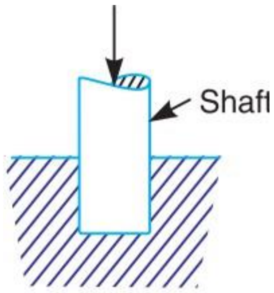
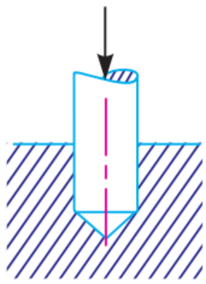
24AG401 THEORY OF MACHINES
UNIT III NOTES

Friction of Pivot and Collar Bearing

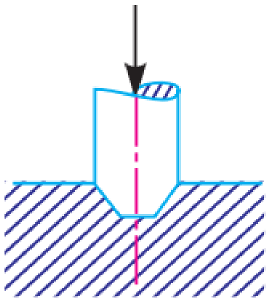
The rotating shafts are frequently subjected to axial thrust. The bearing surfaces such as pivot and collar bearings are used to take this axial thrust of the rotating shaft. The propeller shafts of ships, the shafts of steam turbines, and vertical machine shafts are examples of shafts which carry an axial thrust. The bearing surfaces placed at the end of a shaft to take the axial thrust are known as pivots. The pivot may have a flat surface or conical surface as shown in Figure a and Figure b respectively. When the cone is truncated, it is then known as truncated or trapezoidal pivot as shown in Figure c.



Flat pivot.

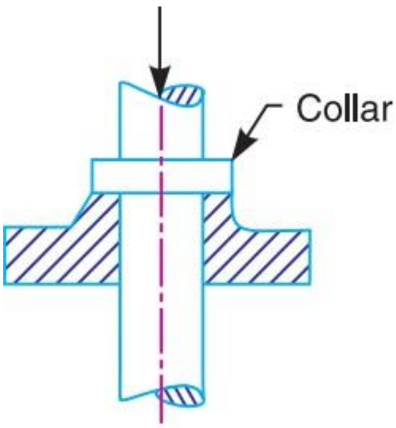


Conical pivot.

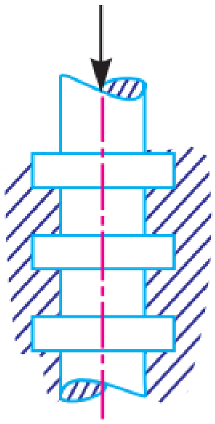


Truncated pivot.

The collar may have flat bearing surface or conical bearing surface, but the flat surface is most commonly used. There may be a single collar, as shown in Figure or several collars along the length of a shaft, as shown in Figure in order to reduce the intensity of pressure.



Single flat collar.



Multiple flat collar.

In modern practice, ball and roller thrust bearings are used when power is being transmitted and when thrusts are large as in case of propeller shafts of ships.

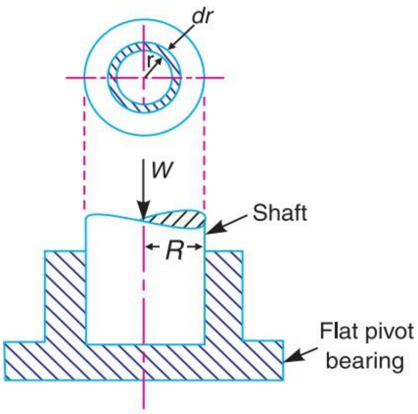
A little consideration will show that in a new bearing, the contact between the shaft and bearing may be good over the whole surface. In other words, we can say that the pressure over the rubbing surfaces is uniformly distributed. But when the bearing becomes old, all parts of the rubbing surface will not move with the same velocity, because the velocity of rubbing surface increases with the distance from the axis of the bearing. This means that wear may be different at different radii and this causes to alter the distribution of pressure. Hence, in the study of friction of bearings, it is assumed that

1. The pressure is uniformly distributed throughout the bearing surface, and
2. The wear is uniform throughout the bearing surface.

Flat Pivot Bearing

When a vertical shaft rotates in a flat pivot bearing (known as foot step bearing), as shown in Figure below .The sliding friction will be along the surface of contact between the shaft and the bearing. Let W = Load transmitted over the bearing surface, R = Radius of bearing surface, p = Intensity of pressure per unit area of bearing surface between rubbing surfaces,

Coefficient of friction.



Flat pivot or footstep bearing.

