

4.3 SHORTEST PATH ALGORITHMS:

An algorithm to find the shortest distance path between the source and destination vertices is called the shortest path algorithm.

Types of shortest path problem

i. Single source shortest path

Given an input graph $G = (V, E)$ and a distinguished vertex S , find the shortest path from S to every other vertex in G .

Example: Dijkstra's algorithm (weighted graph and unweighted graph).

ii. All pairs shortest path problem

Given an input graph $G = (V, E)$. Find the shortest path from each vertex to all vertices in a graph.

4.3.1 Dijkstra's algorithm

Weighted Graph

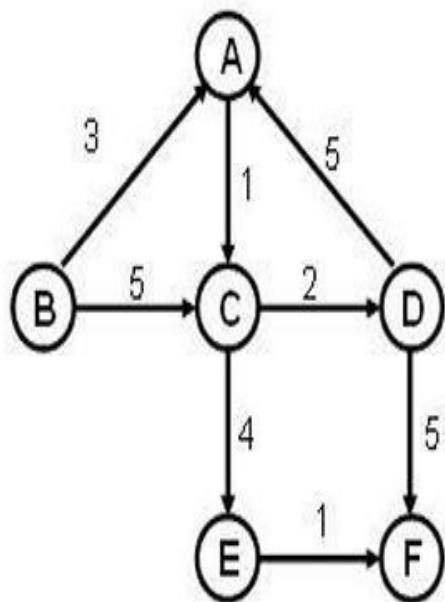
The general method to solve the single source shortest path problem is known as Dijkstra's algorithm. It applied to weighted graph.

Procedure

- It uses greedy technique.
- It proceeds in stages.
- It selects a vertex v , which has the smallest d_v among all the unknown vertices and declares the shortest path from s to v is known.
- The remainder consists of updating the value of d_w .

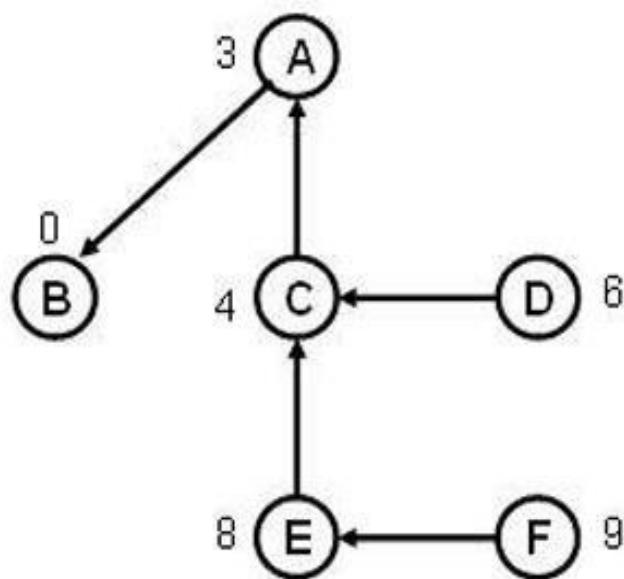
- We should set $dw = dv + Cv$, if the new value for dw would be an improvement.

Example: Find the shortest path for the following graph. Tracing Dijkstra's algorithm starting at vertex B:



Pass:	initially	1	2	3	4	5	6	Shortest distance	Predecessor
		B	A	C	D	E	F		
A	∞	3						3	B
B	0							0	-
C	∞	5	4					4	A
D	∞	∞	∞	6				6	C
E	∞	∞	∞	8	8			8	C
F	∞	∞	∞	∞	11	9		9	E

The resulting vertex-weighted graph is:



Algorithm Analysis

Time complexity of this algorithm $O(|E| + |V|^2) = O(|V|^2)$