UNIT – I – RANDOM VARAIBLES

Random Experiment

An experiment whose output is uncertain even though all the outcomes are known.

Example: Tossing a coin, Throwing a fair die, Birth of a baby.

Sample Space:

The set of all possible outcomes in a random experiment. It is denoted by S.

Example:

For tossing a fair coin, $S = \{H, T\}$

For throwing a fair die, $S = \{1, 2, 3, 4, 5, 6, \}$

For birth of a baby, $S = \{M, F\}$

Event:

A subset of sample space is event. It is denoted by A.

Mutually Exclusive Events:

Two events A and B are said to be mutually exclusive events if they do not occur simultaneously. If A and B are mutually exclusive, then $A \cap B = \Phi$

Example:

Tossing two unbiased coins $S = \{HH, HT, TH, TT\}$

(i) Let $A = \{HH\}, B = \{HT\}$

$$A \cap B = \{H\} \neq \Phi$$

Then A and B are not mutually exclusive.

(i) Let $A = \{HH\}, B = \{TT\}$

$$A \cap B = \Phi$$

Then A and B are mutually exclusive.

Probability:

Probability of an event A is $P(A) = \frac{n(A)}{n(S)}$

i.e., $P(A) = \frac{number\ of\ cases\ favourable\ to\ A}{Total\ number\ of\ cases}$

Axioms of Probability:

(i) $0 \leq P(A) \leq 1$

(ii) $P(S) = 1$

(iii) $P(A \cup B) = P(A) + P(B)$, if A and B are mutually exclusive.

Note:

(i) $P(\phi) = 0$

(ii) $P(\overline{A}) = 1 - P(A)$, for any event A

(iii) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$, for any two events A and B.

Independent events:

Two events A and B are said to be independent if occurrence of A does not affect the occurrence of B.

Condition for two events and B are independent:

$$P(A \cap B) = P(A)\,P(B)$$

Conditional Probability:

If the probability of the event A provided the event B has already occurred is called the conditional probability and is defined as

$$P(A|B) = \frac{P(A \cap B)}{P(B)}, \text{ provided } P(B) \neq 0$$

The probability of an event B provided A has occurred already is given by

$$P(B|A) = \frac{P(A \cap B)}{P(A)}, \text{ provided } P(A) \neq 0$$

Random Variables:

A random variable is a function that assign a real number for all the outcomes in the sample space of a random experiment.

Example:

Toss two coins then the sample space $S = \{HH, HT, TH, TT\}$

Now we define a random variable X to denote the number of heads in 2 tosses.

X(HH) = 2

X(HT) = 1

X(TH) = 1

X(TT) = 0

Types of Random Variables:

(i) Discrete Random Variables

(ii) Continuous Random Variables

Probability mass function (PMF):

Let X be discrete random variable. Then $P(X = x_i) = p(x_i) = p_i$is said to be a Probability mass function of X, if

(i) $0 \le p(x_i) \le 1$

(ii) $\sum_i p(x_i) = 1$

The collection of pairs $\{x_i, p_i\}, i = 1, 2, 3, \ldots$ is called the probability distribution of the random variable X, which is sometimes in the form of a table as given below:

$X = x_i$	x_1	x_2	$\cdots$	x_r	$\cdots$
P($X = x_i$)	p_1	p_2	$\cdots$	p_r	$\cdots$

Problems on Discrete Random Variables

1.A Discrete Random Variable X has the following probability distribution

X	0	1	2	3	4	5	6	7	8
P(x)	a	3a	5a	7a	9a	11a	13a	15a	17a

(i) Find the value of "a".

(ii) Find $P[X < 3], P[0 < X < 3], P\,[X \geq 3]$

(iii) Find the distribution of X.

Solution:

(i) We know that $\sum P(x) = 1$

$$\Rightarrow a + 3a + 5a + 7a + 9a + 11a + 13a + 15a + 17a = 1$$

$$\Rightarrow 81a = 1$$

$$\Rightarrow a = \frac{1}{81}$$

(ii) $P[X < 3] = P[X = 0] + P[X = 1] + P[X = 2]$

$$= a + 3a + 5a$$

$$= 9a$$

$$= \frac{9}{81}$$

$P[0 < X < 3] = P[X = 1] + P[X = 2]$

$$= 3a + 5a$$

$$= 8a$$

$$= \frac{8}{81}$$

$P\,[X \geq 3] = 1 - P[X < 3]$

$$= 1 - \frac{9}{81}$$

$$= \frac{72}{81}$$

(iii) Distribution of X:

X	P(x)	F(X) = P[X≤ x]
0	a	F(0) = P[X≤ 0] = $\frac{1}{81}$
1	3a	F(1) = P[X≤ 1] = $F(0) + P(1) = \frac{1}{81} + \frac{3}{81} = \frac{4}{81}$
2	5a	F(2) = P[X≤ 2] = $F(1) + P(2) = \frac{4}{81} + \frac{5}{81} = \frac{9}{81}$
3	7a	F(3) = P[X≤ 3] = $F(2) + P(3) = \frac{9}{81} + \frac{7}{81} = \frac{16}{81}$
4	9a	F(4) = P[X≤ 4] = $F(3) + P(4) = \frac{16}{81} + \frac{9}{81} = \frac{25}{81}$
5	11a	F(5) = P[X≤ 5] = $F(4) + P(5) = \frac{25}{81} + \frac{11}{81} = \frac{36}{81}$
6	13a	F(6) = P[X≤ 6] = $F(5) + P(6) = \frac{36}{81} + \frac{13}{81} = \frac{49}{81}$
7	15a	F(7) = P[X≤ 7] = $F(6) + P(7) = \frac{49}{81} + \frac{15}{81} = \frac{64}{81}$
8	17a	F(8) = P[X≤ 8] = $F(7) + P(8) = \frac{64}{81} + \frac{17}{81} = \frac{81}{81}$

2. A Discrete Random Variable X has the following probability distribution

X	0	1	2	3	4	5	6	7
P(x)	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

(i) Find the value of "k".

