1.3 Properties of DFT - periodicity, symmetry, circular convolution,Linear filtering using DFT, Filtering long data sequences - overlap save and overlap add method

PROPERTIES OF DFT

1. Periodicity

$$x[n] = x[n+N], \quad X[k] = X[k+N]$$

Meaning:

Both time-domain and frequency-domain signals are **periodic with period** N.

2. Symmetry Property

For **real-valued signals:**

$$X[k] = X^*[N-k]$$

- Magnitude spectrum → **Even symmetry**
- Phase spectrum → **Odd symmetry**

3. Circular Convolution

$$y[n] = x[n] \circledast h[n]$$

DFT Property:

$$\mathrm{DFT}\{x[n] \circledast h[n]\} = X[k]H[k]$$

♦ **Note:** DFT multiplication corresponds to **circular convolution** in time domain.

4. Linear Filtering Using DFT

To perform **linear convolution** using DFT:

1. Zero-pad $x[n]$ and $h[n]$
2. Compute DFTs
3. Multiply spectra
4. Compute IDFT

Condition:

$$N \geq L_x + L_h - 1$$

FILTERING LONG DATA SEQUENCES

1. Overlap-Add (OLA) Method

Steps:

1. Split input into blocks
2. Zero-pad each block
3. Perform DFT-based convolution
4. Add overlapping output samples

Advantage:

Simple implementation

2. Overlap-Save (OLS) Method

Steps:

1. Overlap input blocks
2. Perform DFT convolution
3. Discard corrupted samples
4. Save valid output samples

Advantage:

More computationally efficient than OLA

Comparison: OLA vs OLS

Feature	Overlap-Add	Overlap-Save
Padding	Required	Not required
Computation	Moderate	Efficient
Output handling	Add overlap	Discard samples

Problem 1:

Find the **4-point DFT** of

$$x(n) = \{1,\ 2,\ 3,\ 4\}$$

Formula:

$$X(k) = \sum_{n=0}^{N-1} x(n)\, e^{-j2\pi kn/N}$$

Solution:

For $N = 4$

- $X(0) = 1 + 2 + 3 + 4 = 10$
- $X(1) = 1 - j2 - 3 + j4 = -2 + j2$
- $X(2) = 1 - 2 + 3 - 4 = -2$
- $X(3) = 1 + j2 - 3 - j4 = -2 - j2$

Answer:

$X(k) = \{10,\ -2 + j2,\ -2,\ -2 - j2\}$

Problem 2:

Verify the periodicity property of DFT for $x(n) = \{1, 1, 1, 1\}$

Property:

$$X(k+N) = X(k)$$

Solution:

DFT:

- $X(0) = 4$
- $X(1) = 0$
- $X(2) = 0$
- $X(3) = 0$

Since values repeat every $N = 4$, periodicity is verified.

Problem 3:

Check symmetry of DFT for real signal

$$x(n) = \{1, 2, 1, 2\}$$

Property:

For real signals:

$$X(N-k) = X^*(k)$$

Solution:

Computed DFT:

$$X = \{6,\ -j,\ 0,\ j\}$$

Since:

- $X(3) = X^*(1)$