$

Problem 1.

An area in the form of L section is shown in fig.



$$I = I_{xx1} + A_1[\bar{y} - y_1]^2 + I_{xx2} + A_2[\bar{y} - y_2]^2$$

$$I_{yy} = I_{yy1} + A_1[\bar{x} - x_1]^2 + I_{yy2} + A_2[\bar{x} - x_2]^2$$

Section (1) Rectangle

$$a_1 = 100 \times 10 = 1000mm^2$$

$$x_1 = \frac{100}{2} = 50mm$$

$$y_1 = \frac{10}{2} = 5mm$$

$$I_{xx_1} = \frac{bd^3}{12} = \frac{100 \times 10^3}{12} = 8333.33mm^4$$

$$I_{yy_1} = \frac{db^3}{12} = \frac{10 \times 100^3}{12} = 833.33 \times 10^3 mm^4$$

Section (2) rectangle

$$a_2 = 10 \times 140 = 1400mm^2$$

$$x_2 = \frac{10}{2} = 5mm$$

$$y_2 = 10 + \frac{140}{2} = 80mm$$

$$I_{xx_1} = \frac{bd^3}{12} = \frac{10 \times 140^3}{12} = 2.286 \times 10^6 mm^4$$

$$I_{yy_1} = \frac{db^3}{12} = \frac{140 \times 10^3}{12} = 11.66 \times 10^3 mm^4$$

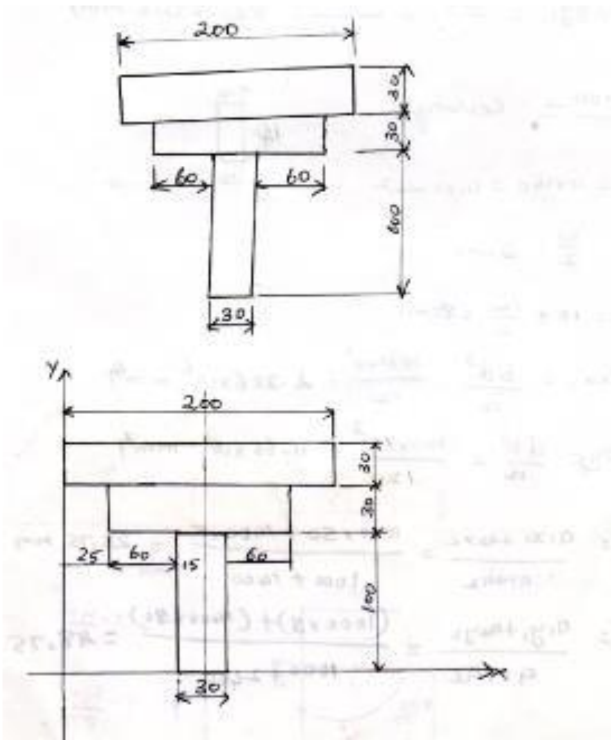
$$\bar{x} = \frac{a_1 x_1 + a_2 x_2}{a_1 + a_2} = \frac{1000 \times 50 + 1400 \times 5}{1000 + 1400} = 23.75mm$$

$$\bar{y} = \frac{a_1 y_1 + a_2 y_2}{a_1 + a_2} = \frac{1000 \times 5 + 1400 \times 80}{1000 + 2400} = 48.75mm$$

$$I_{xx} = 8333.33 + 1000[48.75 - 5]^2 + 2.286 \times 10^6[1400(48.75 - 80)^2]$$
$$= 575 \times 10^6 mm^4$$

$$I_{yy} = 833.33 \times 10^3 + 1000[23.75 - 50]^2 + 11.66 \times 10^3[1400(23.75 - 5)^2]$$
$$= 2.02 \times 10^6 mm^4$$

2. Find the moment of inertia of the built up section shown in fig. about the axes passing through the centre of gravity parallel to the flange plate.



$I = I_{xx1} + A_1[\bar{y} - y_1]^2 + I_{xx2} + A_2[\bar{y} - y_2]^2 + I_{xx3} + A_3[\bar{y} - y_3]^2$

$I_{yy} = I_{yy1} + A_1[\bar{x} - x_1]^2 + I_{yy2} + A_2[\bar{x} - x_2]^2 + I_{yy3} + A_3[\bar{x} - y_3]^2$

Section (1) Rectangle

$a_1 = 30 \times 100 = 3000mm^2$

$x_1 = 25 + 60 + \frac{30}{2} = 100mm$

$y_1 = \frac{100}{2} = 50mm$

$I_{xx_1} = \frac{bd^3}{12} = \frac{30 \times 100^3}{12} = 2.5 \times 10^6 mm^4$

$I_{yy_1} = \frac{db^3}{12} = \frac{100 \times 30^3}{12} = 2.25 \times 10^5 mm^4$

Section (2) Rectangle

$a_2 = 150 \times 30 = 4500 mm^2$

$x_2 = 25 + \frac{150}{2} = 100 mm$

$y_2 = 100 + \frac{30}{2} = 115 mm$

$I_{xx_1} = \frac{bd^3}{12} = \frac{150 \times 30^3}{12} = 3.37 \times 10^5 mm^4$

$I_{yy_1} = \frac{db^3}{12} = \frac{30 \times 150^3}{12} = 8.43 \times 10^5 mm^4$

Section (3)

$a_3 = 300 \times 30 = 9000 mm^2$

$x_2 = \frac{200}{2} = 100 mm$

$y_2 = 100 + 30 + \frac{30}{2} = 145 mm$

$I_{xx_1} = \frac{bd^3}{12} = \frac{300 \times 30^3}{12} = 6.75 \times 10^5 mm^4$

$I_{yy_1} = \frac{db^3}{12} = \frac{30 \times 300^3}{12} = 67.5 \times 10^5 mm^4$

$$\bar{x} = \frac{a_1 x_1 + a_2 x_2 + a_3 x_3}{a_1 + a_2 + a_3} = \frac{(3000 \times 100) + (4500 \times 100) + (9000 \times 100)}{3000 + 4500 + 9000}$$

$\bar{X} = 100 mm$

$$\bar{Y} = \frac{a_1 x_1 + a_2 x_2 + a_3 y_3}{a_1 + a_2 + a_3} = \frac{(3000 \times 50) + (4500 \times 115) + (9000 \times 145)}{3000 + 4500 + 9000}$$

$\bar{Y} = 119.54 mm$

$$I_{xx} = 2.5 \times 10^6 + 3000[119.54 - 50]^2 + 3.37 \times 10^5 + 4500[119.54 - 115]^2 + 6.75 \times 10^5 + 9000[119.54 - 145]^2$$

$$I_{xx} = 23.94 \times 10^6 mm^4$$

$$I_{yy} = 2.25 \times 10^5 + 3000[100 - 100]^2 + 8.43 \times 10^6 + 4500[100 - 100]^2 + 6.75 \times 10^5 + 9000[100 - 100]^2$$

$$I_{yy} = 76.15 \times 10^6 mm^4$$

Product of Inertia:

The moment of inertia of a plane fig. above a set of perpendicular axis is called product of inertia.

$$I_{xy} = \int_A xy \; da$$

$$= \sum axy$$