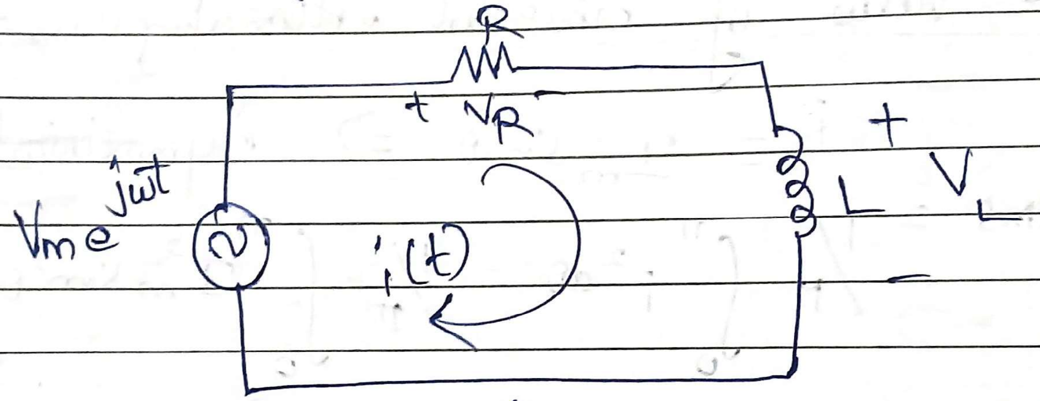
RMS value = 0.707 × Max value.

$$\text{RMS value} = \sqrt{\frac{\text{Area of the squared curve}}{\text{Base}}}$$

3.2 Complex Forcing Function

Euler's identity gives sinusoidal & exponentials are related through complex numbers.

$$e^{j\theta} = \cos\theta + j\sin\theta$$



Basic first order circuit.

$$V_m e^{j\omega t} = V_m \cos(\omega t) + j V_m \sin\omega t.$$

Apply KVL to the above circuit:

$$V_L + V_R - V_m e^{j\omega t} = 0$$

$$L\frac{di(t)}{dt} + Ri(t) = V_m e^{j\omega t} \quad \text{--- (1)}$$

Let $i(t) = I_m e^{j(\omega t + \phi)}$

$$\frac{di(t)}{d(t)} = j\omega I_m e^{j(\omega t + \phi)} \quad \text{--- (2)}$$

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Substituting equation ② in equation ①.

$$j\omega L I_m e^{j(\omega t+\phi)} + R I_m e^{j(\omega t+\phi)} = V_m e^{j\omega t}$$

$$j\omega L I_m e^{j\omega t} e^{j\phi} + R I_m e^{j\omega t} e^{j\phi} = V_m e^{j\omega t}$$

$$\because \left[e^{a+b} = e^{a} \cdot e^{b}\right]$$

$$e^{j\omega t}\left(j\omega L I_m e^{j\phi} + R I_m e^{j\phi}\right) = V_m e^{j\omega t}.$$

$$I_m (R + j\omega L) e^{j\phi} = V_m$$

$$I_m e^{j\phi} = V_m / (R + j\omega L)$$

$$\left|I_m e^{j\phi}\right| = V_m / \sqrt{R^2+\omega^2 L^2} \quad \text{— ③}$$

$$\angle\left(I_m e^{j\phi}\right) = -\tan^{-1}\frac{\omega L}{R} \quad \text{— ④}$$

We know that,

$$i(t) = I_m e^{j\phi} e^{j\omega t} = I_m e^{j(\omega t+\phi)}$$

$$= I_m\left[\cos(\omega t+\phi) + j\sin(\omega t+\phi)\right] \quad ⑤$$

Substituting equation ④ & ③ in equation ⑤

$$i(t) = \frac{V_m}{\sqrt{R^2+\omega^2 L^2}}\left\{\left[\cos\left(\omega t - \tan^{-1}\frac{\omega L}{R}\right)\right] + j\left[\sin\left(\omega t - \tan^{-1}\frac{\omega L}{R}\right)\right]\right\}$$

Real of Part $i(t)$:

$$i(t) = V_m / \sqrt{R^2+\omega^2 L^2} \cos\left(\omega t - \tan^{-1}\frac{\omega L}{R}\right)$$

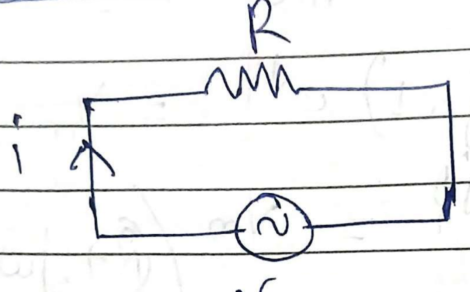
Analysis of AC Circuit : Phasor Relationship of R, L & C

(a) Purely Resistive

(b) Purely Inductive

(c) Purely Capacitive

Purely Resistive



Purely resistive circuit

$V = V_m \sin\omega t$ → Applied voltage

R = Resistance of Circuit

i = Instantaneous Current flowing in the Circuit

(i) By Ohm's law.

$$i = V/R$$

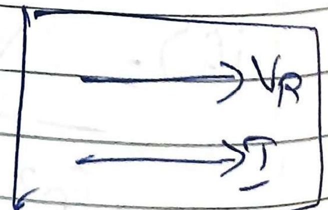
$$i = \frac{V_m \sin\omega t}{R} = I_m \sin\omega t$$

$$i = I_m \sin\omega t$$

$$I_m = V_m / R$$

$I = I_m \sin\omega t$ — (1)

$V = V_m \sin\omega t$ — (2)



Phasor Diagram.

i & V are in phase.

