



ROHINI COLLEGE OF ENGINEERING & TECHNOLOGY

DEPARTMENT OF MATHEMATICS

LECTURE NOTES ON

BA4201 / QUANTITATIVE TECHNIQUES FOR DECISION MAKING

UNIT II : LINEAR PROGRAMMING EXTENSIONS

ASSIGNMENT PROBLEM

Introduction

Assignment problem is one of the special cases of transportation problems. The goal of the assignment problem is to minimize the cost or time of completing a number of jobs by a number of persons. An important characteristic of the assignment problem is the number of sources is equal to the number of destinations. It is explained in the following way.

1. Only one job is assigned to person.
2. Each person is assigned with exactly one job.

Balanced assignment problem: This is an assignment where the number of persons is equal to the number of jobs.

Unbalanced assignment problem: This is the case of assignment problem where the number of persons is not equal to the number of jobs. A dummy variable, either for a person or job (as it required) is introduced with zero cost or time to make it a balanced one.

Dummy job/ person: Dummy job or person is an imaginary job or person with zero cost or time introduced in the unbalanced assignment problem to make it balanced one.

An infeasible Assignment: Infeasible assignment occurs when a person is incapable of doing certain job or a specific job cannot be performed on a particular machine. These restrictions should be taken in to account when finding the solutions for the assignment problem to avoid infeasible assignment

Hungarian method

Step I

(A) **Row reduction:** Subtract the minimum entry of each row from all the entries of the respective row in the cost matrix.

(B) **Column reduction:** After completion of row reduction, subtract the minimum entry of each column from all the entries of the respective column.

Step II Zero assignment:

(A) Starting with first row of the matrix received in first step, examine the rows one by one until a row containing exactly one zero is found. Then an experimental assignment indicated by " " is marked to that zero. Now cross all the zeros in the column in which the assignment is made. This procedure should be adopted for each row assignment.

(B) When the set of rows has been completely examined, an identical procedure is applied successively to columns. Starting with column 1, examine all columns until a column containing exactly one zero is found. Then make an experimental assignment in that position and cross other zeros in the row in which the assignment was made.

Continue these successive operations on rows and columns until all zero's have either been assigned or crossed-out

Now there are two possibilities:

(a) Either all the zeros are assigned or crossed out. Or

(b) At least two zeros are remained by assignment or by crossing out in each row or column. This completes the second step. After this step we can get two situations.

(i) Total assigned zero's = n , the assignment is optimal.

(ii) Total assigned zero's < n , Use step III and onwards.

Step III:

Draw of minimum lines to cover zero's In order to cover all the zero's at least once you may adopt the following procedure.

Marks ($\checkmark$) to all rows in which the assignment has not been done.

(ii) See the position of zero in marked ($\checkmark$) row and then mark ($\checkmark$) to the corresponding column.

(iii) See the marked ($\sqrt$ column and find the position of assigned zero 's and then mark ($\sqrt$) to the corresponding rows which are not marked till now.

(iv) Repeat the procedure (ii) and (iii) till the completion of marking.

(v) Draw the lines through unmarked rows and marked columns.

Note: If the above method does not work then make an arbitrary assignment and then follow step IV.

Step IV:

Select the smallest element from the uncovered elements.

(i) Subtract this smallest element from all those elements which are not covered.

(ii) Add this smallest element to all those elements which are at the intersection of two lines. And continue the assignment

Problem : Consider the problem of assigning four jobs to four persons. The assignment costs are given as follows:

	I	II	III	IV
A	44	80	52	60
B	60	56	40	72
C	36	60	48	48
D	52	76	36	40

Determine the optimum assignment schedule.

Soln.

Given,

	I	II	III	IV
A	44	80	52	60
B	60	56	40	72
C	36	60	48	48
D	52	76	36	40

In Row Reduction

	I	II	III	IV
A	0	36	8	16
B	20	16	0	32
C	0	24	12	12
D	16	40	0	4

In Column Reduction

	I	II	III	IV
A	0	20	8	12
B	20	0	0	28
C	0	8	12	8
D	16	24	0	0

Allocation according to Hungarian method

	I	II	III	IV
A	0	20	8	12
B	20	0	0	28
C	0 X	8	12	8
D	16	24	0	0 X

Mark the unallocated rows (i.e. C) and mark the columns where there are zeros in the marked rows. (i.e.). Mark the rows where allocation is there in the marked columns (i.e. A). Now draw the lines through unmarked rows i.e.B and D and marked columns (i.e.) Selected the minimum element(8) among uncovered elements (where lines are not passing) and deduct it from the rest of the uncovered elements .Add the same where ever there is intersection of the lines i.e. cell (2,1) and cell (4,1). The revised table is as follows with allocations

