

Evaluation Metrics: MSE, MAE, and R² Score

Evaluation metrics are essential in machine learning and statistical modeling because they help measure how well a model performs. In regression problems, where the objective is to predict continuous numerical values, evaluation metrics compare the predicted values generated by the model with the actual values present in the dataset. Among the most commonly used regression evaluation metrics are Mean Squared Error (MSE), Mean Absolute Error (MAE), and R-squared (R² Score).

These metrics provide insights into the accuracy and reliability of a predictive model. Choosing the right metric helps data scientists and researchers understand the strengths and weaknesses of their models and make necessary improvements.

1. Mean Squared Error (MSE)

Definition

Mean Squared Error (MSE) is a regression evaluation metric that calculates the average of the squared differences between actual values and predicted values. Squaring the errors ensures that negative and positive errors do not cancel each other out and gives greater importance to larger errors.

Formula

$$[MSE = \frac{1}{n} \sum_{i=1}^n (y_i - \hat{y}_i)^2]$$

Where:

- (n) = Number of observations
- (y_i) = Actual value
- (\hat{y}_i) = Predicted value

Working

The MSE calculation involves the following steps:

1. Find the difference between actual and predicted values.
2. Square each difference.

3. Add all squared differences.
4. Divide the sum by the total number of observations.

Example

Actual Values: 10, 20, 30

Predicted Values: 12, 18, 33

Errors:

- $10 - 12 = -2$
- $20 - 18 = 2$
- $30 - 33 = -3$

Squared Errors:

- 4
- 4
- 9

MSE: $\left[\frac{4+4+9}{3} = \frac{17}{3} = 5.67\right]$

Advantages

- Penalizes large errors heavily.
- Useful when large mistakes are particularly undesirable.
- Widely used in machine learning optimization algorithms.

Disadvantages

- Sensitive to outliers.
- Units become squared, making interpretation less intuitive.

Applications

- House price prediction
- Sales forecasting
- Weather prediction
- Financial analysis

2. Mean Absolute Error (MAE)

Definition

Mean Absolute Error (MAE) measures the average absolute difference between actual and predicted values. Unlike MSE, MAE does not square the errors. It treats all errors equally regardless of their magnitude.

Formula

$$[MAE = \frac{1}{n} \sum_{i=1}^n |y_i - \hat{y}_i|]$$

Where:

- (n) = Number of observations
- (y_i) = Actual value
- (\hat{y}_i) = Predicted value

Working

The MAE calculation involves:

1. Finding the difference between actual and predicted values.
2. Taking the absolute value of each difference.
3. Adding all absolute differences.
4. Dividing by the total number of observations.

Example

Actual Values: 10, 20, 30

Predicted Values: 12, 18, 33

Absolute Errors:

- $|10 - 12| = 2$
- $|20 - 18| = 2$
- $|30 - 33| = 3$

MAE:

$$\left[\frac{2+2+3}{3}=\frac{7}{3}=2.33\right]$$

Advantages

- Easy to understand and interpret.
- Less sensitive to outliers than MSE.
- Expressed in the same units as the target variable.

Disadvantages

- Does not heavily penalize large errors.
- May not be suitable when large prediction errors are critical.

Applications

- Demand forecasting
- Inventory management
- Economic predictions
- Performance measurement of regression models

3. R-Squared (R² Score)

Definition

R² Score, also known as the Coefficient of Determination, measures how well the regression model explains the variability in the target variable. It indicates the proportion of variance in the dependent variable that can be predicted from the independent variables.

Formula

$$[R^2 = 1 - \frac{\sum(y_i - \hat{y}_i)^2}{\sum(y_i - \bar{y})^2}]$$

Where:

- (y_i) = Actual value
- (\hat{y}_i) = Predicted value
- (\bar{y}) = Mean of actual values

Interpretation

- $R^2 = 1$: Perfect prediction
- $R^2 = 0$: Model performs no better than the mean
- $R^2 < 0$: Model performs worse than a simple average prediction

Example

Suppose a model has an R² score of 0.85.

This means that 85% of the variation in the target variable is explained by the model, while the remaining 15% is unexplained.

Advantages

- Provides an overall measure of model fit.
- Easy to compare multiple regression models.
- Indicates explanatory power of the model.

Disadvantages

- Does not indicate whether predictions are unbiased.

- Can be misleading when used alone.
- May increase when unnecessary variables are added.

Applications

- Linear regression analysis
- Predictive analytics
- Business forecasting
- Scientific research

Comparison of MSE, MAE, and R² Score

Metric	Full Form	Best Value	Error Sensitivity	Interpretation
MSE	Mean Squared Error	0	High	Lower value indicates better performance
MAE	Mean Absolute Error	0	Moderate	Lower value indicates better performance
R ² Score	Coefficient of Determination	of 1	Not Based	Error- Higher value indicates better performance

Key Differences

1. MSE squares errors, whereas MAE uses absolute errors.
2. MSE penalizes larger errors more heavily than MAE.
3. MAE is easier to interpret because it remains in the original unit.
4. R² measures explained variance rather than prediction error.
5. MSE and MAE should be minimized, while R² should be maximized.