We will consider the following two cases :

1. When there is a uniform pressure ; and
2. When there is a uniform wear.

When there is a uniform pressure

Consider a ring of radius r and thickness dr of the bearing area. ∴ Area of bearing surface, $A = 2\pi r.dr$ Load transmitted to the ring, $\delta W = p \times A = p \times 2\ \pi$

r.dr ...(i)

Frictional resistance to sliding on the ring acting tangentially at radius r,

$F_r = \mu.\delta W = \mu\, p \times 2\pi\, r.dr = 2\pi\, \mu.p.r.dr$

Frictional torque on the ring, $T_r = F_r \times r = 2\pi\, \mu\, p\, r.dr \times r = 2\, \pi\, \mu\, p\, r^2\, dr$ ...(ii)

Integrating this equation within the limits from 0 to R for the total frictional torque on the pivot bearing. Collar bearing.

$$\therefore \text{ Total frictional torque, } T = \int_0^R 2\pi\mu\, p r^2\, dr = 2\pi\mu\, p \int_0^R r^2\, dr$$

$$= 2\pi\mu\, p\left[\frac{r^3}{3}\right]_0^R = 2\pi\mu\, p \times \frac{R^3}{3} = \frac{2}{3} \times \pi\mu.p.R^3$$

$$= \frac{2}{3} \times \pi\mu \times \frac{W}{\pi R^2} \times R^3 = \frac{2}{3} \times \mu.W.R \qquad \ldots\left(\because\ p = \frac{W}{\pi R^2}\right)$$

When the shaft rotates at ω rad/s, then power lost in friction,

$$P = T.\omega = T \times 2\pi N/60 \qquad \ldots(\because\ \omega = 2\pi N/60)$$

where N = Speed of shaft in r.p.m.

2. Considering uniform wear

The rate of wear depends upon the intensity of pressure (p) and the velocity of rubbing surfaces (v). It is assumed that the rate of wear is proportional to the product of intensity of pressure and the velocity of rubbing surfaces (i.e. p.v..). Since the velocity of rubbing surfaces increases with the distance (i.e. radius r) from the axis of the bearing, therefore for uniform wear.

p.r = C (a constant) or p = C/r and the load transmitted to the ring, $\delta W = p \times 2\pi r.dr$ ...[From equation (i)

∴ Total load transmitted to the bearing

$$W = \int_0^R 2\pi\, C.dr = 2\pi\, C[r]_0^R = 2\pi C.R \quad \text{or} \quad C = \frac{W}{2\pi R}$$

We know that frictional torque acting on the ring,

$$T_r = 2\pi\mu\, p\, r^2\, dr = 2\pi\mu \times \frac{C}{r} \times r^2\, dr \qquad \ldots\left(\because\ p = \frac{C}{r}\right)$$

$$= 2\pi\, \mu.C.r\, dr$$

∴ Total frictional torque on the bearing,

$$T = \int_0^R 2\pi\, \mu.C.r.dr = 2\pi\mu.C\left[\frac{r^2}{2}\right]_0^R$$

$$= 2\pi\mu.C \times \frac{R^2}{2} = \pi\mu.C.R^2$$

$$= \pi\mu \times \frac{W}{2\pi R} \times R^2 = \frac{1}{2} \times \mu.W.R \qquad \ldots\left(\because\ C = \frac{W}{2\pi R}\right)$$

Problem

A vertical shaft 150 mm in diameter rotating at 100 r.p.m. rests on a flat end footstep bearing. The shaft carries a vertical load of 20 kN. Assuming uniform pressure distribution and coefficient of friction equal to 0.05, estimate power lost in friction.

Solution.

Given : D = 150 mm or R = 75 mm = 0.075 m ; N = 100 r.p.m or ω = 2 π × 100/60 = 10.47 rad/s ; W = 20 kN = 20 × 103 N ; μ = 0.05

We know that for uniform pressure distribution, the total frictional torque,

$$T = \frac{2}{3} \times \mu.W.R = \frac{2}{3} \times 0.05 \times 20 \times 10^3 \times 0.075 = 50\ \text{N-m}$$

∴ Power lost in friction,

$$P = T.\omega = 50 \times 10.47 = 523.5\ \text{W}$$

Problem

A conical pivot supports a load of 20 kN, the cone angle is 120º and the intensity of normal pressure is not to exceed 0.3 N/mm2. The external diameter is twice the internal diameter. Find the outer and inner radii of the bearing surface. If the shaft rotates at 200 r.p.m. and the coefficient of friction is 0.1, find the power absorbed in friction. Assume uniform pressure.

Solution.

Given : W = 20 kN = 20 × 103 N ; 2 α = 120º or α = 60º ; pn = 0.3 N/mm2 ; N = 200 r.p.m. or ω = 2 π × 200/60 = 20.95 rad/s ; μ = 0.1

Outer and inner radii of the bearing surface

Let r 1 and r 2 = Outer and inner radii of the bearing surface, in mm. Since the external diameter is twice the internal diameter, therefore r1 = 2 r2

