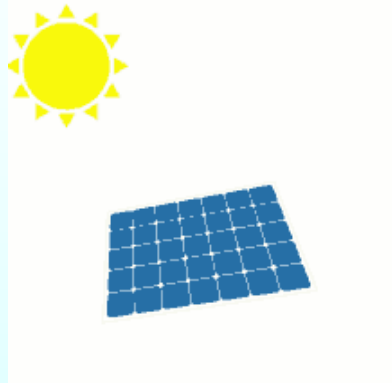


# Mutual Inductance of a coil

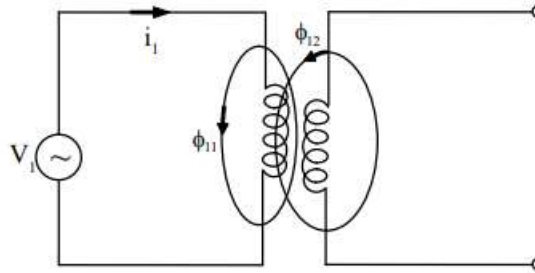


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## Mutual Inductance [M]

The mutual inductance is defined as the ability of one coil to produce emf in other coil by induction when the current in the first changes.



*Fig. 5.14 (a) Mutual Inductance*

It is also defined as the weber turns in one coil per ampere current in other coil. It is measured in Henry.

Consider the circuit shown in fig. 5.14(a). If a varying voltage source  $V_1$  is connected to coil 1, it produces a varying current  $i_1$ . Let the number of turns in coil 1 be  $N_1$ . The number of turns in coil 2 be  $N_2$ . The varying current  $i_1$  in coil produces a changing flux  $\Phi_1$  in it.

The flux  $\Phi_1$  is divided into two parts,  $\Phi_1 = \Phi_{11} + \Phi_{12}$  where  $\Phi_{11} \rightarrow$  part of flux  $\Phi_1$  which links only with coil 1,  $\Phi_{12} \rightarrow$  part of flux  $\Phi_1$  which links with both coil 1 & coil 2.

Let the induced emf in coil 2 be  $e_2$ . The induced emf  $e_2$  is proportional to the rate of change of flux in the coil 2.

$$\begin{aligned} \text{i.e; } e_2 &\propto \frac{d\Phi_{12}}{dt} \\ e_2 &= -N_2 \frac{d\Phi_{12}}{dt} \end{aligned} \quad \text{-----(20)}$$

where,  $N_2 \rightarrow$  Number of turns in coil 2.

$\frac{d\Phi_{12}}{dt} \rightarrow$  rate of change of flux linking coil 1 & coil 2.



The induced emf is also proportional to rate of change of current causing the flux  $\phi_{12}$ .  
 The current causing the flux  $\phi_{12}$  is  $i_1$ ,

$$e_2 \propto \frac{di_1}{dt}$$

$$e_2 = -M \frac{di_1}{dt} \quad \text{-----(21)}$$

where,  $M \rightarrow$  Mutual inductance between coil 1 & coil 2.

$\frac{di_1}{dt} \rightarrow$  Rate of change of current  $i_1$ ,

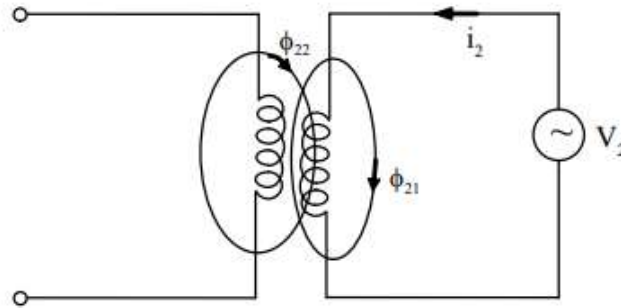
Equating equation (20) & (21),  $-N_2 \frac{d\phi_{12}}{dt} = -M \frac{di_1}{dt}$

$$\Rightarrow M = N_2 \frac{d\phi_{12}}{di_1}$$

For constant permeability,  $M = N_2 \frac{\phi_{12}}{i_1} \quad \text{-----(22)}$

If the voltage source is connected to coil 2, the circuit becomes, as shown in fig. 5.14 (b).  
 The varying voltage  $V_2$  produces a varying current  $i_2$ . Let the number of turns in coil 1 be  $N_1$  & the number of turns in coil 2 be  $N_2$ . The varying current  $i_2$  in coil 2 produces a changing flux  $\phi_2$  in it.





