

### 5.3 CPM AND PERT NETWORK MODELLING AND TIME ANALYSIS

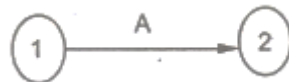
Any project involves planning, scheduling and controlling a number of interrelated activities with use of limited resources like, men, machines, materials, money and time. The projects may be extremely large and complex such as construction of a power plant, a highway, a shopping complex, ships and aircraft. It is required that managers must have a dynamic planning and scheduling system to produce the best possible results and also to react immediately to the changing conditions and make necessary changes in the plan and schedule.

A convenient analytical and visual technique of PERT and CPM prove extremely valuable in assisting the managers in managing the projects. PERT and CPM are basically time-oriented methods in the sense that they both lead to determination of a time schedule for the project. The significant difference between two approaches is that CPM is an activity oriented network while PERT is event oriented. CPM has single time estimate which is assumed to be deterministic and PERT has three time estimates for activities and uses probability theory to find the chance of reaching the scheduled time.

#### NETWORK DIAGRAM REPRESENTATION

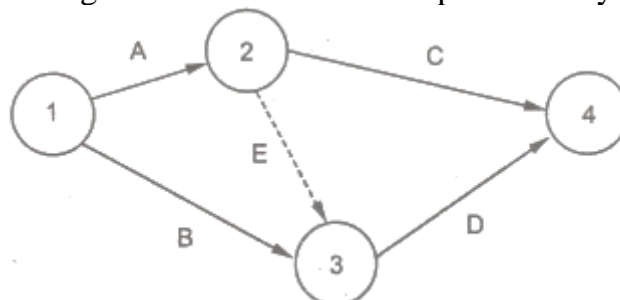
In a network representation of a project the following representations are used:

**Activity:** Any individual operation which utilizes resources and has an end and a beginning is called activity. An arrow is commonly used to represent an activity with its head indicating the direction of progress in the project.



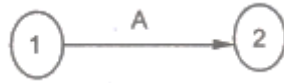
Here 'A' is the activity. These are classified into four categories:

- (i) **Predecessor activity** - Activities that must be completed immediately prior to the start of another activity are called predecessor activities.
- (ii) **Successor activity** - Activities that cannot be started until one or more of other activities are completed but immediately succeed them are called successor activities.
- (iii) **Concurrent activity** - Activities which can be accomplished concurrently are known as concurrent activities. It may be noted that an activity can be a predecessor or a successor to an event or it may be concurrent with one or more of other activities.
- (iv) **Dummy activity** An activity which does not consume any kind of resource and time is called a dummy activity. Dummy activities are simply used to represent a connection between events in order to maintain logic in the network. It is represented by a dotted line in a network.



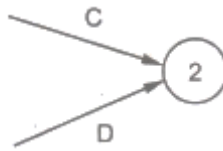
In the above example, A and B are preceding activities. C is dependent on activity A and D is dependent on activity B. Also A and B are concurring activities, since they are starting at the same time. Activity E is the dummy activity and it is marked as dotted line.

❖ **Event:** An event represents a point in time signifying the completion of some activities and the beginning of new ones. This is usually represented by a circle in a network which is also called a node or connector.

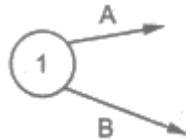


Here '1' and '2' are called events. 1 is the tail event and 2 is the head event. The events are classified in to three categories:

(i) **Merge event** - When more than one activity comes and joins an event such an event is known as merge event.



(ii) **Burst event** - When more than one activity leaves an event such an event is known as burst event.



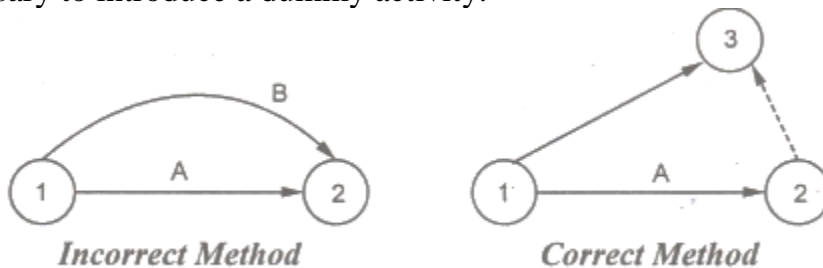
(iii) **Merge and Burst event** - An event may be merge and burst event at the same time as with respect to some activities it can be a merge event and with respect to some other activities it may be a burst event.