(ii) Find $P[X<6], P[1<X<5], P\ [X\geq 6], P[X>2]$

(iii) Find $P[1.5<X<4.5\ /\ X>2]$

(iv) Find the distribution of X and find the value of k if $P[X<k]>\frac{1}{2}$

Solution:

(i) We know that $\sum P(x)=1$

$$\Rightarrow 0+k+2k+2k+3k+k^2+2k^2+7k^2+k=1$$

$$\Rightarrow 10k^2+9k=1$$

$$\Rightarrow 10k^2+9k-1=0$$

$$\Rightarrow (k+1)(10k-1)=0$$

$$\Rightarrow k=\ -1\ (or)k=\frac{1}{10}$$

(ii) $P\ [X\geq 6]=\ P[X=6]+P[X=7]$

$$=2k^2+7k^2+k$$

$$=9k^2+k$$

$$=\ \frac{9}{100}+\frac{1}{10}$$

$$=\ \frac{19}{100}$$

(iii) $P[X<6]=1-\ P[X\geq 6]$

$$= 1 - \frac{19}{100}$$

$$= \frac{81}{100}$$

(iv) $P[1 < X < 5] = P[X = 2] + P[X = 3] + P[X = 4]$

$$= 2k + 2k + 3k$$

$$= 7k$$

$$= \frac{7}{10}$$

(v) $P[1.5 < X < 4.5 \,/\, X > 2] = \frac{P[1\cdot5 < X < 4\cdot5 \cap X > 2]}{P[X>2]}$

$$= \frac{P[2 < X < 4\cdot5\ ]}{P[X>2]}$$

$$= \frac{P[X=3]+P[X=4]}{P[X>2]}$$

$$= \frac{\frac{5}{10}}{\frac{7}{10}}$$

$$= \frac{5}{7}$$

Distribution of X:

X	**P(x)**	**F(X) = P[X≤ x]**
0	0	F(0) = P[X≤ 0] = 0
1	k	F(1) = P[X≤ 1] $= F(0) + P(1) = 0 + \frac{1}{10} = \frac{1}{10}$

2	2k	F(2) = P[X≤ 2] = $F(1) + P(2) = \frac{1}{10} + \frac{2}{10} = \frac{3}{10}$
3	2k	F(3) = P[X≤ 3] = $F(2) + P(3) = \frac{3}{10} + \frac{2}{10} = \frac{5}{10}$
4	3k	F(4) = P[X≤ 4] = $F(3) + P(4) = \frac{5}{10} + \frac{3}{10} = \frac{8}{10}$
5	k^2	F(5) = P[X≤ 5] = $F(4) + P(5) = \frac{8}{10} + \frac{1}{100} = \frac{81}{100}$
6	$2k^2$	F(6) = P[X≤ 6] = $F(5) + P(6) = \frac{81}{100} + \frac{2}{100} = \frac{83}{100}$
7	$7k^2 + k$	F(7) = P[X≤ 7] = $F(6) + P(7) = \frac{83}{100} + \frac{7}{100} + \frac{1}{10} = \frac{100}{100}$

The value of k = 4 when $P[X < k] > \frac{1}{2}$

3. If the random variable X takes the values 1, 2, 3 and 4 such that $2P(X = 1) = 3P(X = 2) = P(X = 3) = 5P(X = 4)$. Find the probability distribution.

Solution:

Let $2P(X = 1) = 3P(X = 2) = P(X = 3) = 5P(X = 4) = k$

$\Rightarrow 2P(X = 1) = k$

$\Rightarrow P(X = 1) = \frac{k}{2}$

$\Rightarrow 3P(X = 2) = k$

$\Rightarrow P(X = 2) = \frac{k}{3}$

$$\Rightarrow P(X = 3) = k$$

$$\Rightarrow 5P(X = 3) = k$$

$$\Rightarrow P(X = 3) = \frac{k}{5}$$

We know that $\sum P(x) = 1$

$$\Rightarrow P(1) + P(2) + P(3) + P(4) = 1$$

$$\Rightarrow \frac{k}{2} + \frac{k}{3} + k + \frac{k}{5} = 1$$

$$\Rightarrow \frac{15k+10k+30k+6k}{30} = 1$$

$$\Rightarrow \frac{61k}{30} = 1$$

$$\Rightarrow k = \frac{30}{61}$$

The Probability Distribution is

X	1	2	3	4
P(x)	$\frac{k}{2} = \frac{1}{2} \times \frac{30}{61} = \frac{15}{61}$	$\frac{k}{3} = \frac{1}{3} \times \frac{30}{61} = \frac{10}{61}$	$k = \frac{30}{61}$	$\frac{k}{5} = \frac{1}{5} \times \frac{30}{61} = \frac{6}{61}$

4. Suppose that the random variable X assumes three values 0, 1 and 2 with probabilities 1/3, 1/6 and ½ respectively. Obtain the distribution function of X.

Solution:

Values of X = x	0	1	2
P(x)	1/3	1/6	1/2
	P(0)	P(1)	P(2)

The distribution of X

X	**P(x)**	**F(X) = P[X≤ x]**
0	0	F(0) = P[X≤ 0] $= \frac{1}{3}$
1	k	F(1) = P[X≤ 1] $= F(0) + P(1) = \frac{1}{3} + \frac{1}{6} = \frac{1}{2}$
2	2k	F(2) = P[X≤ 2] $= F(1) + P(2) = \frac{1}{2} + \frac{1}{2} = 1$

Mathematical expectation for discrete random variable

Note:

(i) $E(c) = c$

(ii) $Var(c) = 0$

(iii) $E(aX) = aE(X)$

(iv) $E(aX + b) = aE(X) + b$

(v) $Var(aX) = a^2Var(X)$

(vi) $Var(aX \pm b) = a^2Var(X)$

Problems:

If $Var(X) = 4$, find $Var(4X+5)$, where X is a random variable.

Solution:

We know that $Var(aX+b) = a^2 Var(X)$

Here $a = 4, Var(X) = 4$

$Var(4X+5) = 4^2 Var(X) = 16 \times 4 = 64$

Continuous Random Variable:

If X is a random variable4 which can take all the values in an interval then X is called continuous random variable.

Properties of Probability Density Function:

The Probability density function of the random variable X denoted by f(x) has the following properties.

(i) $f(x) \geq 0$

(ii) $\int_{-\infty}^{\infty} f(x)dx = 1$

Cumulative Distribution Function (CDF):

$$F_X(x) = P(X \leq x) = \int_{-\infty}^{x} f(x)dx$$

Properties of CDF:

(i) $F(-\infty) = 0$

(ii) $F(\infty) = 1$

(iii) $\frac{d}{dx}[F(x)] = f(x)$

(iv) $P(X \leq a) = F(a)$

(v) $P(X > a) = 1 - F(a)$

(vi) $P(a \leq X \leq b) = F(b) - F(a)$

Problems on Continuous Random Variables:

1. A continuous random variable X has a density function $f(x) = \frac{K}{1+x^2}$, $-\infty \leq X \leq \infty$. Find the values of K.

