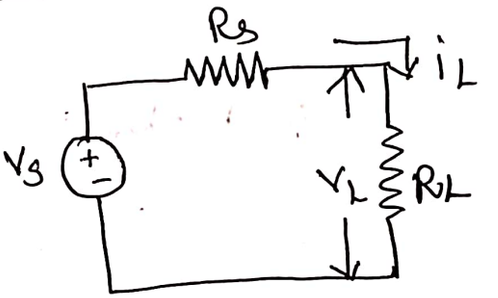
maximum power transfer theorem.

Statement:- An independent voltage source in series with a resistance R_S or an independent current source in parallel with a resistance R_S. delivers maximum power to a load resistance R_L such that $R_L = R_S$.

Proof Let us consider the below ckt



Power delivered to the Load,

$$P_L = i_L^2 R_L$$

$$= \left(\frac{V_S}{R_S+R_L}\right)^2 \cdot R_L = \frac{V_S^2 R_L}{(R_S+R_L)^2}$$

Diff with respect to R_L

$$\frac{dP_L}{dR_L} = \frac{(R_S+R_L)^2 V_S^2 - V_S^2 R_L 2(R_S+R_L)}{(R_S+R_L)^4}$$

$\left[\because d\left(\frac{u}{v}\right) = \frac{v\,du - u\,dv}{v^2}\right]$

equating $\frac{dP_L}{dR_L} = 0$

$$\frac{(R_S+R_L)^2 V_S^2 - V_S^2 R_L 2(R_S+R_L) = 0}{(R_S+R_L)^4}$$

$$= \left[(R_S+R_L)^2 V_S^2 - 2V_S^2 R_L (R_S+R_L)\right]$$

(27)

$$(R_S+R_L)^2 V_S^2 = 2V_S^2 R_L (R_S+R_L)$$

$$V_S^2 R_S + R_L = 2R_L$$

$$\boxed{R_S = R_L}$$

$$P_L = I_L^2 R_L$$

$$= \left(\frac{V_S}{R_S+R_L}\right)^2 \cdot R_L$$

if $V_S = V_{th}$, $R_S = R_{th}$

$$= \left(\frac{V_{th}}{R_{th}+R_L}\right)^2 \cdot R_L$$

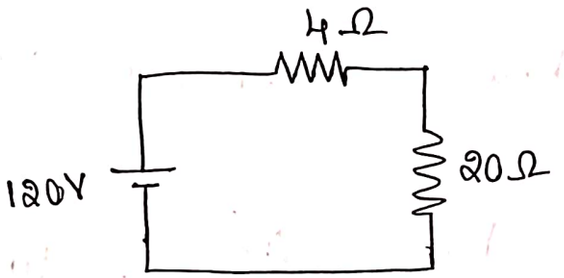
if $R_L = R_{th}$.

$$= \left(\frac{V_{th}}{2R_{th}}\right)^2 R_{th}$$

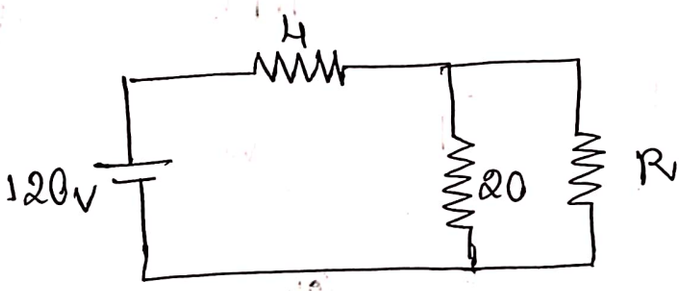
$$= \frac{V_{th}^2}{4R_{th}^2} \cdot R_{th}$$

$$\boxed{P_L^2 = \frac{V_{th}^2}{4R_{th}}}$$

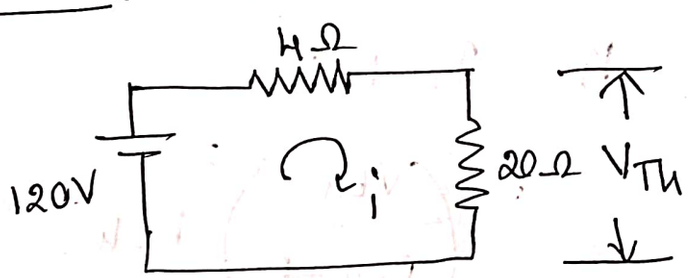
1) A Resistor R is connected in parallel with 20Ω resistor in the given circuit. For what value of R will its power be maximum? Find the maximum power.



