UNIT IV – QUANTUM INFORMATION THEORY

Quantum Information Concepts – Quantum Entropy – Data Compression – Shannon's Noiseless Channel Coding Theorem – Schumacher Quantum Compression – Quantum Channels – Noisy Quantum Communication – Quantum Error Detection – Decoherence – Quantum Fidelity – Quantum Repeaters – Quantum Internet Fundamentals – Fault-Tolerant Quantum Computing.

1. Quantum Information Concepts

Introduction

Quantum information theory combines quantum mechanics with information theory. It studies how information is represented by physical quantum systems and how it can be processed, transmitted and measured. Unlike a classical bit, a qubit can be in a coherent superposition of basis states. Quantum information also uses entanglement and interference as resources.

Definition

Quantum information is information encoded in the state of a quantum system, such as a photon, electron, atom or superconducting circuit. Its basic unit is the quantum bit, or qubit.

1.1 Classical bit and qubit

A classical bit takes the value 0 or 1. A pure qubit is represented by $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$, where α and β are complex probability amplitudes and $|\alpha|^2 + |\beta|^2 = 1$. Measurement in the computational basis gives 0 with probability $|\alpha|^2$ and 1 with

probability $|\beta|^2$. A qubit is not simply an ordinary bit that stores both values; the relative phase of its amplitudes can affect later interference.

1.2 Quantum states and notation

The symbols $|0\rangle$ and $|1\rangle$ are computational basis states, often represented as column vectors $[1,0]^T$ and $[0,1]^T$. A pure state is described by a state vector. A mixed state is described by a density operator ρ, which is positive, has trace one, and can represent statistical uncertainty or a subsystem of an entangled state.

1.3 Superposition

Superposition is the linear-combination principle of quantum mechanics. If $|0\rangle$ and $|1\rangle$ are possible states, then $\alpha|0\rangle + \beta|1\rangle$ is also a valid state when normalised. A Hadamard gate transforms $|0\rangle$ into $(|0\rangle + |1\rangle)/\sqrt{2}$. Superposition does not mean that a measurement reveals both answers; one outcome is obtained, with probabilities determined by the amplitudes.

1.4 Entanglement

Entanglement is a property of a joint quantum state that cannot be written as a product of separate states for each subsystem. A Bell state such as $(|00\rangle + |11\rangle)/\sqrt{2}$ gives perfectly correlated results when both qubits are measured in the computational basis. Entanglement is a resource for teleportation, quantum networking and some quantum algorithms. It does not allow faster-than-light signalling.

1.5 Quantum interference

Quantum amplitudes, rather than probabilities alone, combine in a quantum process. Amplitudes can reinforce one another (constructive interference) or cancel

(destructive interference). Quantum algorithms arrange interference so that desirable outcomes become more likely when measured.