Solution:

We know that $\int_{-\infty}^{\infty} f(x)dx = 1$

$$\Rightarrow \int_{-\infty}^{\infty} \frac{K}{1+x^2} dx = 1$$

$$\Rightarrow K \int_{-\infty}^{\infty} \frac{dx}{1+x^2} = 1$$

$$\Rightarrow K \int_{-\infty}^{\infty} \frac{dx}{1+x^2} = 1$$

$$\Rightarrow K[tan^{-1}x]_{-\infty}^{\infty} = 1$$

$$\Rightarrow K[tan^{-1}\infty - tan^{-1}(-\infty)] = 1$$

$$\Rightarrow K\left[\frac{\pi}{2}+\frac{\pi}{2}\right] = 1$$

$$\Rightarrow K\left[\frac{2\pi}{2}\right] = 1$$

$$\Rightarrow K = \frac{1}{\pi}$$

2. If a random variable X has PDF $f(x) = \begin{cases}\frac{1}{4}, |x| < 2 \\ 0, |x| > 2\end{cases}$ Find (i) $P[X < 1]$

(ii) $P[|X| > 1]$ $(iii) P[2X + 3 > 5]$

Solution:

(i) $P[X < 1] = \int_{-2}^{1} f(x)dx$

$$= \int_{-2}^{1} \frac{1}{4} dx$$

$$= \frac{1}{4}\ [x]_{-2}^{1}$$

$$= \frac{1}{4}[1-(-2)]$$

$$= \frac{3}{4}$$

(ii) $P[|X| > 1] = 1 - P[-1 < X < 1]$

$$= 1 - \int_{-1}^{1} f(x)dx$$

$$= 1 - \int_{-1}^{1} \frac{1}{4} dx$$

$$= 1 - \frac{1}{4}\ [x]_{-1}^{1}$$

$$= 1 - \frac{1}{4}[1-(-1)]$$

$$= 1 - \frac{2}{4}$$

$$= \frac{2}{4}$$

(iii) $P[2X + 3 > 5] = P[2X > 5 - 3]$

$$= P\left[X > \frac{5-3}{2}\right]$$

$$= P\left[X > \frac{2}{2}\right]$$

$$= P[X > 1]$$

$$= \int_1^2 f(x)dx$$

$$= \int_1^2 \frac{1}{4} dx$$

$$= \frac{1}{4}\ [x]_1^2$$

$$= \frac{1}{4}[2 - (1)] = \frac{1}{4}$$

Mathematical expectation of continuous random variables

(i) $E(X) = \int_{-\infty}^{\infty} x\, f(x)dx$

(ii) $E(X^2) = \int_{-\infty}^{\infty} x^2\, f(x)dx$

(iii) $Var(X) = E(X^2) - E[(X)]^2$

Problems:

1. Let X be a continuous random variable with probability density function $f(x) = kx(2 - x), 0 < x < 2$. Find (i) k (ii) mean (iii) variance (iv) cumulative distribution function of X (v) rth moment.

Solution:

(i) To find k

$$\int_0^2 f(x)dx = 1 \Rightarrow k\int_0^2 (2x - x^2)dx = 1$$

$$\Rightarrow k\left[\frac{2x^2}{2} - \frac{x^3}{3}\right]_0^2 = 1$$

$$\Rightarrow k\left[4 - \frac{8}{3}\right] = 1$$

$$\Rightarrow k\left(\frac{4}{3}\right) = 1$$

$$\Rightarrow k = \frac{3}{4}$$

(ii) To calculate mean of X

$$E(X) = \int_0^2 x\, f(x)dx$$

$$= \int_0^2 x^2\, \frac{3}{4}(2 - x)dx$$

$$= \frac{3}{4}\int_0^2 (2x^2 - x^3)dx$$

$$= \frac{3}{4}\left[\frac{2x^3}{3} - \frac{x^4}{4}\right]_0^2$$

$$= \frac{3}{4}\left(\frac{16}{3} - 4\right)$$

$$= \frac{3}{4} \times \frac{4}{3} = 1$$

(iii) To calculate variance of X

$$E(X^2) = \int_0^2 x^2\, f(x)dx$$

$$= \int_0^2 x^3\, \frac{3}{4}(2-x)dx$$

$$= \frac{3}{4}\int_0^2 (2x^3 - x^4)dx$$

$$= \frac{3}{4}\left[\frac{2x^4}{4} - \frac{x^5}{5}\right]_0^2$$

$$= \frac{3}{4}\left(8 - \frac{32}{5}\right)$$

$$= \frac{3}{4} \times \frac{8}{5} = \frac{6}{5}$$

$$Var(X) = E(X^2) - E[(X)]^2$$

$$= \frac{6}{5} - 1 = \frac{1}{5}$$

(iv) To calculate CDF of X

$$F_X(x) = P(X \le x) = \int_{-\infty}^{x} f(x)dx$$

$$= \int_0^x f(x)dx$$

$$= \int_0^x \frac{3}{4}x(2-x)dx$$

$$= \frac{3}{4}\int_0^x (2x - x^2)dx$$

$$= \frac{3}{4}\left[\frac{2x^2}{2} - \frac{x^3}{3}\right]_0^x$$

$$= \frac{3}{4}\left(x^2 - \frac{x^3}{2}\right)$$

$$= \frac{1}{4}(3x^2 - x^3)$$

$$F(x) = \begin{cases} 0; & x < 0 \\ \frac{1}{4}(3x^2 - x^3); & 0 \le x < 2 \\ 1; & x \ge 2 \end{cases}$$

(v) To find the rth moment:

$$E(x^r) = \int_{-\infty}^{\infty} x^r\, f(x)dx$$

$$= \int_0^2 x^r\, \frac{3}{4}x(2-x)dx$$

$$= \frac{3}{4}\int_0^2 (2x^{r+1} - x^{r+2})\, dx$$

$$= \frac{3}{4}\left[\frac{2x^{r+2}}{r+2} - \frac{x^{r+3}}{r+3}\right]_0^2$$

$$= \frac{3}{4}\left[\left(2\frac{2^{r+2}}{r+2} - \frac{2^{r+3}}{r+3}\right) - (0-0)\right]$$

$$= \frac{3}{4} \times 2^r 2^2 \left[\frac{1}{r+2} - \frac{1}{r+3}\right]$$

$$= 6 \cdot \frac{2^r}{(r+2)(r+3)}$$

2. The probability distribution function of a random variable X is

$$\boldsymbol{f(x) = \begin{cases} x; & 0 < x < 1 \\ 2 - x; & 1 < x < 2 \\ 0; & x > 2 \end{cases}}$$ **Find the cdf of X.**

