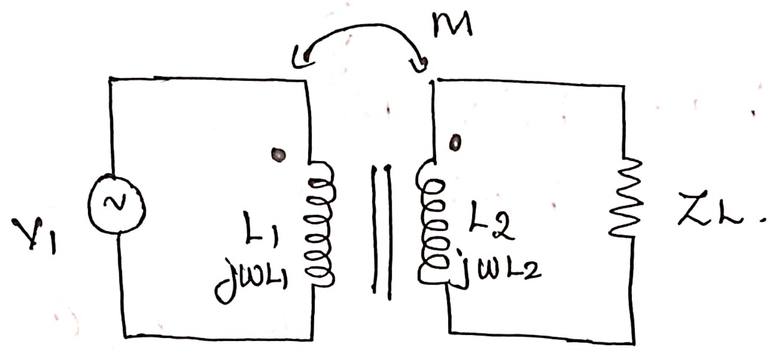
The ideal Transformer

Transformer is a device used to transfer energy from primary winding connected at source to the secondary winding connected at load R_L



characteristic of ideal transformer

1) Resistance in the coil is assumed to be zero
2) Zero power dissipation in primary and secondary windings.
3. self inductance is very large than Z_L
4. co-efficient of coupling is unity

The mutual inductance is replaced by a dependent source

The voltage equation of the loop is,

$V_1 - (-j\omega M I_2) - j\omega L_1 I_1 = 0$

$V_1 + j\omega M I_2 = j\omega L_1 I_1$

$$\boxed{V_1 = j\omega L_1 I_1 - j\omega M I_2}$$

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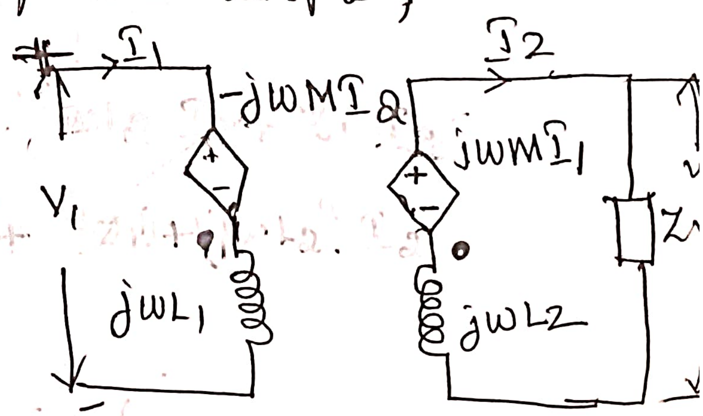


At loop 2,

$$j\omega M I_1 - Z_L I_2 - j\omega L_2 I_2$$

$$j\omega M I_1 - (Z_L + j\omega L_2) I_2$$

$$j\omega M I_1 = (Z_L + j\omega L_2) I_2 \Rightarrow \boxed{I_2 = \frac{j\omega M I_1}{Z_L + j\omega L_2}} \quad - (2).$$

$$\frac{I_1}{I_2} = \frac{Z_L + j\omega L_2}{j\omega M}$$

$$= \frac{\cancel{Z}\cancel{j\omega} L_2}{\cancel{j\omega} M} \quad [\because Z_L \text{ is very small}]$$

$$\boxed{\frac{I_1}{I_2} = \frac{L_2}{M}}$$

we know that $M = \sqrt{L_1 L_2}$

$$\therefore \frac{I_1}{I_2} = \frac{L_2}{\sqrt{L_1 L_2}} = \sqrt{\frac{L_2^{\cancel{2}}}{L_1 \cancel{L_2}}} = \sqrt{\frac{L_2}{L_1}}$$

$$\frac{I_1}{I_2} = \sqrt{\frac{L_2}{L_1}} = \sqrt{\frac{N_2^2}{N_1^2}} = \frac{N_2}{N_1} \qquad \left[\because \frac{L_2}{L_1} = \frac{N_2^2}{N_1^2}\right]$$

sub (2) in (1) $\Rightarrow$

$$V_1 = j\omega L_1 I_1 - j\omega M \cdot \frac{j\omega M I_1}{Z_L + j\omega L_2}.$$

$$= I_1 \left[j\omega L_1 + \frac{\omega^2 M^2}{Z_L + j\omega L_2} \right]$$

12 Equivalent inductance

$$Z_{in} = \frac{V_1}{I_1} = j\omega L_1 + \frac{\omega^2 M^2}{Z_L + j\omega L_2}.$$

