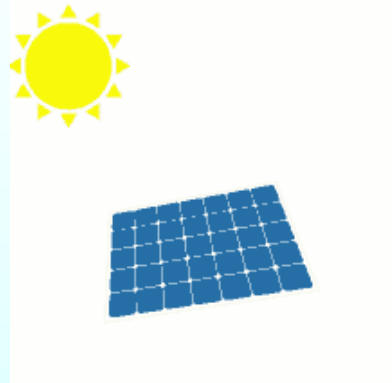# DC – Transient Response of RLC Series Circuit

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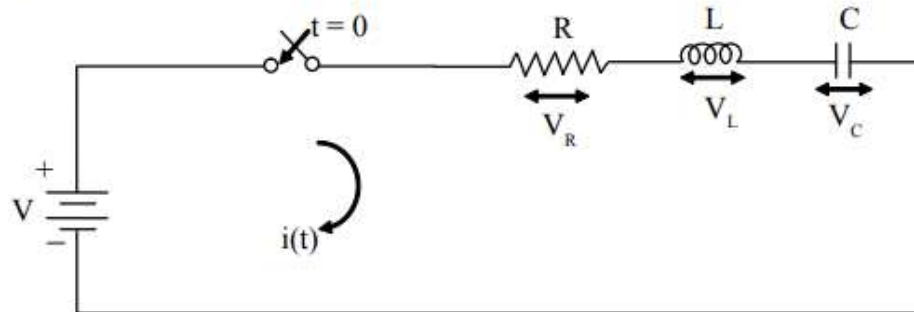
DC Response of RLC Series Circuit :

Fig. 4.10 RLC series circuit

The RLC series circuit is shown above which is excited by a DC source. Assume that at $t = 0$, the switch S is closed. While closing the switch, the voltage drop across the capacitor and current flowing through the inductor is zero.

Applying KVL to the circuit,

$$V = V_R(t) + V_L(t) + V_C(t)$$

$$V = Ri(t) + L\frac{di(t)}{dt} + \frac{1}{C}\int i(t)dt \qquad \text{---------(13)}$$

By applying laplace transform to equation (13), we get

$$\frac{V}{S} = RI(s) + L[SI(s) - I(0)] + \frac{1}{C}\left[\frac{I(s)}{S} + \frac{q(0)}{S}\right]$$

Assume $I(0) = q(0) = 0$

$$\therefore \ \frac{V}{S} = RI(S) + LSI(S) + \frac{I(S)}{CS}$$

$$= I(S)\left[R + LS + \frac{1}{CS} \right]$$

$$\frac{V}{S} = I(S)\left[\frac{RCS + LS^2C + 1}{CS} \right] = I(S)\left[\frac{S^2LC + RCS + 1}{CS} \right]$$

$$V = I(S)\left[\frac{S^2LC + SRC + 1}{C} \right]$$

$$V = I(S)\frac{LC}{C}\left[S^2 + \frac{SR}{L} + \frac{1}{LC} \right]$$

$$\therefore \ \ I(S) = \frac{V}{L\left[S^2 + \frac{R}{L}S + \frac{1}{LC} \right]} \qquad\qquad \text{---------(14)}$$

The roots of the denominator for equation (14)

$$S_1, S_2 = \dfrac{-R/L \pm \sqrt{\left(\dfrac{R}{L}\right)^2 - \dfrac{4}{LC}}}{2 \times 1}$$

$$= \dfrac{-R}{2L} \pm \sqrt{\dfrac{R^2}{4L^2} - \dfrac{1}{LC}}$$

$$S_1, S_2 = \dfrac{-R}{2L} \pm \sqrt{\left(\dfrac{R}{2L}\right)^2 - \dfrac{1}{LC}} \qquad \text{----------(15)}$$

Equation (15) can be represented by

$$S_1, S_2 = \alpha \pm \beta$$

where, $\alpha = \dfrac{-R}{2L}$ and $\beta = \sqrt{\left(\dfrac{R}{2L}\right)^2 - \dfrac{1}{LC}}$

Here, $S_1 = \alpha + \beta$, $\qquad S_2 = \alpha - \beta$

4

The natural frequency, $\omega_n = \dfrac{1}{\sqrt{LC}}$

Equation (14) can be written as

$$I(s) = \dfrac{V/L}{(s - s_1)(s - s_2)}$$

i.e, $\quad I(s) = \dfrac{V/L}{\left[s - (\alpha + \beta)\right]\left[s - (\alpha - \beta)\right]}$ ....(16)

There are three possibilities.

