1.5 SOLVED PROBLEMS

1. **Determine whether the following systems are linear or not ▯(▯) = ▯ (▯ − ▯) + ▯(▯▯).**

   Solution:

Output due to weighted sum of inputs:

$$y_3(n) = ax_1(n-2) + bx_2(n-2) + ax_1(n^2) + bx_2(n^2) \text{ ------ } (1)$$

Weighted sum of outputs:

For input $x_1(n)$:

$$y_1(n) = x_1(n-2) + x_1(n^2)$$

For input $x_2(n)$:

$$y_2(n) = x_2(n-2) + x_2(n^2)$$

$$ay_1(n) + by_2(n) = ax_1(n-2) + ax_1(n^2) + bx_2(n-2) + bx_2(n^2) \text{ ------------ } (2)$$

$$\because\ y_3(n) = ay_1(n) + by_2(n)$$

(1)=(2)

∴∴∴∵∴The system is Linear.

2. **Determine whether the following systems are linear or not**

   $\frac{dy(t)}{dt} + 3ty(t) = t^2x(t).$

   Solution:

   Condition for Linearity:

$$T[ax_1(t) + bx_2(t)] = ay_1(t) + by_2(t)$$

$T[ax1(t) + bx2(t)]$→

$$\frac{d}{dt}[ay_1(t) + by_2(t)] + 3t[ay_1(t) + by_2(t)]$$

$$= t^2[ax_1(t) + bx_2(t)] ----(1)$$

$$ay_1(t) \rightarrow \frac{ady_1(t)}{dt} + 3tay_1(t) = t^2ax_1(t) ----(2)$$

$$ay_2(t) \rightarrow \frac{ady_2(t)}{dt} + 3tay_2(t) = t^2ax_2(t) ----(3)$$

Adding equ (2) and (3) we get,

$$\frac{d}{dt}[ay_1(t) + by_2(t)] + 3t[ay_1(t) + by_2(t)]$$
$$= t^2[ax_1(t) + bx_2(t)] - - - -(4)$$

(1)=(4)

∴ This is a Linear system.

3. Determine whether the following systems are static or dynamic

□(□) = □(□□) + □□(□).

Solution:

For t=0, $y(0) = x(0) + 2x(0) \Rightarrow$ present inputs

For t=-1, $y(-1) = x(-2) + 2x(-1) \Rightarrow$ past and present inputs

For t=1, $y(1) = x(2) + 2x(1) \Rightarrow$ future and present inputs

Since output depends on past and future inputs the given system is dynamic system.

4. Determine whether the following systems are static or dynamic

□(□) = □□□□(□).

Solution:

For n=0, $y(0) = sinx(0) \Rightarrow$ present input

For n=-1, $y(-1) = sinx(-1) \Rightarrow$ present input

For n=1, $y(1) = sinx(1) \Rightarrow$ present input

Since output depends on present input the given system is Static system

5. Determine whether the following systems are time invariant or not

□(□) = □(□)□□□□□.

Solution:

Output due to input delayed by T seconds

$$y(t, T) = x(t - T)sinwt$$

Output delayed by T seconds

$$y(t - T) = x(t - T)sinw(t - T)$$

$\because\ y(t,T) \neq\ y(t-T)$

The given system is time variant

6. Determine whether the following systems are time invariant or not

□(□) = □(−□ + □).

Solution:

Output due to input delayed by k seconds

$$y(n,k) = x(-n+2-k)$$

Output delayed by k seconds

$$y(n-k) = x(-(n-k)+2) = x(-n+k+2)$$

$$\because\ \boldsymbol{y(n,k) \neq\ y(n-k)}$$

The given system is time variant

7. Determine whether the following systems are causal or not

$$\boldsymbol{y(t) = \frac{dx(t)}{dt} + 2x(t)}$$

Solution:

The given equation is differential equation and the output depends on past input. Hence thegiven system is **Causal**.

8. Determine whether the following systems are causal or not

□(□) = □□□□(□)

Solution:

For n=0, $y(0) =\ sinx(0)\ \Rightarrow$ present input

For n= -1, $y(-1) =\ sinx(-1)\quad \Rightarrow$ present input

For n=1, $y(1) =\ sinx(1)\ \Rightarrow$ present input

Since output depends on present input the given system is Causal system

9. Determine whether the following system is stable or not. $y(n) = 3x(n)$.

Solution:

$$\text{Let } x(n) = \delta(n), y(n) = h(n)$$
$$\Rightarrow h(n) = 3\delta(n)$$
$$\text{Condition for stability } \sum_{k=-\infty}^{\infty} |h(k)| < \infty$$
$$\sum_{k=-\infty}^{\infty} |h(k)| = \sum_{k=0}^{\infty} |3\delta(k)| = \sum_{k=0}^{\infty} 3\delta(k) = 3$$
$$\because \delta(k) = 0 \; for \; k \neq 0 \quad and \quad \delta(k) = 1 \; for \; k = 0$$
$$\because \sum_{k=-\infty}^{\infty} |h(k)| < \infty \text{ the given system is } \textbf{stable}$$

10. Determine whether the following system is stable or not

$h(t) = e^{3t}u(t-2)$

Solution:

$$\text{Condition for stability } \int_{-\infty}^{\infty} |\, h(t) \,|\, dt < \infty$$

$$\int_{-\infty}^{\infty} |\, h(t) \,|\, dt = \int_{2}^{\infty} e^{3t} dt$$
$$= \left[\frac{e^{3t}}{3}\right]_2^{\infty} = \infty$$

$\therefore$ The system is unstable.