Soln.



Find V_{TH} Remove R.



Apply KVL.

$120 - 4i - 20i = 0$

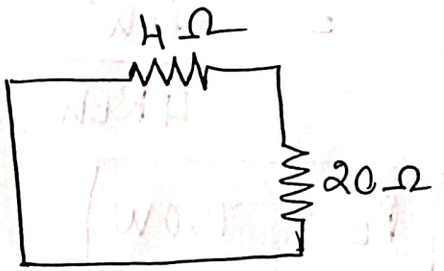
$120 = 24i$

$\boxed{i = 5\,A}$

$\therefore V_{TH} = 20 \times 5$

$\boxed{V_{TH} = 100\,V}$

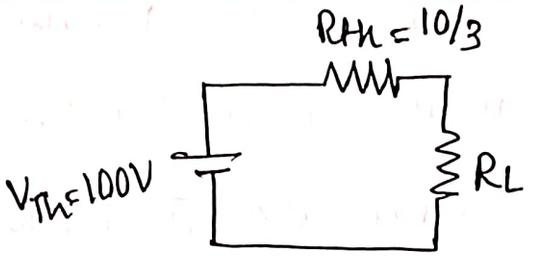
To find R_{TH}.



$$R_{TH} = \frac{4 \times 20}{4 + 20} = \frac{\cancel{4} \times \cancel{20}^{10}}{\cancel{24}^{\cancel{6}}_{3}}$$

$$R_{TH} = \frac{10}{3}$$

(27) Thevenins equivalent ckt



when $R_L = R_{Th}$ max. power will be transfered.

$$P_L = \frac{V_{Th}^2}{4R_{Th}} = \frac{100 \times 100}{4 \times \frac{10}{3}} = 5 \times 3 \times 50$$

$$\boxed{P_L = 750W}$$

(OR)

$$P_L = I_L^2 R_{Th}$$

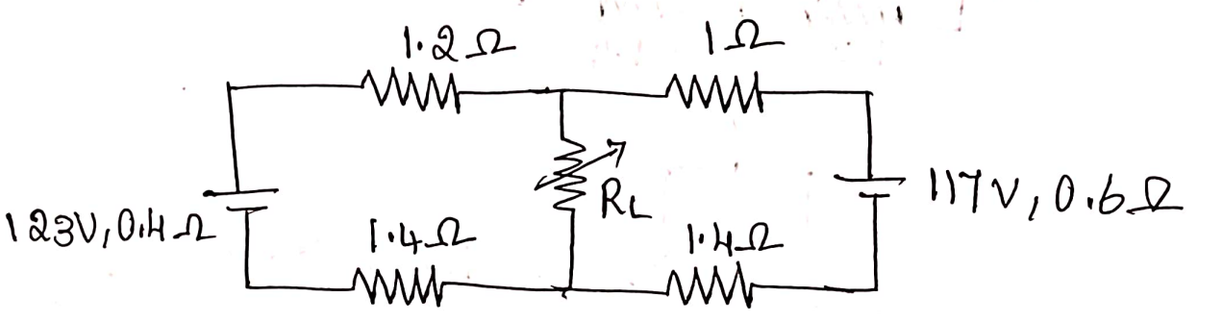
$$= \left(\frac{V_{Th}}{R_{Th} + R_L}\right)^2 R_{Th}.$$

$$= \left(\frac{V_{Th}}{2R_{Th}}\right)^2 \cdot R_{Th}.$$

$$= \frac{V_{Th}^2}{4R_{Th}}.$$

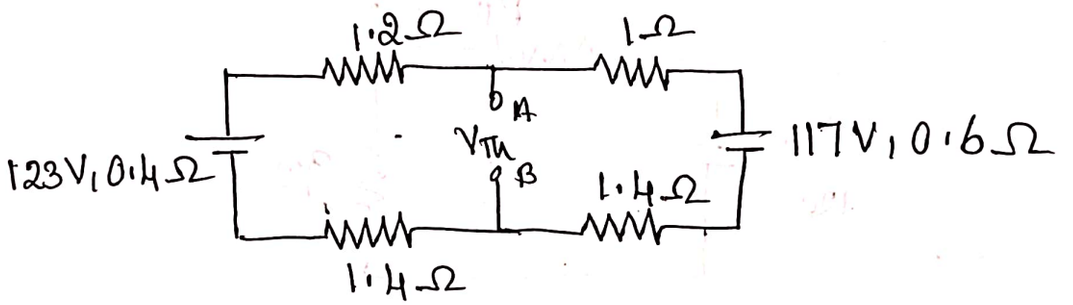
$$\boxed{P_L = 750W}.$$

2) Find the ohmic value of R_L when the power is maximum. also find i) max load power.