Case 1 : when $\left(\dfrac{R}{2L}\right)^2 > \dfrac{1}{LC}$

The two roots are real and distinct. The denominator has the roots $(\alpha + \beta)$ and $(\alpha - \beta)$ and we may write

$$I(s) = \dfrac{k_1}{s - (\alpha + \beta)} + \dfrac{k_2}{s - (\alpha - \beta)}$$

Taking inverse laplace transform,

$$i(t) = k_1 e^{(\alpha+\beta)t} + k_2 e^{(\alpha-\beta)t}$$

$$= e^{\alpha t}\left[k_1 e^{\beta t} + k_2 e^{-\beta t}\right]$$ ---------(17)

The value of k_1 and k_2 can be find by using partial fraction method. The current is said to be overdamped as in below fig. 4.11.
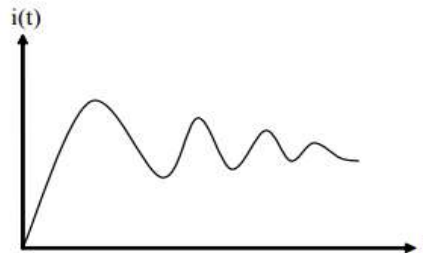

Fig. 3.11 Overdamped response

5

Case 2 : When $\left(\dfrac{R}{2L}\right)^2 = \dfrac{1}{LC}$

The root are equal and the oscillation in the circuit are just eliminated. The solution is the critically damped case.

$$I(S) = \frac{k_1}{(s-\alpha)^2} + \frac{k_2}{(s-\alpha)}$$

Taking inverse laplace transform,

$$i(t) = k_1 t e^{\alpha t} + k_2 e^{\alpha t} = e^{\alpha t} \left[k_1 t + k_2\right]$$

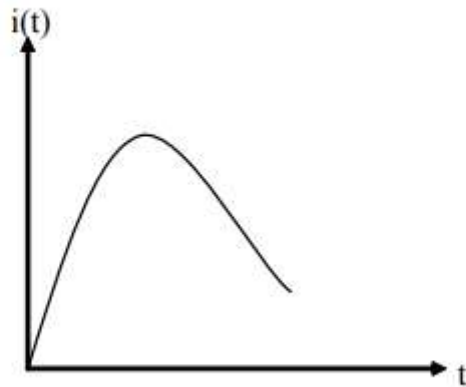The current response of i(t) for critically damped case is shown below.

Fig. 3.12 Critically damped response

Case 3 :

When $\left(\dfrac{R}{2L}\right)^2 < \dfrac{1}{LC}$

The roots are complex conjugate and the circuit is under damped as shown below.

$$I(S) = \frac{k_1}{s - (\alpha + j\beta)} + \frac{k_2}{s - (\alpha - j\beta)}$$

Taking inverse laplace transform,

$$i(t) = k_1 e^{(\alpha + j\beta)t} + k_2 e^{(\alpha - j\beta)t}$$

$$= e^{\alpha t}\left[k_1 e^{j\beta t} + k_2 e^{-j\beta t}\right]$$

Where, k_1 and k_2 are complex and are also conjugate of one another.

$$k_2 = k_1^*$$

$\therefore$ i can be rewritten as, $i = e^{\alpha t}\left[A\cos\beta t + B\sin\beta t\right]$

This solution shows that the current is oscillatory and at the same time decays in a short time as $\alpha = -R/2L$ is always negative.
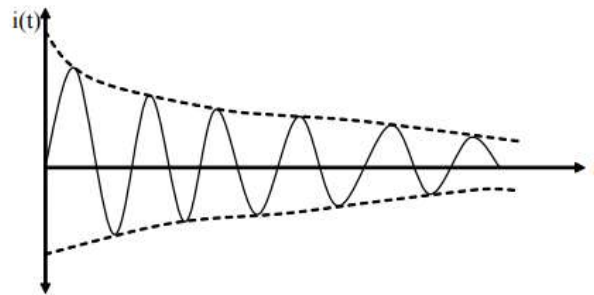

Fig. 3.13 Oscillatory response

Note : When the terms $\left(\dfrac{R}{2L}\right)^2$ and $\dfrac{1}{LC}$ are equal the oscillations are just eliminated and this condition is called "critical damping".

7

Thank You