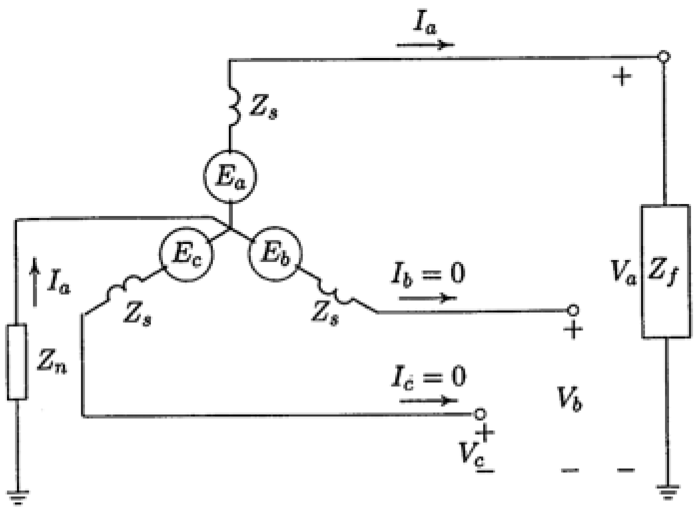
Single Line to Ground Fault:

Single Line to Ground Fault at F in a power system through a fault impedance Z^f. The phases are so labelled that the fault occurs on phase a.

At the fault point F, the currents out of the power system and the Single Line to Ground Fault are constrained as follows



Suppose a line-to-ground fault occurs on phase a through impedance Z_f. Assuming the generator is initially on no-load, the boundary conditions at the fault point are

$$V_a = Z_f I_a$$
$$I_b = I_c = 0$$

Substituting for V_a^0, V_a^1, and V_a^2 from (10.54) and noting $I_a^0 = I_a^1 = I_a^2$,

$$V_a = E_a - (Z^1 + Z^2 + Z^0) I_a^0$$

Substituting for $I_b = I_c = 0$, the symmetrical components of currents are

$$\begin{bmatrix} I_a^0 \\ I_a^1 \\ I_a^2 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_a \\ 0 \\ 0 \end{bmatrix}$$

From the above equation, we find that

$$I_a^0 = I_a^1 = I_a^2 = \frac{1}{3} I_a$$

Phase a voltage in terms of symmetrical components is

$$V_a = V_a^0 + V_a^1 + V_a^2$$

Substituting for V_a^0, V_a^1, and V_a^2 from (10.54) and noting $I_a^0 = I_a^1 = I_a^2$

$$V_a = E_a - (Z^1 + Z^2 + Z^0)I_a^0$$

where $Z^0 = Z_s + 3Z_n$. Substituting for V_a from (10.55), and noting I_a
get

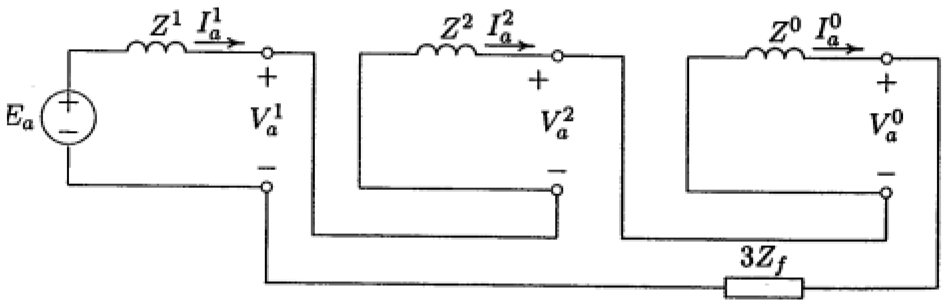
$$3Z_f I_a^0 = E_a - (Z^1 + Z^2 + Z^0)I_a^0$$

or

$$I_a^0 = \frac{E_a}{Z^1 + Z^2 + Z^0 + 3Z_f}$$

The fault current is

$$I_a = 3I_a^0 = \frac{3E_a}{Z^1 + Z^2 + Z^0 + 3Z_f}$$



LINE –LINE FAULT

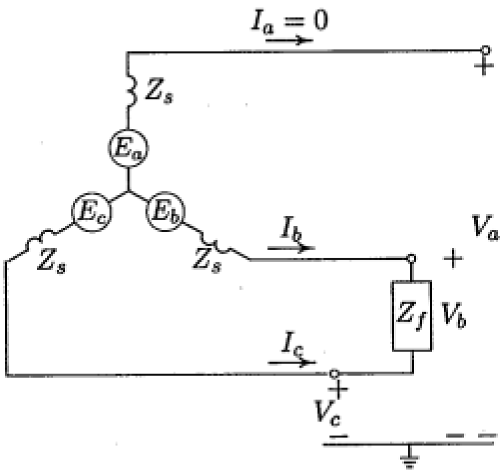
$$V_b - V_c = Z_f I_b$$
$$I_b + I_c = 0$$
$$I_a = 0$$