Solution:

We know that c.d.f $F(x) = \int_{-\infty}^{x} f(x)dx$

(i) When $0 < x < 1$

$$F(x) = \int_{-\infty}^{0} f(x)dx + \int_{0}^{x} f(x)dx$$

$$= 0 + \int_{0}^{x} x\,dx$$

$$= \left[\frac{x^2}{2}\right]_0^x$$

$$= \frac{x^2}{2} - 0$$

$$= \frac{x^2}{2}$$

(ii) When $1 < x < 2$

$$F(x) = \int_{-\infty}^{0} f(x)dx + \int_{0}^{1} f(x)dx + \int_{1}^{x} f(x)dx$$

$$= 0 + \int_{0}^{1} f(x)dx + \int_{1}^{x} x\,dx$$

$$= \left[\frac{x^2}{2}\right]_0^1 + \left[2x - \frac{x^2}{2}\right]_1^x$$

$$= \left(\frac{1}{2} - 0\right) + \left[\left(2x - \frac{x^2}{2}\right) - \left(2 - \frac{1}{2}\right)\right]$$

$$= \frac{1}{2} + 2x - \frac{x^2}{2} - \frac{3}{2}$$

$$= 2x - \frac{x^2}{2} - 1$$

(iii) When $x > 2$

$$F(x) = \int_{-\infty}^{0} f(x)dx + \int_{0}^{1} f(x)dx + \int_{1}^{2} f(x)dx + \int_{2}^{x} f(x)dx$$

$$= 0 + \int_0^1 f(x)dx + \int_1^2 f(x)dx + \int_2^x x\,dx$$

$$= 0 + \int_0^1 x\,dx + \int_1^2 (2-x)dx + 0$$

$$= \left[\frac{x^2}{2}\right]_0^1 + \left[2x - \frac{x^2}{2}\right]_1^2$$

$$= \left(\frac{1}{2} - 0\right) + \left[(4-2) - \left(2 - \frac{1}{2}\right)\right]$$

$$= \frac{1}{2} + 2 - \frac{3}{2} \quad = 1$$

$$F(x) = \begin{cases} \frac{x^2}{2}; & 0 < x < 1 \\ 2x - \frac{x^2}{2} - 1; & 1 < x < 2 \\ 1; & x > 2 \end{cases}$$

3. The Cumulative distribution function of a random variable X is given by

$$\boldsymbol{F(x) = \begin{cases} 0; & x < 0 \\ x^2; & 0 \le x < \frac{1}{2} \\ 1 - \frac{3}{25}(3-x)^2; & \frac{1}{2} \le x < 3 \\ 1, & x \ge 3 \end{cases}}$$

Find the pdf of X and evaluate $P(|X| \le 1)$ using both pdf and cdf.

Solution:

Given

$$F(x) = \begin{cases} 0; & x < 0 \\ x^2; & 0 \le x < \frac{1}{2} \\ 1 - \frac{3}{25}(3-x)^2; & \frac{1}{2} \le x < 3 \\ 1, & x \ge 3 \end{cases}$$

Pdf id $f(x) = \frac{d}{dx}[F(x)]$

$$f(x) = \begin{cases} 0; & x < 0 \\ 2x; & 0 \leq x < \frac{1}{2} \\ \frac{6}{25}(3-x); & \frac{1}{2} \leq x < 3 \\ 0, & x \geq 3 \end{cases}$$

To find $P(|X| \leq 1)$ using cdf:

$$P(|X| \leq 1) = P(-1 \leq X \leq 1)$$

$$= F(1) - F(-1)$$

$$= \left[1 - \frac{3}{25}(3-1)^2\right] - 0$$

$$= 1 - \frac{12}{25}$$

$$= \frac{25-12}{25} = \frac{13}{25}$$

To find $P(|X| \leq 1)$ using pdf:

$$P(|X| \leq 1) = P(-1 \leq X \leq 1)$$

$$= \int_{-1}^{1} f(x)dx$$

$$= \int_{-1}^{0} f(x)dx + \int_{0}^{\frac{1}{2}} f(x)dx + \int_{\frac{1}{2}}^{1} f(x)dx$$

$$= 0 + \int_{0}^{\frac{1}{2}} 2x\,dx + \int_{\frac{1}{2}}^{1} \frac{6}{25}(3-x)dx$$

$$= 2\left(\frac{x^2}{2}\right)_0^{\frac{1}{2}} + \frac{6}{25}\left[3x - \frac{x^2}{2}\right]_{\frac{1}{2}}^{1}$$

$$= \frac{1}{4} + \frac{6}{25}\left[3 - \frac{1}{2} - \frac{3}{2} + \frac{1}{8}\right]$$

$$= \frac{13}{25}$$

Moment Generating Function:

The moment generating function (MGF) of a random variable "X" (about origin) whose probability function f(x) is given by $M_X(t) = E(e^{tx})$

$$= \begin{cases} \int_{-\infty}^{\infty} e^{tx} f(x)dx, for\ a\ continuous\ random\ variable \\ \sum_{x=-\infty}^{\infty} e^{tx} p(x), for\ a\ discrete\ probability\ distribution \end{cases}$$

Problems:

1. If a random variable "X" has the MGF, $M_X(t) = \frac{2}{2-t}$, find the variance of X.

Solution:

Given $M_X(t) = \frac{2}{2-t} = 2(2-t)^{-1}$

$$M_X'(t) = -2(2-t)^{-2}(-1)$$

$$= 2(2-t)^{-2}$$

$M_X'(t=0) = 2(2-0)^{-2} = \frac{2}{4} = \frac{1}{2}$

$M_X''(t) = -4(2-t)^{-3}(-1)$

$$= 4(2-t)^{-3}$$

$M_X''(t=0) = 4(2-0)^{-3} = \frac{4}{8} = \frac{1}{2}$

$Var(X) = E(X^2) - E[(X)]^2$

$$= \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$$

Moments

The rth moment about origin is $\mu_r' = E[\,x_r']$

First moment about origin $\mu_1' = E[X]$= $E(X^2) - [E(X)]^2$

Variance $\sigma^2 = \mu_2' - (\mu_1')^2$

The rth moment about mean is $\mu_r = E[(X-\mu)^r], where\ \mu\ is\ mean\ of\ X.$