We know that intensity of normal pressure (p_n),

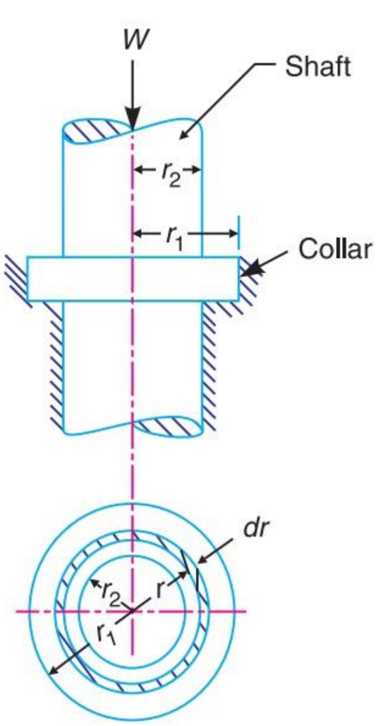
$$0.3 = \frac{W}{\pi[(r_1)^2 - (r_2)^2]} = \frac{20 \times 10^3}{\pi[(2r_2)^2 - (r_2)^2]} = \frac{2.12 \times 10^3}{(r_2)^2}$$

$$\therefore \quad (r_2)^2 = 2.12 \times 10^3 / 0.3 = 7.07 \times 10^3 \quad \text{or} \quad r_2 = 84 \text{ mm}$$

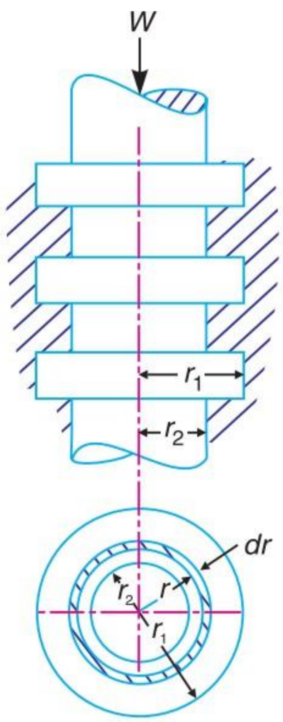
$$r_1 = 2\, r_2 = 2 \times 84 = 168 \text{ mm}$$

Flat Collar Bearing

Collar bearings are used to take the axial thrust of the rotating shafts. There may be a single collar or multiple collar bearings as shown in Fig. 10.20 (a) and (b) respectively. The collar bearings are also known as thrust bearings. The friction in the collar bearings may be found as discussed below :



Single collar bearing



Multiple collar bearing.

Consider a single flat collar bearing supporting a shaft as shown in Fig. 10.20 (a). Let r1 = External radius of the collar, and r2 = Internal radius of the collar.

∴

Area of the bearing surface, $A = \pi [(r1)^2 – (r2)^2]$

1. Considering uniform pressure

When the pressure is uniformly distributed over the bearing surface, then the intensity of pressure,

$$p = \frac{W}{A} = \frac{W}{\pi[(r_1)^2 - (r_2)^2]}$$

We have seen in Art. 10.25, that the frictional torque on the ring of radius r and thickness dr,

$$T_r = 2\pi\mu . p . r^2 . dr$$

Integrating this equation within the limits from r_2 to r_1 for the total frictional torque on the collar.

$$T = 2\pi\mu \times \frac{W}{\pi[(r_1)^2 - (r_2)^2]}\left[\frac{(r_1)^3 - (r_2)^3}{3}\right]$$

$$= \frac{2}{3} \times \mu.W\left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right]$$

Problem

A shaft has a number of a collars integral with it. The external diameter of the collars is 400 mm and the shaft diemater is 250 mm. If the intensity of pressure is 0.35 N/mm2 (uniform) and the coefficient of friction is 0.05, estimate : 1. power absorbed when the shaft runs at 105 r.p.m. carrying a load of 150 kN ; and 2. number of collars required.

Solution.

Given : d1 = 400 mm or r 1 = 200 mm ; d2 = 250 mm or r2 = 125 mm ; p = 0.35 N/mm2 ; μ = 0.05 ; N = 105 r.p.m or ω = 2 π × 105/60 = 11 rad/s ; W = 150 kN = 150 × 103 N

1. **Power absorbed**

We know that for uniform pressure, total frictional torque transmitted,

$$T = \frac{2}{3} \times \mu.W\left[\frac{(r_1)^3 - (r_2)^3}{(r_1)^2 - (r_2)^2}\right] = \frac{2}{3} \times 0.05 \times 150 \times 10^3\left[\frac{(200)^3 - (125)^3}{(200)^2 - (125)^2}\right] \text{N-mm}$$

$$= 5000 \times 248 = 1240 \times 10^3 \text{ N-mm} = 1240 \text{ N-m}$$

∴ Power absorbed,

$$P = T.\omega = 1240 \times 11 = 13640 \text{ W} = 13.64 \text{ kW}$$

2.Number of collars required

Let n = Number of collars required.

We know that the intensity of uniform pressure (p),

$$0.35 = \frac{W}{n.\pi[(r_1)^2 - (r_2)^2]} = \frac{150 \times 10^3}{n.\pi[(200)^2 - (125)^2]} = \frac{1.96}{n}$$

$\therefore$ $n = 1.96 / 0.35 = 5.6$ say 6 **Ans.**