Substituting for $I_a = 0$, and $I_c = -I_b$, the symmetrical c
from (10.14) are

$$\begin{bmatrix} I_a^0 \\ I_a^1 \\ I_a^2 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} 0 \\ I_b \\ -I_b \end{bmatrix}$$

From the above equation, we find that

$$I_a^0 = 0$$
$$I_a^1 = \frac{1}{3}(a - a^2)I_b$$
$$I_a^2 = \frac{1}{3}(a^2 - a)I_b$$



$$I_a^1 = -I_a^2$$

From (10.16), we have

$$V_b - V_c = (a^2 - a)(V_a^1 - V_a^2)$$
$$= Z_f I_b$$

Substituting for V_a^1 and V_a^2 from (10.54) and noting $I_a^2 = -I_a^1$, we get

$$(a^2 - a)[E_a - (Z^1 + Z^2)I_a^1] = Z_f I_b$$

Substituting for I_b from (10.69), we get

$$E_a - (Z^1 + Z^2)I_a^1 = Z_f \frac{3I_a^1}{(a - a^2)(a^2 - a)}$$

Since $(a - a^2)(a^2 - a) = 3$, solving for I_a^1 results in

$$I_a^1 = \frac{E_a}{Z^1 + Z^2 + Z_f}$$

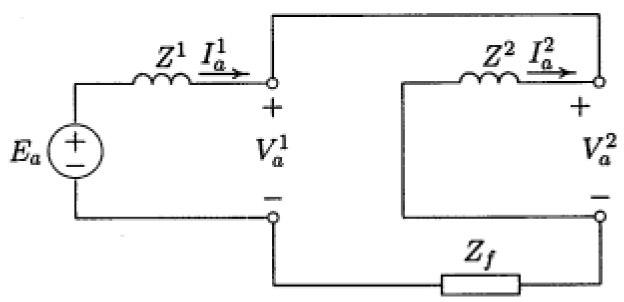
The phase currents are

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ I_a^1 \\ -I_a^1 \end{bmatrix}$$

$$I_b = -I_c = (a^2 - a)I_a^1$$

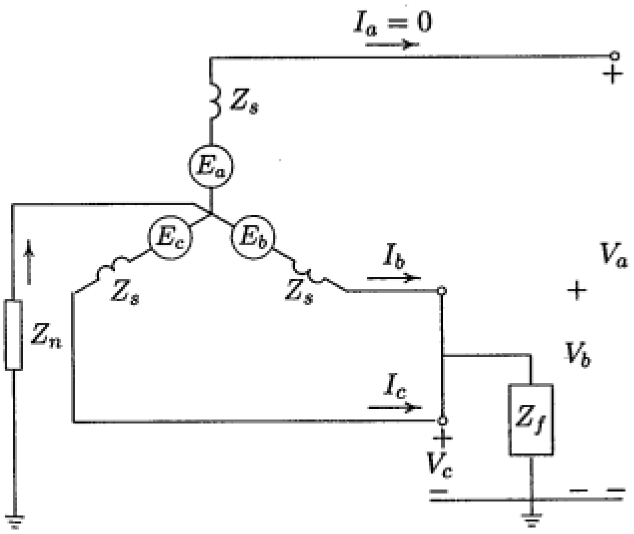
or

$$I_b = -j\sqrt{3}\, I_a^1$$



DOUBLE LINE TO GROUND FAULT

$$V_b = V_c = Z_f(I_b + I_c)$$
$$I_a = I_a^0 + I_a^1 + I_a^2 = 0$$



$$V_b = V_a^0 + a^2 V_a^1 + a V_a^2$$
$$V_c = V_a^0 + a V_a^1 + a^2 V_a^2$$

Since $V_b = V_c$, from above we note that

$$V_a^1 = V_a^2$$

$$\begin{aligned} V_b &= Z_f(I_a^0 + a^2 I_a^1 + a I_a^2 + I_a^0 + a I_a^1 + a^2 I_a^2) \\ &= Z_f(2I_a^0 - I_a^1 - I_a^2) \\ &= 3Z_f I_a^0 \end{aligned}$$

$$I_a^1 = \frac{E_a}{Z^1 + \frac{Z^2(Z^0 + 3Z_f)}{Z^2 + Z^0 + 3Z_f}}$$

$$\begin{aligned} 3Z_f I_a^0 &= V_a^0 + (a^2 + a)V_a^1 \\ &= V_a^0 - V_a^1 \end{aligned}$$