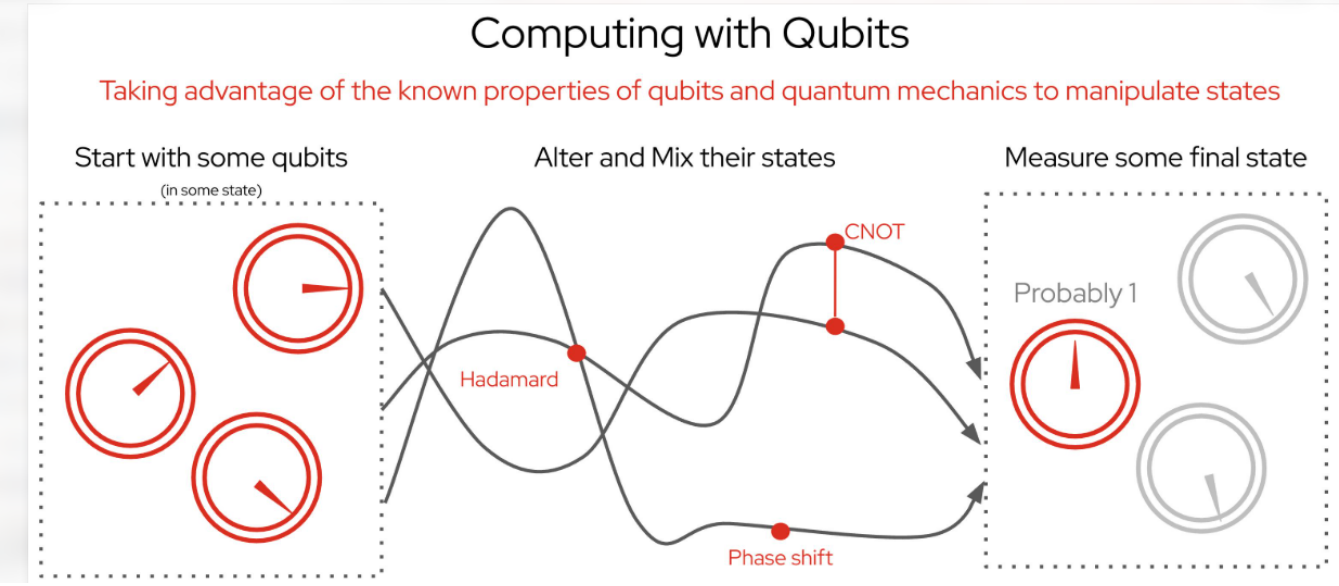
1.6 Quantum gates and circuits

Quantum gates are reversible operations represented by unitary matrices. Common single-qubit gates include Pauli-X (bit flip), Pauli-Z (phase flip) and Hadamard (superposition). The controlled-NOT gate acts on two qubits and flips the target when the control is 1. A quantum circuit is a sequence of gates applied to initialized qubits, followed by measurement.

1.7 Measurement and the no-cloning principle

Measurement converts quantum information into a classical result and generally changes the state. The Born rule gives the probabilities of outcomes. The no-cloning theorem states that no universal physical operation can make a perfect copy of every arbitrary unknown quantum state. This differs from classical data, which can be copied exactly.

Quantum Information Processing

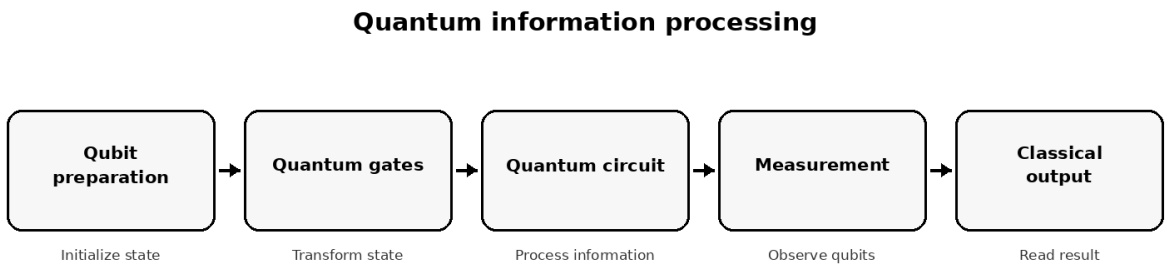


1. **Qubit Preparation:** The quantum system is initialized into a required state.
2. **Quantum Gates:** Operations are applied to change the quantum state.
3. **Quantum Processing:** The qubits interact through quantum circuits.
4. **Measurement:** The quantum state is measured.
5. **Output:** The measurement produces classical information.

1.8 Types of quantum state

Pure states are represented by a single state vector and have zero Von Neumann entropy. Mixed states are statistical mixtures and are represented by density matrices. Multipartite states may be separable or entangled, depending on whether they can be expressed as mixtures or products of subsystem states.

Diagram



The diagram shows the usual processing sequence: prepare qubits, apply gates, process the state, measure and obtain a classical output. The boxes represent functional stages, not necessarily separate physical machines.

1. **Pure Quantum State:** A quantum state that can be described by a single state vector.
2. **Mixed Quantum State:** A statistical mixture of different quantum states.
3. **Quantum Entanglement:** Information represented through correlated quantum systems.

4. **Quantum Classical Information:** Classical information obtained from measuring a quantum system.
5. **Quantum Coherent Information:** A quantity used to study the transmission of quantum information through quantum channels.

Advantages

- Provides a framework for quantum computing and communication.
- Enables quantum algorithms that offer advantages for selected tasks.
- Entanglement supports protocols such as teleportation and quantum key distribution.
- Offers new ways to represent and process physical information.

Disadvantages / limitations

- Qubits are fragile and sensitive to noise.
- Measurement can disturb an unknown state.
- Large-scale control and error correction are difficult.
- Quantum advantage is problem-dependent, not universal.

Applications

- Quantum computing and quantum algorithm design.
- Quantum key distribution and secure communication.
- Quantum teleportation and entanglement-based networks.
- Quantum simulation of molecules, materials and physical systems.
- Quantum sensing and scientific measurement.

Thus Quantum information theory provides the foundation for quantum computing and communication. Qubits, superposition, entanglement, interference, gates and measurement are its central ideas.