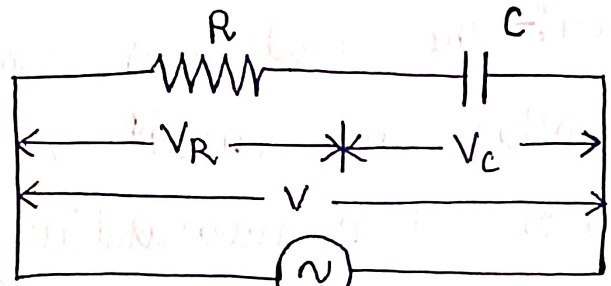
Resistance - Capacitance (R-C) series circuit

A circiut that contains pure resistance R Ω connected in series with a pure capacitor of capacitance c farads is known as RC series circuit. A sinusoidal voltage is applied and current I flows through the circuit



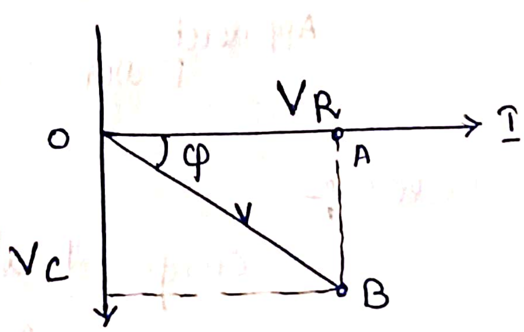
$V = V_m \sin \omega t$

where, V_R = Voltage across the resistance R

V_C = Voltage across capacitor c.

V = Total voltage across RC series ckt

<u>Phasor diagram</u>:-



+ Take the current I (rms value) as a reference vector
* Voltage drop in resistance $V = IR$ is inphase with current.

Voltage drop in capacitor is 90° behind the current

In Δle OAB

$$V^2 = (V_R)^2 + (V_C)^2$$

$$V = \sqrt{(V_R)^2 + (V_C)^2}$$

$$= \sqrt{(IR)^2 + (IX_C)^2}$$

$$= I\sqrt{R^2 + (X_C)^2}$$

$$V = IZ$$

$$I = \frac{V}{Z}$$

Phase angle :-

The current leads the ~~ax~~ voltage by an angle φ. This angle is called phase angle.

$$\tan\varphi = \frac{V_C}{V_R} = \frac{IX_C}{IR}$$

$$\varphi = \tan^{-1}\left(\frac{X_C}{R}\right)$$

Let $V = V_m \sin(\omega t)$ be the instantaneous voltage

& $i = I_m \sin(\omega t + \varphi)$ be the instantaneous current

Instantaneous power is

$$P = V_m \sin\omega t \; I_m \sin(\omega t + \varphi)$$

$$= V_m I_m \sin\omega t \sin(\omega t + \varphi)$$

$$= \frac{V_m I_m}{2}\left[\cos(-\varphi) - \cos(2\omega t+\varphi)\right]$$

$C_{A-B} = C_A C_B + S_A S_B$

$$= \frac{V_m I_m}{2}\left[\cos\varphi - \cos(2\omega t+\varphi)\right]$$

$C_{A+B} = C_A C_B - S_A S_B$.

$C(A-B) - C_{A+B} = 2 S_A S_B$

$S_A S_B = \frac{C_{A-B} - C_{A+B}}{2}$

Average power is

$$= \text{avg } \frac{V_m I_m}{2}\cos\varphi - \text{avg } \frac{V_m I_m}{2}\cancel{\cos(2\omega t+\varphi)} \to 0$$

$$\boxed{P_{avg} = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}\cos\varphi}$$

$P = VI\cos\varphi$

active power

$\cos\varphi$ ⟶ power factor.