$\Rightarrow \mu_1 = E[(X-\mu)^1]$

$$= E[X] - E[\mu] = \mu - \mu = 0$$

$\Rightarrow \mu_1 = 0$

$\Rightarrow \mu_2 = E[(X-\mu)^2]$

$= E[X^2 + \mu^2 - 2X\mu]$

$= E[X^2] + \mu^2 - 2E[X]\mu$

$= E(X^2) + [E(X)]^2 - 2E(X)E(X)$

$= E(X^2) + [E(X)]^2 - 2[E(X)]^2$

$= E(X^2) - [E(X)]^2 = \sigma^2$

$\Rightarrow \mu_2 = \sigma^2$

1. If the probability density of X is given $f(x) = \begin{cases} 2(1-x) & 0 < x < 1 \\ 0 & otherwise \end{cases}$ Find its r^th^ moment about origin. Hence find evaluate $E[(2X+1)^2]$

Solution:

The r^th^ moment about origin is given by

$\mu_r' = E[\,x_r'] = \int_0^1 x^r\, f(x)dx$

$= \int_0^1 x^r\, 2(1-x)dx$

$= 2\int_0^1 \,(x^r - x^{r+1})dx$

$= 2\left[\frac{x^{r+1}}{r+1} - \frac{x^{r+1+1}}{r+2}\right]_0^1$

$= 2\left[\frac{1}{r+1} - \frac{1}{r+2}\right]$

$$= 2\left[\frac{(r+2)-(r+1)}{(r+2)(r+1)}\right] = \frac{2}{r^2+3r+2}$$

$E[(2X+1)^2] = E[4X^2 + 4X + 1]$

$$= 4E[X^2] + 4E[X] + 1$$

$$= 4\mu_2' + 4\mu_1' + 1$$

$$= 4\frac{2}{2^2+3(2)+2} + 4\frac{2}{2^2+3(2)+2} + 1$$

$$= \frac{8}{12} + \frac{8}{6} + 1 = 3$$

$\therefore E[(2X+1)^2] = 3$

Moment Generating Function (MGF)

Let X be a random variable. Then the MGF of X is $M_X(t) = E[e^{tx}]$

If X is a discrete random variable, then the MGF is given by

$$M_X(t) = \sum_x e^{tx} p(x)$$

If X is a continuous random variable, then the MGF is given by

$$M_X(t) = \int_{-\infty}^{\infty} e^{tx} f(x) dx$$

Define MGF and why it is called so?

Solution:

Let X be a random variable. Then the MGF of X is $M_x(t) = E[e^{tX}]$.

Let X be a continuous random variable. Then

$M_X(t)$= $\int_{-\infty}^{\infty} e^{tx} f(x)dx = \int_{-\infty}^{\infty} \left[1 + \frac{tx}{1!} + \frac{t^2x^2}{2!} + \cdots \frac{t^r x^r}{r!} + \cdots\right] f(x)dx$

$=\int_{-\infty}^{\infty} \left[f(x) + \frac{tx}{1!} f(x) + \frac{t^2x^2}{2!} f(x) + \cdots \frac{t^r x^r}{r!} f(x) + \cdots\right] dx$

$=\int_{-\infty}^{\infty} f(x)dx + \frac{t}{1!}\int_{-\infty}^{\infty} xf(x)dx + \frac{t^2}{2!}\int_{-\infty}^{\infty} x^2 f(x)dx \ldots + \frac{t^r}{r!}\int_{-\infty}^{\infty} x^r f(x)dx + \cdots$

$M_X(t) = 1 + \frac{t}{1!}\mu_1' + \frac{t^2}{2!}\mu_2' + \cdots \frac{t^r}{r!}\mu_r' + \cdots \ldots \ldots$

$\because$ $M_X(t)$ generates moments therefore it is moment generation function

Note:

If X is a discrete RV and if $M_X(t)$ is known, then $\mu_r' = \left[\frac{d^r}{dt^r}[M_X(t)]\right]_{t=0}$

If X is a continuous RV and if $M_X(t)$ is known, then μ_r'

$$= r! \times \text{ coeff of } t^r \text{ in } M_X(t)$$

Problems under MGF of discrete random variable

$$M_X(t) = \sum_x e^{tx} p(x)$$

If X is a discrete RV and if $M_X(t)$ is known, then $\mu_r' = \left[\frac{d^r}{dt^r}[M_X(t)]\right]_{t=0}$

1. Let X be the number occur when a die is thrown. Find the MGF mean and variance of X.

Solution:

x	1	2	3	4	5	6
$p(x)$	1/6	1/6	1/6	1/6	1/6	1/6

(i) $M_X(t) = \sum_{x=1}^{6} e^{tx} p(x)$

$$= e^t P(1) + e^{2t} P(2) + e^{3t} P(3) + e^{4t} P(4) + e^{5t} P(5) + e^{6t} P(6)$$

$$= e^t \frac{1}{6} + e^{2t} \frac{1}{6} + e^{3t} \frac{1}{6} + e^{4t} \frac{1}{6} + e^{5t} \frac{1}{6} + e^{6t} \frac{1}{6}$$

$$M_X(t) = \frac{1}{6}[e^t + e^{2t} + e^{3t} + e^{4t} + e^{5t} + e^{6t}]$$

(ii) $E(X) = \left[\frac{d}{dt} M_X(t)\right]_{t=0} = \frac{1}{6}[e^t + 2e^{2t} + 3e^{3t} + 4e^{4t} + 5e^{5t} + 6e^{6t}]_{t=0}$

$$= \frac{1}{6}[1 + 2 + 3 + 4 + 5 + 6] = \frac{21}{6}$$

$$\Rightarrow E(X) = 3.5$$

$$E(X^2) = \left[\frac{d^2}{dt^2}[M_X(t)]\right]_{t=0}$$

$$= \frac{1}{6}[e^t + 4e^{2t} + 9e^{3t} + 16e^{4t} + 25e^{5t} + 36e^{6t}]$$

$$= \frac{1}{6}(1 + 4 + 9 + 16 + 25 + 36) = \frac{91}{6}$$

$$= 15.1$$

(iii) Variance of $X = E(X^2) - [E(X)]^2 = 15.1 - 12.25$

$$\sigma_X = 2.85$$

2. Find the moment generating function for the distribution

where $(\boldsymbol{X} = \boldsymbol{x}) = \begin{cases} \frac{2}{3}; \boldsymbol{x} = \mathbf{1} \\ \frac{1}{3}; \boldsymbol{x} = \mathbf{2} \\ \mathbf{0} \text{ ; otherwise} \end{cases}$ **. Also find its mean & variance,**

Solution:

The probability distribution of X is given by

x	1	2
$p(x)$	2/3	1/3

$$\Rightarrow M_X(t) = E[e^{tx}] = \sum_{x=1}^{2} e^{tx} p(x)$$

$$= e^t p(X = 1) + e^{2t} p(X = 2) = e^t \frac{2}{3} + e^{2t} \frac{1}{3}$$

$$\Rightarrow M_X(t) = \frac{1}{3}(2e^t + e^{2t})$$

$$\Rightarrow E(X) = M_X'(0)$$

$$= \left[\frac{d}{dt}\left[\frac{1}{3}(2e^t + e^{2t})\right]\right]_{t=0}$$

$$= \frac{1}{3}(2e^t + 2e^{2t})$$

$$\Rightarrow E(X) = \frac{4}{3}$$

$$\Rightarrow E(X^2) = M_X''(0) = \left[\frac{d^2}{dt^2}\left[\frac{1}{3}(2e^t + e^{2t})\right]\right]_{t=0}$$

$$= \left[\frac{d}{dt}\left[\frac{1}{3}(2e^t + 2e^{2t})\right]\right]_{t=0}$$

$$= \left[\frac{1}{3}(2e^t + 4e^{2t})_{t=0}\right] = \frac{6}{3} = 2$$

Variance of $X = E(X^2) - [E(X)]^2 = 2 - \left(\frac{4}{3}\right)^2$

$$\Rightarrow \text{Var}(X) = \frac{2}{9}$$

3. Let X be a RV with PMF $P(x) = \left(\frac{1}{2}\right)^x$; $x = 1, 2, 3, \dots$. Find MGF and hence find mean and variance of X.

Solution:

(i) $M_X(t) = E[e^{tX}]$

$$= \sum_{x=1}^{\infty} e^{tx} p(x)$$

$$= \sum_{x=1}^{\infty} e^{tx} \left(\frac{1}{2}\right)^x$$

$$= \sum_{x=1}^{\infty} \left(\frac{e^t}{2}\right)^x$$

$$= \left[\frac{e^t}{2} + \left(\frac{e^t}{2}\right)^2 + \left(\frac{e^t}{2}\right)^3 + \cdots\right]$$

$$= \frac{e^t}{2}\left(1 + \frac{e^t}{2} + \left(\frac{e^t}{2}\right)^2 + \cdots\right)$$

$$= \frac{e^t}{2}\left(1 - \frac{e^t}{2}\right)^{-1}$$

$$= \frac{e^t}{2}\frac{e^t}{2-e^t}$$

$$\Rightarrow M_X(t) = \frac{e^t}{2-e^t}$$

(ii) $E(X) = \left[\frac{d}{dt}[M_X(t)]\right]_{t=0}$

$$= \left[\frac{d}{dt}\left(\frac{e^t}{2-e^t}\right)\right]_{t=0}$$

$$= \left[\frac{(2-e^t)e^t - e^t(0-e^t)}{(2-e^t)^2}\right]_{t=0}$$

$$= \left[\frac{2e^t - e^{2t} + e^{2t}}{(2-e^t)^2}\right]_{t=0} \quad \because d\left(\frac{u}{v}\right) = \frac{vu' - uv'}{v^2}$$

$$= \left[\frac{2e^t}{(2-e^t)^2}\right]_{t=0}\Bigg] = \frac{2}{1}$$

$$\Rightarrow E(X) = 2$$

$$E(X^2) = \left[\frac{d^2}{dt^2}[M_X(t)]\right]_{t=0}$$

$$= \left[\frac{d}{dt}\left(\frac{2e^t}{(2-e^t)^2}\right)\right]_{t=0}$$

$$= \left[\frac{(2-e^t)^2 2e^t - 2e^t 2(2-e^t)(-e^t)}{(2-e^t)^4}\right]_{t=0} = \frac{2+4}{1} = 6$$

(iii) Variance $= E(X^2) - [E(X)]^2 = 6 - 4$

$$\mathrm{Var}(X) = 2$$

Problems under MGF of discrete random variable

$$M_X(t) = \int_{-\infty}^{\infty} e^{tx} f(x)dx$$

If X is a continuous RV and if $M_X(t)$ is known, then μ_r'

$$= r! \times \text{ coeff of } t^r \text{ in } M_X(t)$$

1. If a random variable "X" has the MGF, $M_X(t) = \frac{2}{2-t}$, find the variance of X.

Solution:

Given $M_X(t) = \frac{2}{2-t} = 2(2-t)^{-1}$

$$M_X{}'(t) = -2(2-t)^{-2}(-1)$$

$$= 2(2-t)^{-2}$$

$M_X{}'(t=0) = 2(2-0)^{-2} = \frac{2}{4} = \frac{1}{2}$

$M_X{}''(t) = -4(2-t)^{-3}(-1)$

$$= 4(2-t)^{-3}$$

$$M_X''(t=0) = 4(2-0)^{-3} = \frac{4}{8} = \frac{1}{2}$$

$$Var(X) = E(X^2) - E[(X)]^2$$

$$= \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$$

2. Let X be a RV with PDF $f(x) = ke^{-2x}, x \geq 0$. Find (i) k, (ii) MGF, (iii) Mean and (iv) variance

Solution:

Given $f(x) = ke^{-2x}; 0 \leq x < \infty$

(i) To find k

$$\Rightarrow \int_0^\infty f(x)dx = 1$$

$$\Rightarrow \int_0^\infty ke^{-2x}dx = 1$$

$$\Rightarrow k\left[\frac{e^{-2x}}{-2}\right]_0^\infty = 1$$

$$\Rightarrow \frac{k}{-2}(e^{-\infty} - 1) = 1$$

$$\Rightarrow \frac{k}{-2}(0-1) = 1$$

$$\Rightarrow \frac{k}{2} = 1$$

$\Rightarrow k = 2$

(ii) $M_X(t) = E[e^{tx}]$

$$= \int_0^\infty e^{tx} f(x) dx$$

$$= 2\int_0^\infty e^{tx} e^{-2x} dx = 2\int_0^\infty e^{tx-2x} dx$$

$$= 2\int_0^\infty e^{-(2-t)x} dx = 2\left[\frac{e^{-(2-t)x}}{-(2-t)}\right]_0^\infty = 2\left(0 + \frac{1}{2-t}\right)$$

$M_X(t) = \frac{2}{2-t}$

(iii) To find Mean and Variance

$$M_X(t) = \frac{2}{2\left(1 - \frac{t}{2}\right)} = \left(1 - \frac{t}{2}\right)^{-1} = 1 + \frac{t}{2} + \frac{t^2}{2^2} + \cdots$$