	I	II	III	IV
A	(0)	12	0 X	4
B	20+8=28	(0)	0 X	28
C	0 X	0 X	4	(0)
D	16+8=24	24	(0)	0 X

Thus the assignment schedule is A – I , B – II, C— IV, D – III,
with the cost of $44+56+48+36 = \text{Rs. } 184$

Problem:

A batch of 4 jobs can be assigned to 5 different machines. The set up time (in hours) for each job on various machines is given below:

		Machine				
Job		1	2	3	4	5
	1	10	11	4	2	8
	2	7	11	10	14	12
	3	5	6	9	12	14
	4	13	15	11	10	7

Find the optimal assignment of jobs to machines which will minimize the total setup cost.

Soln.

The matrix of the given assignment problem is $\begin{bmatrix} 10 & 11 & 4 & 2 & 8 \\ 7 & 11 & 10 & 14 & 12 \\ 5 & 6 & 9 & 12 & 14 \\ 13 & 15 & 11 & 10 & 7 \end{bmatrix}$	since the number of rows is less than the number of columns in the cost matrix, the given assignment problem is unbalanced. $\begin{bmatrix} 10 & 11 & 4 & 2 & 8 \\ 7 & 11 & 10 & 14 & 12 \\ 5 & 6 & 9 & 12 & 14 \\ 13 & 15 & 11 & 10 & 7 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$
Row reduction $\begin{bmatrix} 8 & 9 & 2 & 0 & 6 \\ 0 & 4 & 3 & 7 & 5 \\ 0 & 1 & 4 & 7 & 9 \\ 6 & 8 & 4 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$	Column reduction $\begin{bmatrix} 8 & 9 & 2 & 0 & 6 \\ 0 & 4 & 3 & 7 & 5 \\ 0 & 1 & 4 & 7 & 9 \\ 6 & 8 & 4 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$
since each row and columns contains at least one zero, we shall make the assignment in rows and columns of this reduced cost matrix is $\begin{bmatrix} 8 & 9 & 2 & (0) & 6 \\ (0) & 4 & 3 & 7 & 5 \\ 0 & 1 & 4 & 7 & 9 \\ 6 & 8 & 4 & 3 & (0) \\ 0 & (0) & 0 & 0 & 0 \end{bmatrix}$	Cover all the zeros by drawing a minimum number of straight lines. $\begin{bmatrix} \cancel{8} & \cancel{9} & \cancel{2} & 0 & \cancel{6} \\ 0 & 4 & 3 & 7 & 5 \\ 0 & (1) & 4 & 7 & 9 \\ \cancel{6} & \cancel{8} & \cancel{4} & 3 & \cancel{0} \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

The current assignment is not optimal.	
Here 1 is the smallest cost element not covered by these straight lines. Subtract this 1 from all the uncovered elements, add this 1 to those elements which lie in the intersection of these straight lines and do not change the remaining elements which lie on the straight lines, we get	
$\begin{bmatrix} 9 & 9 & 2 & 0 & 6 \\ 0 & 3 & 2 & 6 & 4 \\ 0 & 0 & 3 & 6 & 8 \\ 7 & 8 & 4 & 3 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$	Since each row and column contains atleast one zero. $\begin{bmatrix} 9 & 9 & 2 & (0) & 6 \\ (0) & 3 & 2 & 6 & 4 \\ 0 & (0) & 3 & 6 & 8 \\ 7 & 8 & 4 & 3 & (0) \\ 1 & 0 & (0) & 0 & 0 \end{bmatrix}$

Therefore the current assignment is optimal. The optimum assignment schedule is given by $J_1 \rightarrow M_4$, $J_2 \rightarrow M_1$, $J_3 \rightarrow M_2$, $J_4 \rightarrow M_5$ and M_3 is left without any assignment. The optimum

total set up time is $2 + 7 + 6 + 7 = 22$ hours.