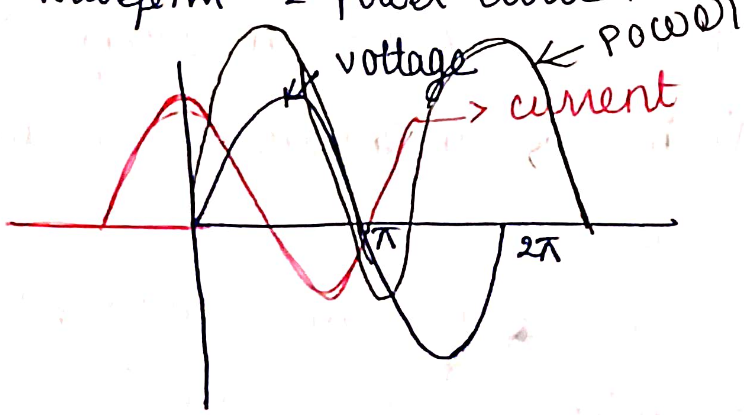
$$= VI \times \frac{V_R}{V}$$

$$= VI \times \frac{\cancel{I}R}{\cancel{I}Z}$$

$$= I\cancel{Z}I \cdot \frac{R}{\cancel{Z}}$$

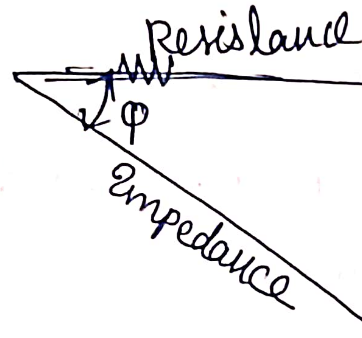
$$\boxed{P_{avg} = I^2 R}$$

waveform & power curve.



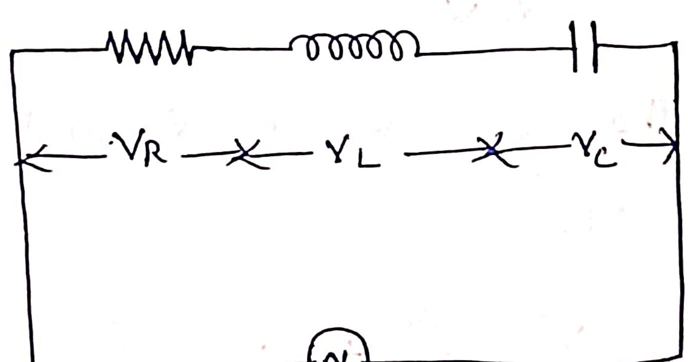
The power is negative between $(180° - \varphi)$ & $(360° - \varphi)$ & $360°$, rest of the cycle the power is positive

Impedance triangle



RLC series circuit

When a pure resistor, inductor and capacitor are connected in series it is called a RLC series circuit. The current flowing through each element will be the same.



$V = V_m \sin \omega t$.

In the circuit, $X_L = 2\pi f L$ and $X_C = \dfrac{1}{2\pi f C}$

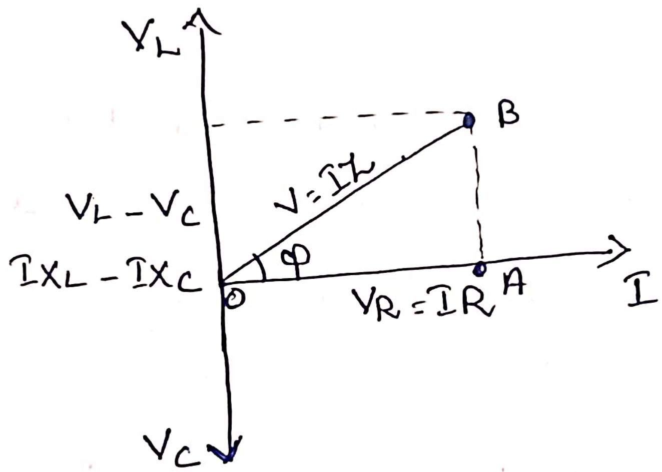
$V_R = IR$ Voltage across resistor & is inphase with I

$V_L = IX_L$ Voltage across inductor L & it leads the current by $90°$.

