Displacement Current:

For static electromagnetic fields, according to Ampere's circuital law,

$$\nabla \times H = J \text{ ....} ①$$

Taking divergence on both the sides,

$$\nabla.(\nabla \times H) = \nabla.J$$

But according to vector identity, divergence of the curl of any vector field is zero,

$$\nabla.(\nabla \times H) = \nabla.J = 0 \text{........................} ②$$

But the equation of continuity is given by

$$\nabla.J = -\frac{\partial \rho v}{\partial t} \text{....................................} ③$$

In ③ when $\frac{\partial \rho v}{\partial t}$=0 then only eqn② becomes true. Thus eqn ②&③ not compatible for time varying fields.

①→ can be modified by adding one unknown term $\overline{N}$.

① ⟹ $\nabla \times H = J + \overline{N}$......................④

Taking divergence on both sides of eqn ④

$$\nabla.(\nabla \times \overline{H}) = \nabla.\overline{J} + \nabla.\overline{N} = 0 \text{......................} ◯$$

As $\nabla.J = -\frac{\partial \rho v}{\partial t}$

$$\nabla.\overline{J} + \nabla.N = -\frac{\partial \rho v}{\partial t} + \nabla.\overline{N} = 0$$

$\nabla.N = +\frac{\partial \rho v}{\partial t}$........................◯

But According to Gauss's law,

$\rho v = \nabla.\mathrm{D}$

○⟹ $\nabla.N = +\frac{\partial(\nabla.D)}{\partial t}$

$\nabla.N = \nabla.\frac{\partial D}{\partial t}$ ....○

Compare two sides of eqn ○

$N=\frac{\partial D}{\partial t}$ ....⑤

Now we can write Ampere's circuital law in point from as

④⟹ $\nabla \times H = Jc + \frac{\partial D}{\partial t}$ ....⑥

J_C→ conduction current density, ie, current due to moving charges.

$\frac{\partial D}{\partial t}$ → ampere per square meter. This is obtained from time varying electric flux density. This is also called displacement density ie, This is called displacement current density $J_{D.}$

⑥⟹ $\boxed{\nabla \times H = \overline{Jc} + \overline{JD}}$ ....⑦

Consider a parallel circuit of a resistor and capacitor driven by a time varying voltage v.

Let the current flowing through resistor R be i1 and the current flowing through the capacitor C be i2. The nature of current flowing through the capacitor is different than that flowing through the resistor R.

The current through the resistor is due to the actual motion of charges. Thus the current to be written as

$i_1 = \frac{V}{R}$......................................⑧

This current is called conduction current as the current is flowing because of actual motion of charges ie, i_C.

Let 'A' be the cross sectional area of resistor, then the condition current density is given by

$$\boxed{J_c = \frac{ic}{A} = \sigma E} \text{..............................} ⑨$$

Assume that the initial charge on a capacitor is zero. Then for time varying voltage applied across parallel plate capacitor, the current through the capacitor is given by

$$i_2 = C\frac{dv}{dt} \text{.................................} ⑩$$

Let the two plates of the area A are separated by distance d with dielectric having permittivity ε in between the plates.

$$⑩ \Longrightarrow \quad i_2 = \frac{\varepsilon A}{d}\frac{dv}{dt} \text{..................................} ⑪$$

This current is called displacement current denoted by i_D. The electric field produced by the voltage applied between the two plates is given by,

$$E = \frac{V}{d} \text{.....................................} ⑫$$

$$V = dE$$

Sub V in eqn ⑪

$$i_D = i_2 = \frac{\varepsilon A}{d}\frac{d[dE]}{dt}$$

$$i_D = \frac{\varepsilon A}{d}\, d\, \frac{dE}{dt}$$

$$i_D = \varepsilon A\frac{dE}{dt}$$

Now the ratio of current to the area of plate is the current density. In this case displacement current density denoted by J_D.

$$J_D = \frac{i_D}{A}$$

$$i_D = \varepsilon A\frac{dE}{dt}$$

$$J_D = \frac{i_D}{A} = \frac{d(\varepsilon E)}{dt}$$

$$J_D = \frac{dD}{dt} = \frac{\partial \overline{D}}{\partial t} \text{...................................} ⑬$$

Thus in a given medium both the types of currents [conduction current and the displacement current] may flow.

$$\boxed{\begin{array}{ll} J_C = \sigma E & conduction\ current\ density \\ J_D = \frac{\partial D}{\partial t} & displacement\ current\ density \end{array}} \quad \text{.................................⑭}$$

The total current density

$J = J_C + J_D$.......................................⑮

Some materials are good conductors while some are perfect dielectrics. But in some materials which are neither good conductor nor perfect dielectrics, both the current conduction current and displacement current may exist.

For the electric field intensity $\vec{E}$, let the time dependence is given by $e^{j\omega t}$, the total current density is

$$J = J_C + J_D$$

$$J = \sigma E + \frac{\partial D}{\partial t}$$

$$= \sigma E + \frac{\partial}{\partial t}[\varepsilon E] = \sigma E + \frac{\partial}{\partial t}[\varepsilon e^{j\omega t} E]$$

$\boxed{\bar{J} = \sigma\bar{E} + j\omega\varepsilon\bar{E}}$.............................⑯

The ratio of the magnitudes of the condition current density to the displacement current density is

$\frac{|J_C|}{|J_D|} = \frac{\sigma}{j\omega\varepsilon}$...........................⑰

Thus this ratio depends on the properties of the medium $[\sigma \& \mu]$ and the frequency $[\omega]$ for a conductor, the value of conductivity σ is very large.

In conductor, the conduction current is very large as compared to the displacement current.

While for a dielectric, the value of conductivity σ is very small.

In dielectric medium displacement current is greater as compared to the conduction current.

⑰ $\Rightarrow$ $\frac{\sigma}{\omega\varepsilon} >> 1$, medium is conductor

$\frac{\sigma}{\omega\varepsilon} << 1$, medium is dielectric

The above ratio depends on frequency a medium which is conductor at low frequency may become insulator at very high frequency.

General field relations for Time Varying Electric and Magnetic fields.

The relation between an electric and magnetic fields is given by faraday's law.

$$\nabla \times E = -\frac{\partial B}{\partial t} \text{..................................} \quad (1)$$

We know that

$$B = \nabla \times \overline{A}$$

Where $\overline{A} \rightarrow$ vector magnetic potential

$$\nabla \times E = -\frac{\partial(\nabla \times \overline{A})}{\partial t} \text{..............................} \quad (2)$$

Interchanging operators at R.H.S of above equation we get,

$$\nabla \times E = -\nabla \times \frac{\partial \overline{A}}{\partial t}$$