Coefficient of $t = \frac{1}{2}$ Coefficient of $t^2 = \frac{1}{2^2}$

Mean $E(X) = \mu_1' = 1! \times$ coefficient of $t \Rightarrow E(X) = \frac{1}{2}$

$E(X^2) = 2! \times$ coefficient of $t^2 = 2 \times \frac{1}{2^2} = \frac{1}{2}$

(iv) Variance $= E(X^2) - [E(X)]^2 = \frac{1}{2} - \frac{1}{4} = \frac{2-1}{4}$

Var$(X) = \frac{1}{4}$

3. Let X be a continuous RV with PDF $f(x) = Ae^{\frac{-x}{3}}; x \geq 0$. Find (i) A, (ii) MGF,(iii) Mean and (iv) variance

Solution:

Given $f(x) = Ae^{\frac{-x}{3}}; 0 \leq x \leq \infty$

(i) To find A

$\Rightarrow \int_0^\infty f(x)dx = 1$

$\Rightarrow \int_0^\infty Ae^{\frac{-x}{3}}dx = 1$

$\Rightarrow A\left[\frac{e^{\frac{-x}{3}}}{-\frac{1}{3}}\right]_0^\infty = 1$

$\Rightarrow -3A(0-1) = 1$

$\Rightarrow 3A = 1 \Rightarrow A = \frac{1}{3}$

$\therefore f(x) = \frac{1}{3}e^{\frac{-x}{3}}; 0 \leq x \leq \infty$

(ii) $M_X(t) = E[e^{tx}] = \int_0^\infty e^{tx} f(x)dx = \frac{1}{3}\int_0^\infty e^{tx}e^{\frac{-x}{3}}dx$

$= \frac{1}{3}\int_0^\infty e^{tx-\frac{x}{3}}dx = \frac{1}{3}\int_0^\infty e^{-\left(\frac{1}{3}-t\right)x}dx = \frac{1}{3}\left[\frac{e^{-\left(\frac{1}{3}-t\right)x}}{-\left(\frac{1}{3}-t\right)}\right]_0^\infty$

$= \frac{1}{3}\left[0 + \frac{1}{\frac{1}{3}-t}\right] = \frac{1}{3}\frac{1}{\frac{1-3t}{3}}$

$= (1-3t)^{-1}$

(iii) To find mean and variance:

$M_X(t) = (1-3t)^{-1}$

$$= 1 + 3t + 9t^2 + 27t^3 + \cdots$$

coefficient of $t = 3$

coefficient of $t^2 = 9$

$E(X) = 1!$ X coefficient of t in $M_X(t) = 1$ X 3

Mean = 3

$E(X) = 2!$ X coefficient of t^2 in $M_X(t)$

$$= 2 \times 9 = 18$$

(iv)Variance = $E(X^2) - [E(X)]^2 = 18-9$

$$Var\,(X) = 9$$

4. Let X be a continuous random variable with the pdf

$\boldsymbol{f(x) = \begin{cases} x & ; 0 < x < 1 \\ 2 - x & ; 1 < x < 2 \\ 0 & ; \text{elsewhere} \end{cases}}$ **. Find (i) MGF, (ii) Mean and variance.**

Solution:

Since X is defined in the region $0 < x < 2, X$ is a continuous RV.

$$M_X(t) = E[e^{tX}] = \int_0^2 e^{tx} f(x) dx$$

$$= \int_0^1 e^{tx} x dx + \int_1^2 e^{tx}(2 - x) dx$$

$$= \int_0^1 xe^{tx}dx + \int_1^2 (2-x)e^{tx}dx$$

$$= \left[x\left(\frac{e^{tx}}{t}\right) - 1\left(\frac{e^{tx}}{t^2}\right)\right]_0^1 + \left[(2-x)\frac{e^{tx}}{t} - (-1)\frac{e^{tx}}{t^2}\right]_1^2$$

$$= \left[1\left(\frac{e^{t}}{t}\right) - 1\left(\frac{e^{t}}{t^2}\right) - \left(\frac{-1}{t^2}\right)\right] + \left[0 + \frac{e^{2t}}{t^2} - \frac{e^t}{t} - \frac{e^t}{t^2}\right]$$

$$= \left[\frac{e^t}{t} - \frac{e^t}{t^2} + \frac{1}{t^2} + \frac{e^{2t}}{t^2} - \frac{e^t}{t} - \frac{e^t}{t^2}\right] = \frac{1}{t^2} - \frac{2e^t}{t^2} + \frac{e^{2t}}{t^2}$$

$$M_X(t) = \frac{1-2e^t+e^{2t}}{t^2}$$

To find Mean and Variance:

$$M_X(t) = \frac{1}{t^2}\left[1 - 2\left(1 + \frac{t}{1!} + \frac{t^2}{2!} + \frac{t^3}{3!} + \frac{t^4}{4!} + \cdots\right)\right.$$
$$\left. + \left(1 + \frac{2t}{1!} + \frac{2^2t^2}{2!} + \frac{2^3t^3}{3!} + \frac{2^4t^4}{4!} + \cdots\right)\right]$$

$$\mu_r' = r! \times \text{coefficient of } t^r$$

Coefficient of $t = -\frac{2}{3!} + \frac{2^3}{31} = \frac{-2}{6} + \frac{8}{6} = 1$

Coefficient of $t^2 = -\frac{2}{4!} + \frac{2^4}{4!} = \frac{14}{24} = \frac{7}{12}$

$$\mu_1' = 1! \times \text{coefficient of } t$$

$$\mu_1' = 1$$

Mean $= 1$

$\mu_2' = 2! \times$ coefficient of t^2; $\mu_2' = 2 \times \frac{7}{12} = \frac{7}{6}$

Variance $= \mu_2' - (\mu_1')^2 = \frac{7}{6} - 1 = \frac{1}{6}$

5. Let X be a continuous random variable with PDF $f(x)\frac{1}{2a}; -a < x < a$. Then find the M.G.F of X.

Solution:

Let X is a continuous random variable defined in $-a < x < a$.

$$M_x(t) = E[e^{tx}]$$

$$= \int_{-a}^{a} e^{tx} f(x) dx$$

$$= \int_{-a}^{a} e^{tx} \frac{1}{2a} dx$$

$$= \frac{1}{2a}\left(\frac{e^{tx}}{t}\right)^{a} \quad e^x - e^{-x} = 2\sin hx$$

$$= \frac{1}{2a}\left(\frac{e^{ta} - e^{-ta}}{t}\right)$$

$$= \frac{1}{2a}\frac{2\sinh at}{t}$$

$$M_X(t) = \frac{\sinh at}{at}$$

Bayes Theorem

State and Prove Bayes Theorem

(OR)

State and Prove Theorem of Probability of Causes.