V_C is voltage across the Capacitor & it lags the current by 90°.

<u>phasor diagram:</u>

The phasor diagram of the RLC series circuit is shown below. when $V_L > V_C$ the circuit acts as an inductive circuit and if $V_L < V_C$ the circuit will behave as a capacitive circuit



~~Ho~~ The two vectors V_L and V_C are opposite to each other.

$$V^2 = V_R^2 + (V_L - V_C)^2$$

$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$= \sqrt{I^2R^2 + (IX_L - IX_C)^2}$$

$$V = I\sqrt{R^2 + (X_L - X_C)^2}$$

$$I = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}}$$

$$I = \frac{V}{Z}$$

Phase angle

From phasor diagram

$$\tan\varphi = \frac{V_L - V_C}{V_R}$$

$$= \frac{IX_L - IX_C}{IR}$$

$$= \frac{\cancel{I}(X_L - X_C)}{\cancel{I}R}$$

$$\boxed{\varphi = \tan^{-1}\frac{X_L - X_C}{R}}$$

Power in RLC series circuit

The product of current & voltage is power

$$P = VI\cos\varphi$$

$$\boxed{P = I^2 R}$$

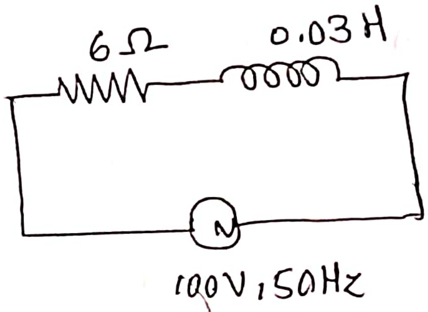
when $X_L > X_C$ $\varphi = +ive$ circuit acts as series RL ckt

$X_L < X_C$ $\varphi = -ive$ circuit acts as a RC ckt.

$X_L = X_C$ $\varphi = 0$ circuit acts as a pure resistive circuit.

(30) A coil having resistance of 6Ω and inductance of 0.03H is connected across 100V, 50Hz supply. Calculate i) the current 2) Phase angle b/w I and V, c) power factor d) power.

soln



$X_L = \omega L = 2\pi f \times L$

$X_L = 2 \times \pi \times 50 \times 0.03$
$= 9.42$

current $I = \dfrac{V}{Z}$

$= \dfrac{V}{\sqrt{X_L^2 + R^2}}$

$= \dfrac{100}{\sqrt{88.73+36}}$

$= \dfrac{100}{\sqrt{124.7}}$

$= \dfrac{100}{11.16}$

$\boxed{I = 8.96A}$

phase angle $\varphi = \tan^{-1} \dfrac{X_L}{R}$

$= \tan^{-1} \left(\dfrac{9.42}{6}\right)$

$= \tan^{-1}(1.57)$

$\boxed{\varphi = 57.3^0 \text{ lag}}$

Power factor $\cos\varphi = \cos 57.3$
$= 0.537$

Power $= |V| |I| \cos\varphi$

$= 100 \times 8.96 \times 0.537$

$= 481.$ watts.

In an ac circuit the applied voltage is given by $v = 200 \sin 314t$ and the expression for the leading current is given by $i = 10 \cos 314t$. Find the circuit current and find power factor of the circuit. Draw the phasor diagram.

Soln

$$v = 200 \sin 314t$$

$$i = 10 \cos 314t.$$

power factor

$$\cos\phi = \cos 90^\circ = 0$$

$$i = 10 \cos 314t$$
$$= 10 \sin(314t + \pi/2)$$
$$= I_m \sin(\omega t + \pi/2)$$