Soln

To find V_{TH}.



current flowing through the ckt is

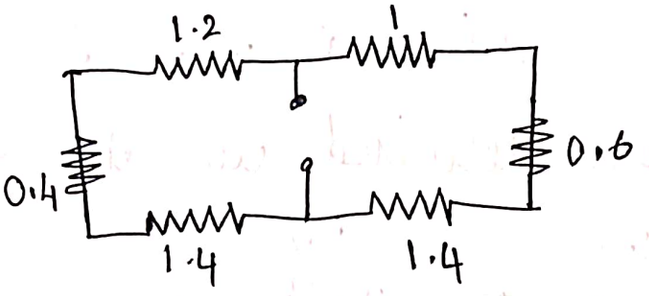
$$i = \frac{123 - 117}{0.4 + 1.2 + 1 + 1.4 + 1.4 + 0.6}$$

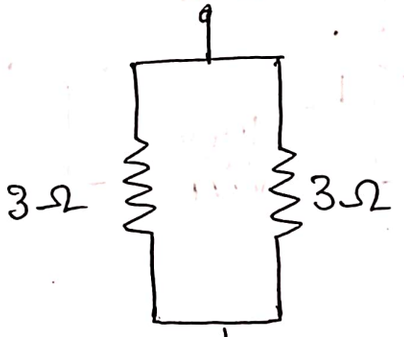
$$= \frac{6}{6} \Rightarrow \boxed{i = 1A}$$

$$V_{TH} = 123 - i(0.4 + 1.2 + 1.4)$$

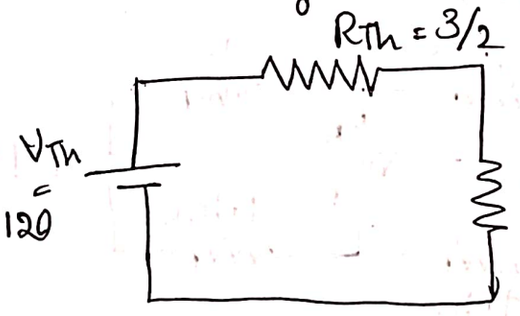
$$= 123 - 3 \Rightarrow \boxed{V_{TH} = 120}$$

(3) To find R_{Th}





$$R_{Th} = \frac{3\times3}{3+3} = \frac{3\times3}{6\times2} = \frac{3}{2}$$



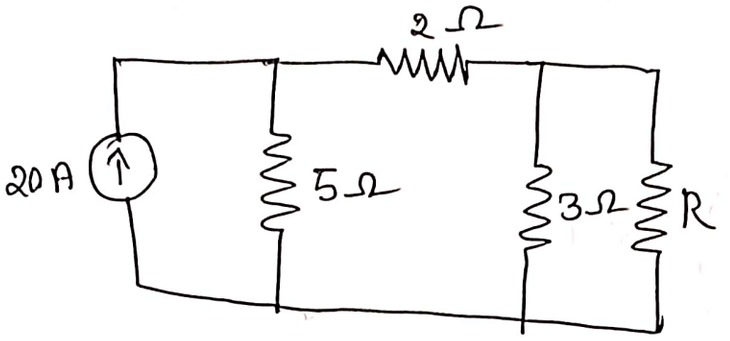
i) $R_L = 3/2\ \Omega$.

2) $$P_L^{\max} = \left(\frac{V_{Th}}{\frac{3}{2}+\frac{3}{2}}\right)^2 \times \frac{3}{2}$$

$$= \frac{120^2}{\frac{6}{2}} \times \frac{3}{2} \Rightarrow \frac{\overset{10}{\cancel{60}}\ \cancel{120}\times\cancel{3}\times\cancel{2}\times 2\times 120}{\cancel{6}\times\cancel{2}\times\cancel{6}}$$

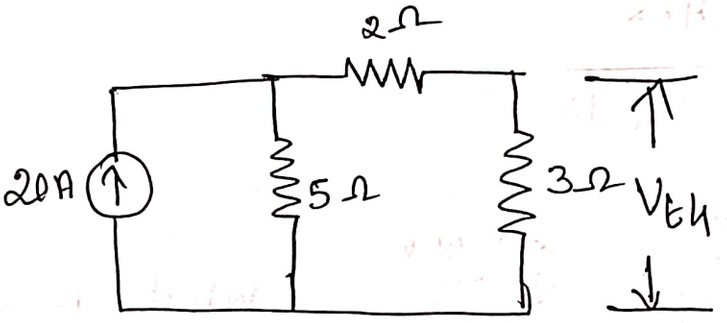
Power = 400 watts

3) In the following circuit R absorb maximum power. Compute R and maximum power.

