1.2 SUMMARY OF ANALYSIS & SYNTHESIS EQUATIONS FOR FT & DTFT, FREQUENCY DOMAIN SAMPLING, DISCRETE FOURIER TRANSFORM (DFT)

1. Fourier Transform (FT)

(a) Analysis Equation (Time → Frequency)

$$X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

(b) Synthesis Equation (Frequency → Time)

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega)\, e^{j\omega t}\, d\omega$$

- Used for continuous-time signals
- Frequency spectrum is continuous
- Applicable for non-periodic signals

2. Discrete-Time Fourier Transform (DTFT)

(a) Analysis Equation

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n}$$

(b) Synthesis Equation

$$x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega})\, e^{j\omega n}\, d\omega$$

Key Points:

- Used for **discrete-time signals**
- Frequency response is **continuous and periodic**
- Periodicity: 2π

3. Frequency-Domain Sampling

Frequency-domain sampling refers to **sampling the DTFT at equally spaced frequency points.**

Concept:

- Sampling $X(e^{j\omega})$ at

$$\omega_k = \frac{2\pi k}{N}, \quad k = 0, 1, 2, \ldots, N-1$$

- Leads directly to the **Discrete Fourier Transform (DFT)**

Importance:

- Converts continuous frequency representation into **discrete frequency samples**
- Enables digital computation using FFT algorithms

4. Discrete Fourier Transform (DFT)

(a) Analysis Equation

$$X[k] = \sum_{n=0}^{N-1} x[n]\, e^{-j\frac{2\pi}{N}kn}$$

(b) Synthesis Equation

$$x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k]\, e^{j\frac{2\pi}{N}kn}$$

Key Points:

- Operates on **finite-length discrete signals**
- Frequency domain is **discrete and periodic**
- Basis for **FFT (Fast Fourier Transform)**

Relationship Between FT, DTFT, and DFT

Transform	Time Domain	Frequency Domain
FT	Continuous	Continuous
DTFT	Discrete	Continuous (Periodic)
DFT	Discrete (Finite)	Discrete

- FT analyzes continuous-time signals
- DTFT extends frequency analysis to discrete-time signals
- Frequency-domain sampling converts DTFT into discrete form
- DFT provides a practical and computable frequency representation for digital systems