1.6 The Predicate Calculus

The predicate calculus deals with the study of predicates.

Consider the following statement.

"Ram is a boy"

In the above statement, **"is a boy"** is the predicate and the name **"Ram"** is the subject.

If we denote **"is a boy"** by B and subject **"Ram"** by r, then the statement **"Ram is a boy"** can be represented as B(r).

Some examples

1." x is a man"

Here, Predicate is **"is a man"** and it is denoted by M. Subject is **"x"** and it is denoted by x.

Hence the given statement **"x is a man"** can be denoted by **M(x).**

2. "Sam is poor and Ram is intelligent"

The statement **"Sam is poor"** can be represented by **P(s)** where P represents predicate **"is poor"** and s represents subject **"Sam"**

The statement "Ram is intelligent" can be represented by I(r) where I represents predicate **"is intelligent"** and r represents subject **"Ram".**

Hence the given statement **"Sam is poor and Ram is intelligent"** can be symbolized as $\boldsymbol{P(s) \wedge I(r)}$**.**

The Theory of Inference for Predicate Calculus

Universal Specification (UG): $A(y) \Rightarrow (x)A(x)$

Existential Generalization (EG): $A(y) \Rightarrow (\exists x)A(x)$

Universal Specification (US): $(x)A(x) \Rightarrow A(y)$

Existential Specification (ES): $(\exists x)A(x) \Rightarrow A(y)$

Problems:

1.Show that $\boldsymbol{(x)\big(H(x) \to M(x)\big) \wedge H(s) \Rightarrow M(s)}$

Solution:

$\{1\}$	1) $(x)\big(H(x) \to M(x)\big)$	Rule P
$\{1\}$	2)$H(s) \to M(s)$	Rule US
$\{3\}$	3 $H(s)$	Rule P
$\{1,3\}$	4)$M(s)$	Rule T $(P, P \to Q \Rightarrow Q)$

2. Show that $(x)(P(x) \rightarrow Q(x)) \wedge (x)(Q(x) \rightarrow R(x)) \Rightarrow (x)(P(x) \rightarrow R(x))$

Solution:

$\{1\}$	1) $(x)(P(x) \rightarrow Q(x))$	Rule P
$\{1\}$	2)$P(y) \rightarrow Q(y)$	Rule US
$\{3\}$	3 $(x)(Q(x) \rightarrow R(x))$	Rule P
$\{1, 3\}$	4)$Q(y) \rightarrow R(y)$	Rule US
$\{1, 3\}$	5) $P(y) \rightarrow R(y)$	Rule T $(P \rightarrow Q, Q \rightarrow R \Rightarrow P \rightarrow R)$
$\{1, 3\}$	6)$(x)(P(x) \rightarrow R(x))$	Rule UG

3. Show that $(\exists x)(P(x) \wedge Q(x)) \Rightarrow (\exists x)P(x) \wedge (\exists x)Q(x)$

Solution:

$\{1\}$	1) $(\exists x)(P(x) \wedge Q(x))$	Rule P
$\{1\}$	2)$P(y) \wedge Q(y)$	Rule ES
$\{3\}$	3 $P(y)$	Rule T $(P \wedge Q \Rightarrow P)$
$\{1, 3\}$	4)$Q(y)$	Rule T $(P \wedge Q \Rightarrow P)$
$\{1, 3\}$	5) $(\exists x)P(x)$	Rule EG
$\{1, 3\}$	6)$(\exists x)Q(x)$	Rule EG
$\{1\}$	7)$(\exists x)P(x) \wedge (\exists x)Q(x)$	Rule T$(P, Q \Rightarrow P \wedge Q)$

4.Show that $(\boldsymbol{x})(\boldsymbol{P}(\boldsymbol{x}) \vee \boldsymbol{Q}(\boldsymbol{x})) \Rightarrow (\boldsymbol{x})\boldsymbol{P}(\boldsymbol{x}) \vee (\exists \boldsymbol{x})\boldsymbol{Q}(\boldsymbol{x})$

Solution:

We shall use the indirect method of proof.

Method of contradiction:

Assume $\neg((\boldsymbol{x})\boldsymbol{P}(\boldsymbol{x}) \vee (\exists \boldsymbol{x})\boldsymbol{Q}(\boldsymbol{x}))$ as an additional premises.

{1}	1) $\neg((x)P(x) \vee (\exists x)Q(x))$	Assumed Premises
{1}	2) $(\exists x)\neg P(x) \wedge (x)Q(x)$	Rule T (D'Morgan's law)
{1}	3) $(\exists x)\neg P(x)$	Rule T $(P \wedge Q \Rightarrow P)$
{1}	4) $(x)Q(x)$	Rule T $(P \wedge Q \Rightarrow P)$
{1}	5) $\neg P(y)$	Rule ES
{1}	6) $\neg Q(y)$	Rule US
{1}	7) $\neg P(y) \wedge \neg Q(y)$	Rule T$(P, Q \Rightarrow P \wedge Q)$
{1}	8) $\neg(P(y) \vee Q(y))$	Rule T (D'Morgan's law)
{1}	9) $(x)(P(x) \vee Q(x))$	Rule P
{1}	10) $P(y) \vee Q(y)$	Rule US
{1}	11) $(P(y) \vee Q(y)) \wedge \neg(P(y) \vee Q(y))$	Rule T$(P, Q \Rightarrow P \wedge Q)$

which is nothing but false value.

5.Show that $(x)\left(P(x) \rightarrow Q(x)\right) \Rightarrow (x)P(x) \rightarrow (x)Q(x)$

Solution:

Assume $\neg\left((x)P(x) \rightarrow (x)Q(x)\right)$

$\{1\}$	1) $\neg\left((x)P(x) \rightarrow (x)Q(x)\right)$	Assumed Premises
$\{1\}$	2) $(x)P(x) \wedge \neg(x)Q(x)$	Rule T $(P \rightarrow Q \Rightarrow \neg P \vee Q)$
$\{1\}$	3) $(x)P(x)$	Rule T $(P \wedge Q \Rightarrow P)$
$\{1\}$	4) $\neg\left((x)Q(x)\right)$	Rule T $(P \wedge Q \Rightarrow P)$
$\{1\}$	5) $(\exists x)\neg Q(x)$	Rule T(Taking $\neg$)
$\{1\}$	6) $P(y)$	Rule US
$\{1\}$	7) $\neg Q(y)$	Rule ES
$\{1\}$	8) $P(y) \wedge \neg Q(y)$	Rule T $(P, Q \Rightarrow P \wedge Q)$
$\{9\}$	9) $\neg\left(P(y) \rightarrow Q(y)\right)$	Rule T$((P \wedge \neg Q) \Leftrightarrow \neg(P \rightarrow Q))$
$\{9\}$	10) $(\exists x)\neg\left(P(x) \rightarrow Q(x)\right)$	Rule EG
$\{1, 9\}$	11) $\neg\left((x)P(x) \rightarrow Q(x)\right)$	Rule T(Taking $\neg$)