(2) **To represent V & I in polar form.**

Let V → Rms Value ⇒ $V = V_m/\sqrt{2}$.

I_m → RMS Value ⇒ $I = I_m/\sqrt{2}$.

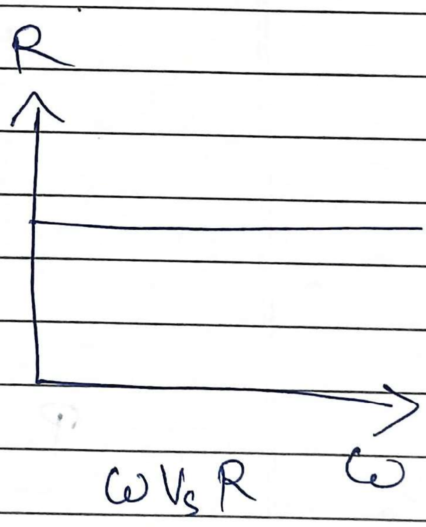
writing equation's (1) & (2) in polar form

$$V = |V| \angle 0^\circ$$
$$I = |I| \angle 0^\circ$$

(3) **Impedance (Z)**

In AC circuit, Ratio of V to I ⇒ Impedance.

$$Z = V/I = \frac{|V| \angle 0^\circ}{|I| \angle 0^\circ}$$



$$Z = \left|\frac{V}{I}\right| \angle 0^\circ$$

$$Z = |Z| \angle 0^\circ$$

(4) **Instantaneous Power (W)**

$$W = VI$$

$$W = V_m \sin \omega t \times I_m \sin \omega t = V_m \times I_m \sin^2 \omega t$$

$$W = V_m \times I_m \left[\frac{1 - \cos 2\theta}{2}\right] \quad \text{— (3)} \quad \left[\because \sin^2\theta = \frac{1 - \cos 2\theta}{2}\right]$$

At $\theta = \pi/2 \Rightarrow w \to$ maximum value $\Rightarrow$

$V_m I_m \left[\frac{1 - \cos 2\frac{\pi}{2}}{2}\right]$

~~At~~ $W = V_m I_m.$

⑤ Average Power (P)

$$P = 1/\pi \int_0^{\pi} w \, d\theta$$

Substituting equation (3) in equation (4)

$$= 1/\pi \int_0^{\pi} V_m I_m \left(\frac{1 - \cos 2\theta}{2}\right) d\theta$$

$$= \frac{V_m I_m}{2\pi} \left[\int_0^{\pi} 1 \, d\theta + 0\right]$$

$$= \frac{V_m I_m}{2\pi} (\pi)$$

$$P = \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}}$$

$$\boxed{P = VI}$$

⑥ Power Factor

The angle between the voltage & current is called phase angle & is denoted by ϕ

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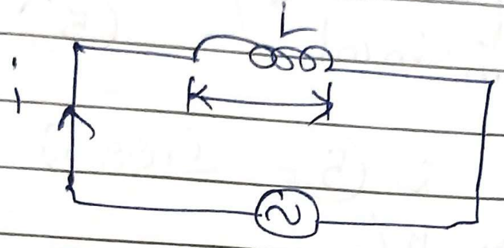
$\cos\phi$ = Power Factor

Here $\phi = 0$

$\cos 0 = 1$

Power factor = $\cos\phi = 1$

Purely Inductive



Purely Inductive Circuit.

Let $V = V_m \sin\omega t$.

i = Instantaneous current

L = Self inductance of coil.

$$V = L\, di/dt. \quad \text{—— (1)}$$

$$di = V/L\; dt.$$

Integrating on both sides

$$i = 1/L \int v\, dt = 1/L \int V_m \sin\omega t\, dt$$

$$= \frac{V_m}{\omega L}\left[-\cos\omega t\right] \quad \left[\because \cos\theta = \sin(\pi/2 - \theta)\right]$$

$$= \frac{V_m}{\omega L}\left[-\sin(\pi/2 - \omega t)\right] = \frac{V_m}{\omega L}\left[\sin(\omega t - \pi/2)\right]$$

$$i = \frac{V_m}{X_L} \sin(\omega t - \pi/2) \quad \text{—— (2)}$$

$[\because$ Inductive reactance $X_L = \omega L]$