1.2. De-Morgan's theorem

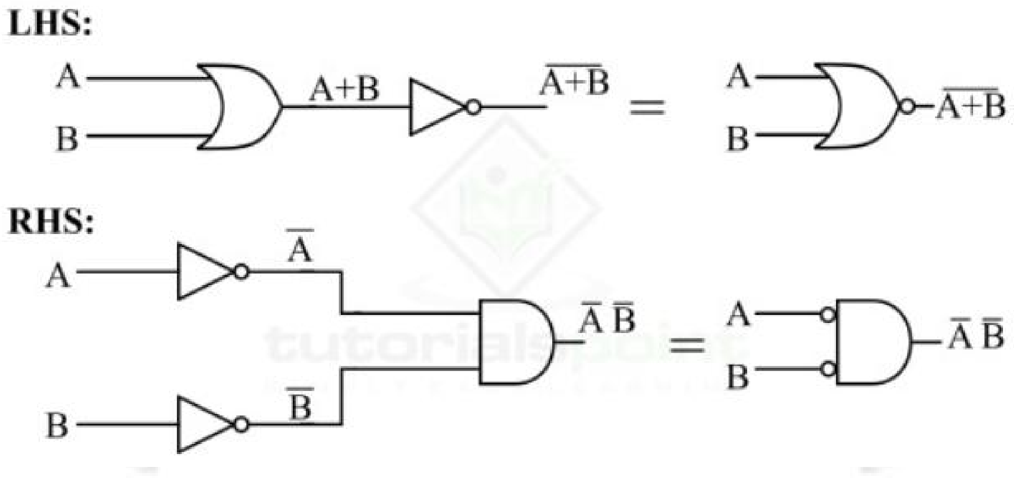
DeMorgan's Theorem is a powerful theorem in Boolean algebra which has a set of two rules or laws. These two laws were developed to show the relationship between two variable AND, OR, and NOT operations. These two rules enable the variables to be negated, i.e. opposite of their original form. Therefore, DeMorgan's theorem gives the dual of a logic function.

DeMorgan's First Theorem (Law 1)

DeMorgan's First Law states that the complement of a sum (ORing) of variables is equal to the product (ANDing) of their individual complements. In other words, the complement of two or more ORed variables is equivalent to the AND of the complements of each of the individual variables, i.e.

$$\overline{A+B} = \bar{A} \cdot \bar{B}$$

The logic implementation of left side and right side of this law is ,



Thus, DeMorgan's first law proves that the NOR gate is equivalent to a bubbled AND gate. The following truth table shows the proof of this law,

Left Side			Right Side		
Input		Output	Input		Output
A	B	(A + B)'	A'	B'	A'· B'
0	0	1	1	1	1
0	1	0	1	0	0
1	0	0	0	1	0
1	1	0	0	0	0

This truth table proves that the Boolean expression on the left is equivalent to that on the right side of the expression of DeMorgan's first law.

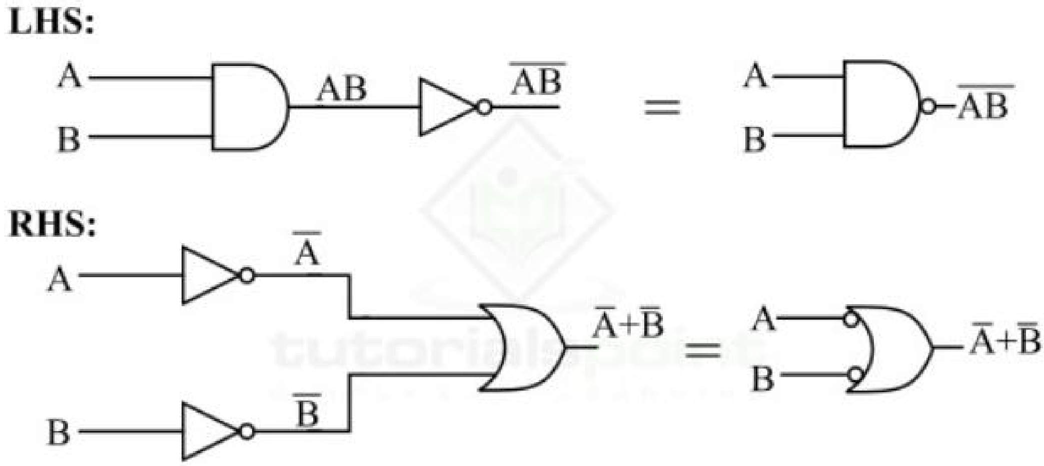
DeMorgan's Second Theorem (Law 2)

DeMorgan's second law states that the complement of the product (ANDing) of variables is equivalent to the sum (ORing) of their individual complements.

In other words, the complement of two or more ANDed variables is equal to the sum of the complement of each of the individual variables, i.e.,

$$\overline{AB} = \overline{A} + \overline{B}$$

The logic implementation of left and right sides of this expression is shown as ,



Hence, DeMorgan's second law proves that the NAND gate is equivalent to a bubbled OR gate. The following truth table shows the proof of this law.

Left Side			Right Side		
Input		Output	Input		Output
A	B	AB	A'	B'	A' + B'
0	0	0	1	1	1
0	1	1	1	0	1
1	0	1	0	1	1
1	1	1	0	0	0

This truth table proves that the Boolean expression on the left side is equivalent to that on the right side of the expression of DeMorgan's second law.

SOP and POS

SOP (Sum of Products) Form

The SOP or Sum of Products form is a form of expressing a logical or Boolean expression. In SOP, different product terms of input variables are logically ORed together. Therefore, in the case of SOP form, we first

logically AND the input variables, and then all these product terms are summed together with the help of logical OR operation.
For example −

$$f(A, B, C) = ABC + \bar{A}BC + AB\bar{C}$$

This is a logical expression in three variables. Here, ABC, A'BC, and ABC' are the three product terms which are summed together to get the expression in SOP form.

POS (Product of Sum) Form

The POS or Product of Sum form is another form used to represent a logical expression. In POS form, different sum terms of input variables are logically ANDed together. Hence, if we want to express a logical expression in POS form, for that we first logically OR all the input variables and then these sum terms are ANDed using AND operation.
For example −

$$f(A, B, C) = (A + B + C)(\bar{A} + B + C)(A + B + \bar{C})$$

Here, f is a logical expression in three variables. From this example, it can be seen that there are three sum terms which are ANDed together to obtain the POS form of the given expression.