Statement

If $\boldsymbol{B_1, B_2}, \ldots$ B_n be a set of exhaustive and mutually exclusive events associated with random experiment and $\boldsymbol{A}$ is another event associated with (or caused)by Bi. Then

$$\boldsymbol{P(A \mid B_i) = \frac{P(B_i) \cdot P(A/B_i)}{\sum_{i=1}^{n} P(B_i) P(A/B_i)}}$$

Proof:

Given $B_1, B_2, \ldots, B_n$ are mutually exclusive events

$A \cap B_1, A \cap B_2, \ldots, A \cap B_n$ are mutually exclusive events

Let $A = (A \cap B_1) \cup (A \cap B_2) \cup \ldots \cup (A \cap B_n)$

By addition theorem,

$\Rightarrow P(A) = P(A \cap B_1) + P(A \cap B_2) + \ldots + P(A \cap B_n)$

$\Rightarrow P(A) = \sum_{i=1}^{n} P(A \cap B_i)$

$$\Rightarrow P(A \cap B_i) = P(B) \cdot P\left({}^{A}/_{B}\right)$$

$$\Rightarrow P(B_i \mid A) = \frac{P(B_i) \cdot P(A/B_i)}{P(A)} \ldots \ldots \ldots (1)$$

Substitute $P[A]$ in eqn (1)

$$(1) \Rightarrow P[B_i/A] = \frac{P(B_i) \cdot P(A/B_i)}{\sum_{i=1}^{n} P(B_i) P(A/B_i)}$$

Hence the proof.

1.(a) Four boxes A, B, C, D contain fuses. The boxes contain 5000, 3000, 2000 and 1000 fuses respectively. The percentages of fuses in boxes which are defective are 3%, 2%, 1% and 0.5% respectively.one fuse in selected at random arbitrarily from one of the boxes. It is found to be defective fuse. Find the probability that it has come from box D.

(OR)

(b) Four boxes A, B, C, D contain fuses. Box A contain 5000 fuses, box B contain 3000 fuses, box C contain 2000 fuses and box D contain 1000 fuses. The percentage of fuses in boxes which are defective are 3%, 2%, 1% and 0.5% respectively. One fuse is select at random from one of the boxes. It is found to be defective fuse. What is the probability that it has come from box *D*.

Solution:

Since selection ratio is not given

Assume selection ratio is $1 : 1 : 1 : 1$

Total $= 1 + 1 + 1 + 1 = 4$

$\Rightarrow P(A) = \frac{1}{4}$

$\Rightarrow P(B) = \frac{1}{4}$

$\Rightarrow P(C) = \frac{1}{4}$

$\Rightarrow P(D) = 1/4$

Let E be the event selecting a defective fuse from any one of the machine

$\Rightarrow P(E/A) = 3\% = 0.03$

$\Rightarrow P(E/B) = 2\% = 0.02$

$\Rightarrow P(E/C) = 1\% = 0.01$

$\Rightarrow P(E/D) = 5\% = 0.05$

$P(E) = P(A)P(E/A) + P(B)P(E/B) + P(C)P(F/\mathrm{C}) + P(D)(F/\mathrm{D})$

$= \frac{1}{4} \times 0.03 + 1/4 \times 0.02 + 1/4 \times 0.01 + 1/4 \times 0.05$

$= 0.0275$

$$P(D/E) = \frac{P(D)P(E/D)}{P(E)}$$

$$= \frac{\frac{1}{4}\times 0.05}{0.0275} = 0.4545$$

$$= 0.4545$$

2. (a) In a bolt Factory, Machines A, B and C manufacture respectively 25%, 35% and 40% of total output. also out of these output of A, B, C are 5, 4, 2 percent respectively are defective. A bolt is drawn at random from the total output and it is found to be defective. What is the probability that it was manufactured by the machine B?

(OR)

(b) In a company machine A, B and C manufactured bolts, 25%, 35% and 40% of total output. also out of these output of A, B, C are 5,4,2 percent respectively are defective. A bolt is taken random from the total output and it is found to be defective. Find the probability that it was manufactured by the machine B?

Solution:

Given ,P(E_1) =P(A) =25% = 0.25

$\Rightarrow P(E_2) = P(B) = 35\% = 0.35$

$\Rightarrow P(E_3) = P(C) = 40\% = 0.40$

Let D be the event of drawing defective bolt

$\Rightarrow P(D/E_1) = 5\% = \frac{5}{100} = 0.05$

$\Rightarrow P(D/E_2) = 4\% = 0.04$

$\Rightarrow P(D/E_3) = 2\%) = 0 \cdot 02$

To find $P(E_2/D)$

By Bayes theorem

$$P(E_2/D) = \frac{P(E_2)P(D/E_2)}{P(E_1)\, P(D/E_1) + P(E_2)P(D/E_2) + P(E_3)P(D/E_3)}$$

$$= \frac{(0.35)(0.04)}{(0.25)(0.05) + (0.35)(0.04) + (0.4)(0.02)}$$

$$= \frac{0.014}{0.0345}$$

$$= 0.406$$

3. (a) A bag A contains 2 white and 3 red balls and a bag B contains 4 white and 5 red balls. One ball is drawn at random from one of the bag and is found to be red. Find the Probability that it was drawn from bag B

(OR)

(b) A box A contains 2 white and 3 red balls and a box B contains 4 white and 5 red balls at random one ball is taking and is found to be red. What is the probability that it was drawn from bag B?

Solution:

Let B_1 be the event that the ball is drawn from the bag A.

Let B_2 be the event that the ball is drawn from the bag B.

Let A be the event that the drawn ball is red

$\Rightarrow P(B_1) = P(B_2) = \frac{1}{2}$

$\Rightarrow P(A/B_1) = \frac{3C_1}{5C_1} = \frac{3}{5}$

$\Rightarrow P(A/B_2) = \frac{5C_1}{9C_1} = \frac{5}{9}$

$$P(B_2/A) = \frac{P(B_2)P(A/B_2)}{P(B_1)P(A/B_1) + P(B_2)P(A/B_2)}$$

$$= \frac{(\frac{1}{2})(\frac{5}{9})}{\left(\frac{1}{2}\right)\left(\frac{3}{5}\right)+(\frac{1}{2})(\frac{5}{9})}$$

$$= \frac{\frac{5}{18}}{\frac{52}{90}}$$

$\Rightarrow P(B_2/A)= \frac{25}{52}$