$$v = \underset{V_m}{200} \sin \underset{\omega}{314} t$$

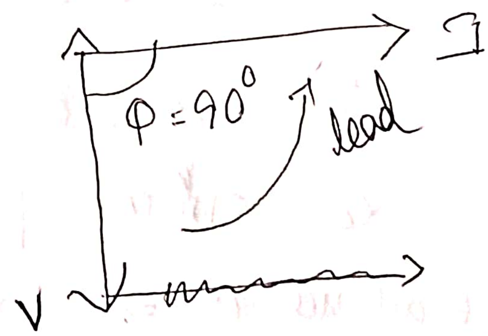
$I_m = 10$ $\omega = 314$ $V_m = 200$

$$X_C = \frac{V_m}{I_m} = \frac{200}{10}$$

$$X_C = 20\,\Omega$$

$$\frac{1}{C\omega} = 20$$

$$C = \frac{1}{\omega \times 20} = \frac{1}{314 \times 20} = 159\,\mu F.$$



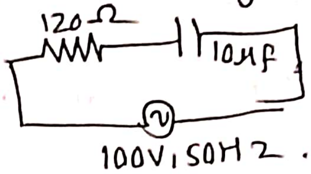
$$V = \frac{V_m}{\sqrt{2}}$$

$$\boxed{V = 141.4\,V}$$

$$I = \frac{I_m}{\sqrt{2}}$$

$$\boxed{I = 7.07A}$$

(32) A Capacitor having a capacitance of $10\mu F$ is connected in series with non-Inductive resistance of 120Ω across $100V, 50Hz$. Calculate current, power, phase difference b/w ct & vlg.



Soln

$$I = \frac{V}{Z} = \frac{100}{\sqrt{X_C^2 + R^2}}$$

$$X_C = \frac{1}{C\omega} = \frac{1}{10\times10^{-6}\times 2\times\pi\times 50} = \frac{10^6}{3140} = 318\,\Omega$$

$$= \frac{100}{\sqrt{318^2 + (120)^2}}$$

$$= \frac{100}{\sqrt{101{,}124 + 14{,}400}} = \frac{100}{\sqrt{115{,}524}} = \frac{100}{339.8} = 0.294\,A.$$

$\boxed{I = 0.294A}$, power $= VI\cos\varphi$

$= 100 \times 0.29 \times \cos 69.3$

$=$

$\boxed{P = 10.4\,\text{watts}}$

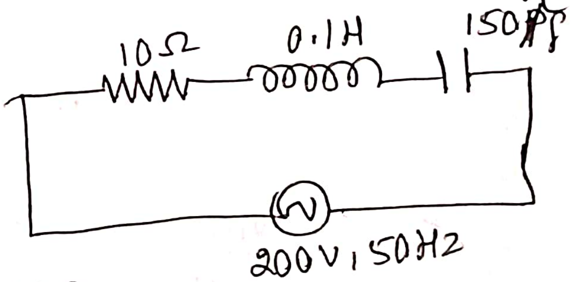
$\varphi = \tan^{-1}\left(\frac{X_C}{R}\right)$

$= \tan^{-1}\left(\frac{318}{120}\right)$

$= \tan^{-1}(2.65)$

$\boxed{\varphi = 69.3^0}$

A coil of $R = 10\Omega$, $L = 0.1H$, $C = 150\mu F$ are connected across a $200V, 50Hz$ supply. Calculate a) X_L b) X_C c) net Reactance, d) current e) power factor



$$X_L = \omega L = 2\times\pi\times f\times L$$

$$= 2\times 3.14\times 50\times 0.1 = 31.4\,\Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{2\times\pi\times f\times C} = \frac{1}{2\times 3.14\times 50\times 150\mu}$$

$X_C = 21\Omega$ $\boxed{X_C = 21\Omega}$

Net Reactance is $X = X_L - X_C = 31.4 - 21 = 10.4\,\Omega$

$$\text{current} = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + X^2}} = \frac{200}{\sqrt{10^2 + (10.4)^2}} = \frac{200}{\sqrt{208.16}} = \frac{200}{14.4}$$

$\boxed{I = 13.88}$

$\varphi = \tan^{-1}\frac{X}{R}$

$\tan^{-1}\left(\frac{10.4}{10}\right) = 46.12$

Power factor, $\cos\varphi = 0.7$