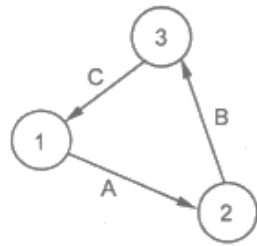
**ERRORS TO BE AVOIDED IN A NETWORK DIAGRAM**

While drawing a network diagram, the following representations should be avoided:

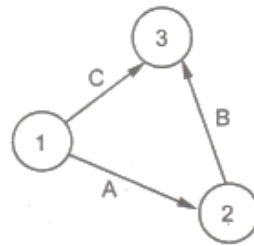
(i) Two activities starting from a tail event must not have a same end event. To ensure this, it is absolutely necessary to introduce a dummy activity.



(ii) **Looping error** should not be formed in a network, as it represents performance of activities repeatedly in a cyclic manner.

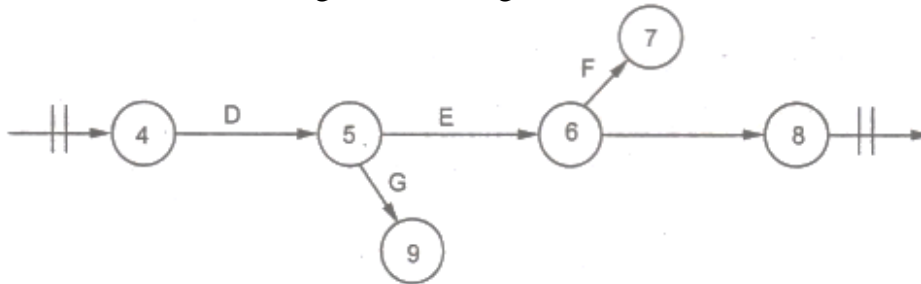


*Incorrect Method*

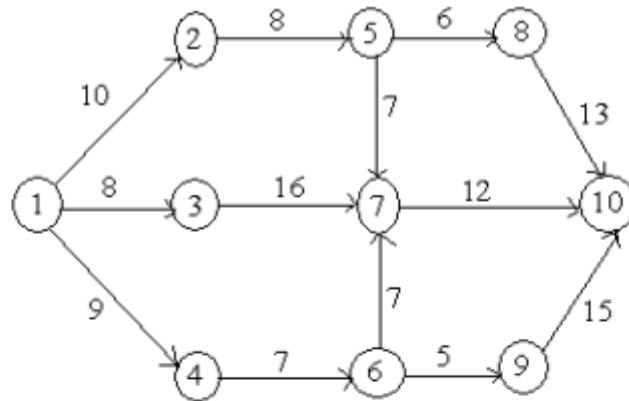


*Correct Method*

(iii) **Dangling:** To disconnect an activity before the completion of all activities in a network diagram is known as dangling. As shown in the figure activities (5-9) and (6-7) are not the last activities in the network. So the diagram is wrong and indicates the error of dangling

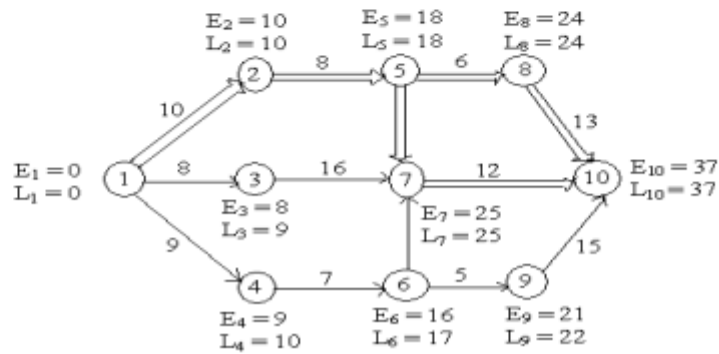


**Example 1 Determine the early start and late start in respect of all node points and identify critical path for the following network.**



