# Bell states

**Bell states** are quantum states of **two** qubits that represent simple examples of quantum entanglement. When one of the two qubits is measured, it takes on a specific value, and the second qubit is forced to *also* take on a specific value, as the entangled state collapses.

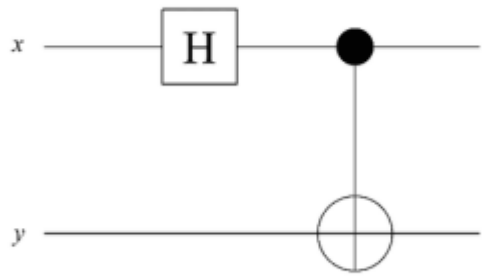
$$|\Phi^+\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}$$
$$|\Phi^-\rangle = \frac{|00\rangle - |11\rangle}{\sqrt{2}}$$
$$|\Psi^+\rangle = \frac{|01\rangle + |10\rangle}{\sqrt{2}}$$
$$|\Psi^-\rangle = \frac{|01\rangle - |10\rangle}{\sqrt{2}}$$

Bell states are also known as **EPR states** or **EPR pairs**.

There are four Bell states, which are as below.

For notations such as and , the first digit in the brackets refers to the first qubit, and the second digit refers to the second qubit.

The Bell states can be produced by a very simple quantum circuit consisting of a Hadamard gate and a CNOT gate, as shown below.



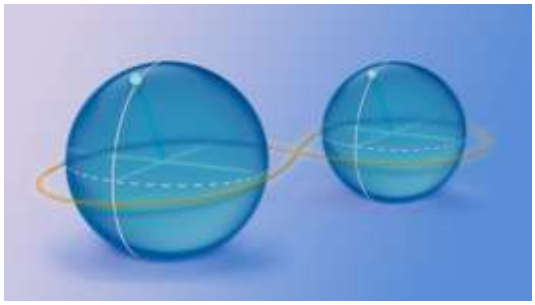

If qubit x starts with a state of , the Hadamard gate puts it into a superposition of and . The CNOT gate flips qubit y depending on the state of qubit x, but since qubit x is in a superposition of and this makes the final state of qubit y dependent on what the final state of qubit x turns out to be — thus the two qubits are part of an entangled state.

# Entanglement

Entanglement is a fundamental concept of quantum mechanics that describes a non-classical correlation, or shared quantum state, between two or more quantum systems (or quantum particles) even if they are separated by a large distance. This phenomenon is also known as quantum non-locality, and it is one of the key features of quantum mechanics that distinguishes it from classical mechanics.

Quantum systems are described by a mathematical object called a wave function, which contains information about the possible outcomes of measurements that can be performed on the systems. When two or more quantum systems are entangled, their wave function cannot be expressed as a product of individual wave functions for each system. Instead, the systems are described by a single wave function that captures the correlation between them.

The fact that entangled systems are described by a single wave function means that any actions or measurements made on one of the systems affect the state of the other systems.



In quantum computing, entanglement is used to enable quantum parallelism, which is the ability of quantum computers to perform multiple calculations simultaneously. Entanglement allows quantum computers to manipulate many qubits in a single operation, instead of manipulating each qubit individually, as in classical computing.

For example, consider two qubits that are initially prepared in an entangled state. If a measurement is made on one of the qubits, and it is found to be in the state $|0\rangle$, then the state of the other qubit immediately collapses to the state $|0\rangle$ as well.

Similarly, if the first qubit is measured to be in the state $|1\rangle$, then the state of the second qubit collapses to the state $|1\rangle$ as well.

Entanglement enables quantum computers to implement various protocols and algorithms that are not possible with classical systems. For example, it is used in quantum teleportation, which allows for the transfer of quantum states between two distant systems.

Entanglement is also a key resource for quantum error correction, which is necessary to protect quantum information from decoherence and other errors. By creating and manipulating entangled states, quantum computers can detect and correct errors in a way that is not possible for classical computers.