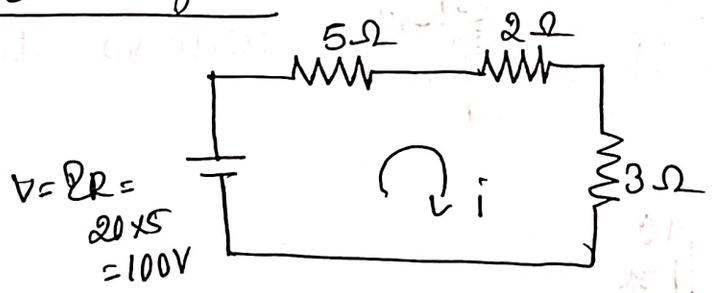


Soln

$V_{TH} = ?$



Source Transformation

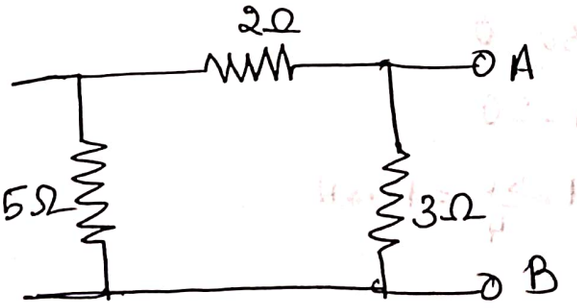


$$i = \frac{100}{5+2+3}$$

$$i = 10A$$

$$V_{TH} = 10 \times 3 = \underline{30\ V}$$

To find R_{th} s.c vtg source, o.c ct source

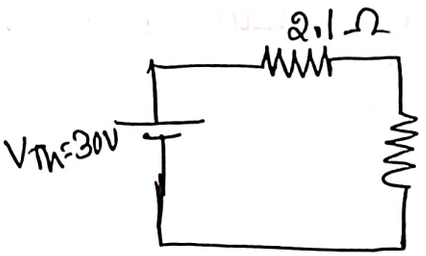


$$R_{th} = (5+2) \| 3$$

$$= \frac{7 \times 3}{7+3} = \frac{21}{10}$$

$$= 2.1\ \Omega$$

(4V)

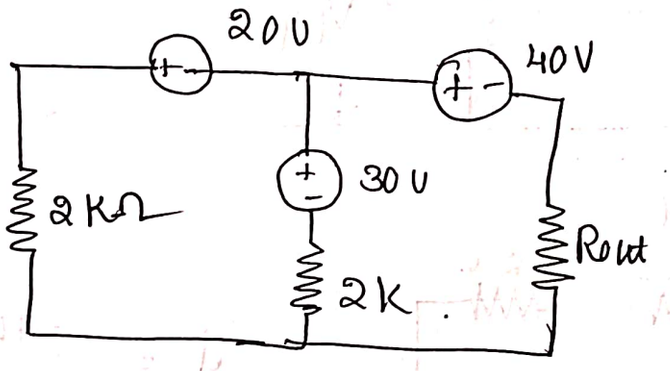


$$P_L = \left(\frac{V_{Th}}{2.1+2.1}\right)^2 2.1$$

$$= \frac{30^2}{4.2^2} \times 2.1$$

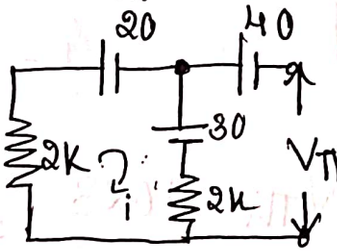
$$\boxed{P_L = 107 \text{ watts}}$$

4)



what is the max. Power that can be delivered to any Rout?

To find V_{Th}.



$$-2i - 20 - 30 - 2i = 0$$

$$-4i - 50 = 0$$

$$-4i = 50$$

$$i = \frac{50}{4} = 12.5A.$$