**Solution**

Calculation of E and L for each node is shown in the network



Activity(i, j)	Normal Time (D <sub>ij</sub> )	Earliest Time		Latest Time		Float Time (L <sub>i</sub> - D <sub>ij</sub> ) - E <sub>i</sub>
		Start (E <sub>i</sub> )	Finish (E <sub>i</sub> + D <sub>ij</sub> )	Start (L <sub>i</sub> - D <sub>ij</sub> )	Finish (L <sub>i</sub> )	
(1, 2)	10	0	10	0	10	0
(1, 3)	8	0	8	1	9	1
(1, 4)	9	0	9	1	10	1
(2, 5)	8	10	18	10	18	0
(4, 6)	7	9	16	10	17	1

(3, 7)	16	8	24	9	25	1
(5, 7)	7	18	25	18	25	0
(6, 7)	7	16	23	18	25	2
(5, 8)	6	18	24	18	24	0
(6, 9)	5	16	21	17	22	1
(7, 10)	12	25	37	25	37	0
(8, 10)	13	24	37	24	37	0
(9, 10)	15	21	36	22	37	1

**Network Analysis Table**

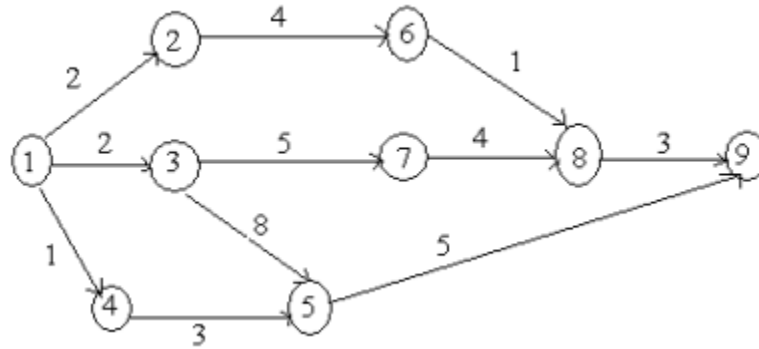
From the table, the critical nodes are (1, 2), (2, 5), (5, 7), (5, 8), (7, 10) and (8, 10)

From the table, there are two possible critical paths

- i.  $1 \rightarrow 2 \rightarrow 5 \rightarrow 8 \rightarrow 10$
- ii.  $1 \rightarrow 2 \rightarrow 5 \rightarrow 7 \rightarrow 10$



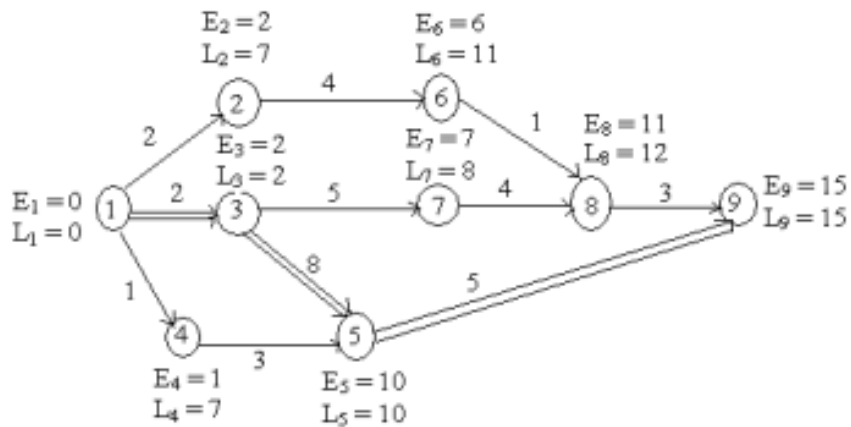
**Example 2 Find the critical path and calculate the slack time for the following network**



The earliest time and the latest time are obtained below

Activity(i, j)	Normal Time (D <sub>ij</sub> )	Earliest Time		Latest Time		Float Time (L <sub>i</sub> - D <sub>ij</sub> ) - E <sub>i</sub>
		Start (E <sub>i</sub> )	Finish (E <sub>i</sub> + D <sub>ij</sub> )	Start (L <sub>i</sub> - D <sub>ij</sub> )	Finish (L <sub>i</sub> )	
(1, 2)	2	0	2	5	7	5
(1, 3)	2	0	2	0	2	0
(1, 4)	1	0	1	6	7	6
(2, 6)	4	2	6	7	11	5
(3, 7)	5	2	7	3	8	1
(3, 5)	8	2	10	2	10	0
(4, 5)	3	1	4	7	10	6
(5, 9)	5	10	15	10	15	0
(6, 8)	1	6	7	11	12	5
(7, 8)	4	7	11	8	12	1
(8, 9)	3	11	14	12	15	1