**Fig. 5.14 (b) Mutual Inductance with source in the second coil**

The flux  $\phi_2$  is divided into two parts,  $\phi_2 = \phi_{22} + \phi_{21}$

where  $\phi_{22} \rightarrow$  part of flux  $\phi_2$  links only with coil 2

$\phi_{21} \rightarrow$  part of flux  $\phi_2$  links with coil 2 and coil 1.

Let  $e_1 \rightarrow$  induced emf in coil 1

The induced emf  $e_1$ , is proportional to the rate of change of flux in coil 1.

$$\text{i.e., } e_1 \propto \frac{d\phi_{21}}{dt}$$

$$\Rightarrow e_1 = -N_1 \frac{d\phi_{21}}{dt} \quad \text{-----(23)}$$

where,  $N_1 \rightarrow$  Number of turns in coil 1.

$\frac{d\phi_{21}}{dt} \rightarrow$  Rate of change of flux linking coil 2 and coil 1.

The induced emf is also proportional to the rate of change of current causing the flux  $\phi_{21}$ .

The current causing the flux  $\phi_{21}$  is  $i_2$ .

$$\text{i.e., } e \propto \frac{di_2}{dt}$$

$$\Rightarrow e = -M \frac{di_2}{dt} \quad \text{-----(24)}$$

where,  $M \rightarrow$  Mutual inductance between coil 2 and coil 1.

$\frac{di_2}{dt} \rightarrow$  Rate of change of current  $i_2$ .

$$\text{Equating eq. (4) and (5) } -N_1 \frac{d\phi_{21}}{dt} = -M \frac{di_2}{dt}$$

$$\Rightarrow M = N_1 \frac{d\phi_{21}}{di_2}$$

$$\text{when permeability is constant, } M = \frac{N_1 \phi_{21}}{i_2} \quad \text{-----(25)}$$



### Coupling Coefficient [K]

It is also known as coefficient of coupling or magnetic coupling coefficient or coefficient of magnetic coupling.

It is defined as the fraction of the total flux produced by one coil linking the other coil.

$$\text{i.e., } K = \frac{\phi_{12}}{\phi_1} = \frac{\phi_{21}}{\phi_2} \quad \text{-----(26)}$$

$$\text{Multiplying equation (22) \& (25), } M^2 = \left( \frac{N_2 \phi_{12}}{i_1} \right) \left( \frac{N_1 \phi_{21}}{i_2} \right) \quad \text{-----(27)}$$

From equation (26), substitute  $\phi_{12} = k\phi_1$ ;  $\phi_{21} = k\phi_2$  in equation (27)

$$M^2 = \left( \frac{N_2 k \phi_1}{i_1} \right) \left( \frac{N_1 k \phi_2}{i_2} \right) \Rightarrow M^2 = K^2 \left( \frac{N_1 \phi_1}{i_1} \right) \left( \frac{N_2 \phi_2}{i_2} \right) \quad \text{-----(28)}$$

$$\text{we know that } L = \frac{N\phi}{i} \Rightarrow L_1 = \frac{N_1 \phi_1}{i_1} \text{ and } L_2 = \frac{N_2 \phi_2}{i_2}$$

Substituting  $L_1$  and  $L_2$  in (28)  $M^2 = K^2 L_1 L_2$

$$\Rightarrow M = K \sqrt{L_1 L_2}$$

K value depends on the spacing between coils. As spacing increases K decreases. It is also depends on coil orientation and permeability. K is always positive and its maximum value is 1.

The maximum mutual inductance occurs when  $K = 1$

$$\text{i.e. } M_{\max} = 1 \sqrt{L_1 L_2} = \sqrt{L_1 L_2}$$

For iron-core coupled circuits, K may be as high as 0.99 and is between 0.4 to 0.8 for air core coupled circuit.



**Thank You**