$$= j\omega L_1 + \frac{\omega^2 (\sqrt{L_1 L_2})^2}{Z_L + j\omega L_2}$$

$$= j\omega L_1 + \frac{\omega^2 L_1 L_2}{Z_L + j\omega L_2}$$

$$= j\omega L_1 + \frac{\omega^2 L_1 \cdot a^2 L_1}{Z_L + j\omega a^2 L_1} \quad \left[\frac{L_2}{L_1} = a^2 \Rightarrow L_2 = a^2 L_1 \right.$$

$$= j\omega L_1 + \frac{\omega^2 a^2 L_1^2}{Z_L + j\omega a^2 L_1}$$

$$= \frac{j\omega L_1 (Z_L + j\omega a^2 L_1) + \omega^2 a^2 L_1^2}{Z_L + j\omega a^2 L_1}$$

$$= \frac{j\omega L_1 Z_L - \omega^2 L_1^2 a^2 + \omega^2 a^2 L_1^2}{Z_L + j\omega a^2 L_1}$$

$$= \frac{j\omega L_1 Z_L}{Z_L + j\omega a^2 L_1}$$

$$Z_{in} = \frac{\cancel{j\omega} \cancel{L_1} Z_L}{\cancel{j\omega} a^2 \cancel{L_1}}$$

$$\boxed{\frac{Z_{in}}{Z_L} = \frac{1}{a^2}}$$

A transformer is made of 20V AC to a higher voltage. The current in the primary side is 6A and secondary is 2A.

1) Calculate secondary voltage, power P.

2) Number of coils in secondary, If there are 50 primary coils.

<u>soln</u>

Given $V_1 = 20V$, $I_1 = 6A$, $I_2 = 2A$, $N_1 = 50$

Find V_2, $N_2 = ?$

$$\frac{I_1}{I_2} = \frac{N_2}{N_1}$$

$$\frac{\cancel{6}^{3}}{\cancel{2}} = \frac{N_2}{50} \Rightarrow \boxed{N_2 = 150}$$

$$\frac{V_1}{V_2} = \frac{I_2}{I_1}$$

$$\cancel{\frac{V_2}{20} = \frac{2}{6} \Rightarrow V_2 = \frac{20 \times 2}{6_3}} =$$

$$\frac{20}{V_2} = \frac{2}{6} \Rightarrow V_2 = \frac{20 \times \cancel{6}}{\cancel{2}}$$

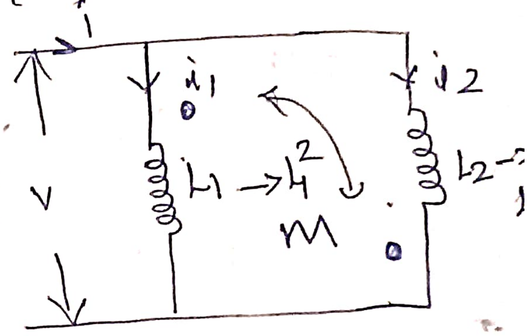
$$\boxed{V_2 = 60V}$$

$$\text{power} = I_1 V_1 \quad (\text{or}) \quad I_2 V_2$$

$$= 6 \times 20$$

$$\boxed{P = 120\,W}$$

Derive the expression for equivalent inductance, L for the circuit shown below L_1, L_2 are self inductance and M is mutual inductance [Apr/May 2023]



$$V = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$$

$$= L_1 \frac{di_1}{dt} - M \frac{di_1}{dt} = L_2 \frac{di_2}{dt} - M \frac{di_2}{dt}$$

$$V = (L_1 - M) \frac{di_1}{dt} = (L_2 - M) \frac{di_2}{dt}$$

$$\frac{1}{L_{eq}} = \frac{1}{L_1'} + \frac{1}{L_2'}$$

$$= \frac{1}{\frac{L_1L_2 - M^2}{L_2 + M}} + \frac{1}{\frac{L_1L_2 - M^2}{L_1 + M}}$$

$$= \frac{L_2 + M + L_1 + M}{L_1L_2 - M^2}$$

$$\frac{1}{L_{eq}} = \frac{L_1 + L_2 + 2M}{L_1L_2 - M^2} \Rightarrow L_{eq} = \frac{L_1L_2 - M^2}{L_1 + L_2 + 2M}$$