From the above table, the critical nodes are the activities (1, 3), (3, 5) and (5, 9)



The critical path is 1 → 3 → 5 → 9

**Example 3**

A project has the following times schedule

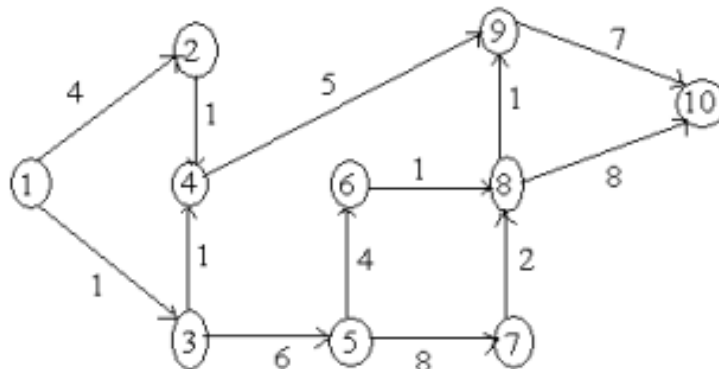
Activity	Times in weeks	Activity	Times in weeks
(1 – 2)	4	(5 – 7)	8
(1 – 3)	1	(6 – 8)	1
(2 – 4)	1	(7 – 8)	2
(3 – 4)	1	(8 – 9)	1
(3 – 5)	6	(8 – 10)	8
(4 – 9)	5	(9 – 10)	7
(5 – 6)	4		

Construct the network and compute

1.  $T_E$  and  $T_L$  for each event
2. Float for each activity
3. Critical path and its duration

**Solution**

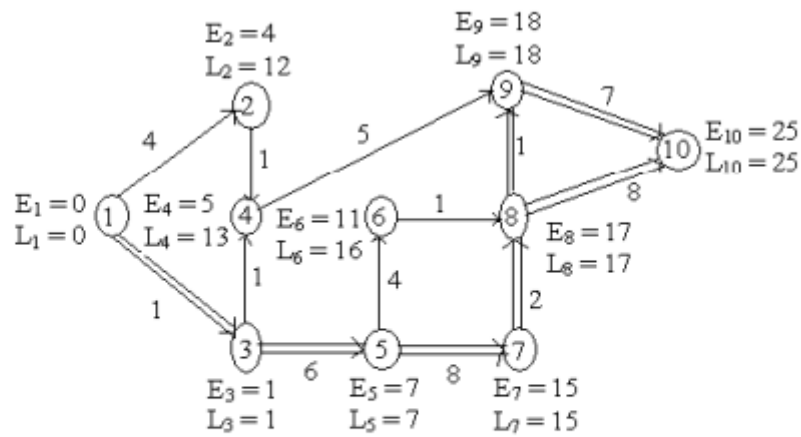
The network is



Event No.:	1	2	3	4	5	6	7	8	9	10
$T_E$ :	0	4	1	5	7	11	15	17	18	25
$T_L$ :	0	12	1	13	7	16	15	17	18	25

Float =  $T_L$  (Head event) –  $T_E$  (Tail event) – Duration

Activity	Duration	$T_E$ (Tail event)	$T_L$ (Head event)	Float
(1-2)	4	0	12	8
(1-3)	1	0	1	0
(2-4)	1	4	13	8
(3-4)	1	1	13	11
(3-5)	6	1	7	0
(4-9)	5	5	18	8
(5-6)	4	7	16	5
(5-7)	8	7	15	0
(6-8)	1	11	17	5
(7-8)	2	15	17	0
(8-9)	1	17	18	0
(8-10)	8	17	25	0
(9-10)	7	18	25	0



The two critical paths are

- i. 1 → 3 → 5 → 7 → 8 → 9 → 10
- ii. 1 → 3 → 5 → 7 → 8 → 10