K- Map

The Karnaugh map or K-Map is an efficient tool for simplifying Boolean expression up to 4 variables. It is an easy method for simplification of logic expression because it does not make the use of Boolean algebra theorems. Another advantage of K-Map is that it is a visual method of simplification. However, the K-map becomes complex and inefficient when the variables in the logical expression are equal to or more than 5.

Some typical examples of K-Map of two variable, three variable, and four variables are shown in Figure-1.

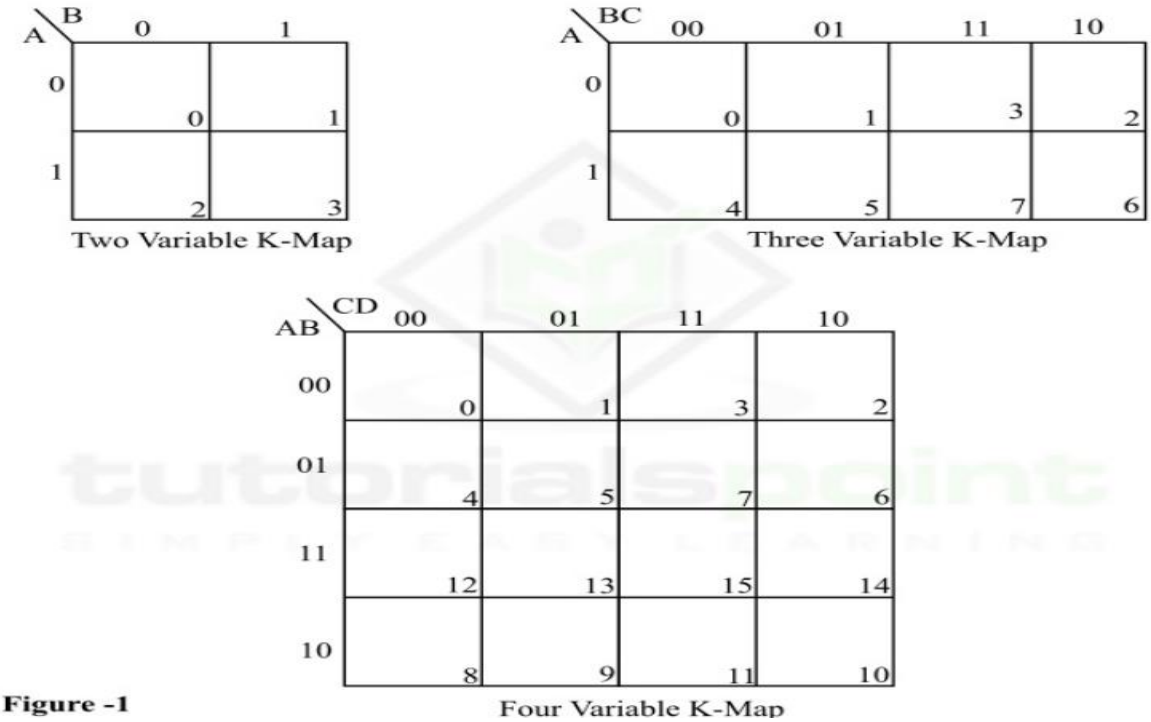


Figure -1

From Figure-1, it is clear that the number of squares or cells in the Karnaugh map depends on the number of variables in the expression.

If n is the number of variables in the given Boolean function, then the corresponding Karnaugh map (K-Map) will have 2^n squares or cells. For examples, if the number of variables in the Boolean function is 3, then the corresponding K-Map will have 8 (= 2^3) cells.

K-Map Simplification

The procedure of K-Map or Karnaugh map simplification is started with the entering the values of the variables, either in their SOP (Sum of Products) form or in POS (Product of Sums) form, in the right K-map cells. After that we need to group the maximum number of 1s (in the case of SOP form) or the maximum number of 0s (in the case of POS form). Each of these groups must be in powers of 2 and must be carried on in decreasing order only.

Once the grouping is done, each group has to be expressed in terms of combinations of input variables which are corresponding to the common binary values along the associated rows and columns. At last, all the combinations express the output expression of the Boolean function.

Advantages of Karnaugh Map

The following are the important advantages of the Karnaugh map –

- For simplifying Boolean expression, the K-map does not require the knowledge of theorems of Boolean algebra.
- Karnaugh map involves less number of steps in simplification process of logical expressions as compared to other simplification techniques.

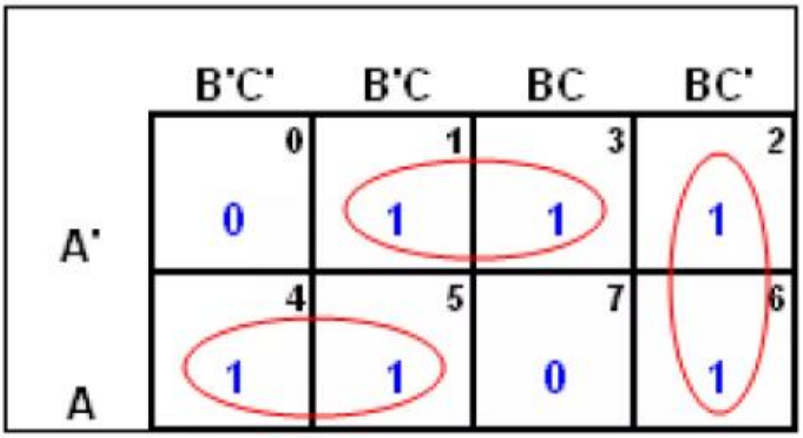
Limitations of Karnaugh Map

The following are the major limitations of the Karnaugh map –

- The most significant limitation of the Karnaugh map is that it is only efficient when the Boolean expression has less number of variables. It becomes quite complicated with the higher number of variables in the logical expression.
- The simplification of a Boolean function having more than or equal to five variables using K-Map is quite complex.
- It is very difficult to get equations correct with more than 5 variables using the K-map.

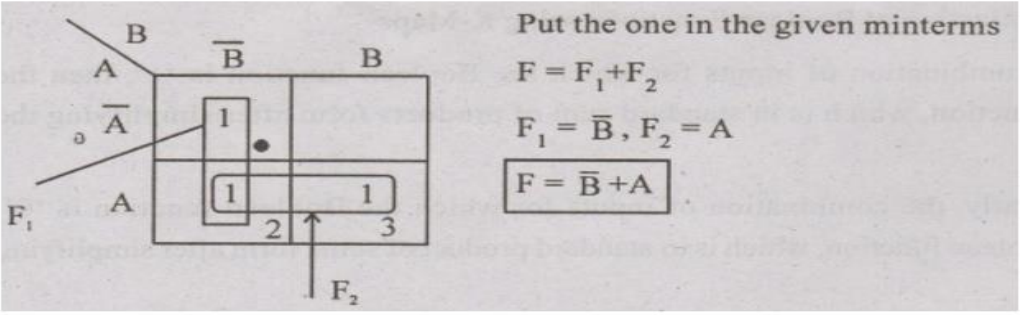
Example of three variables of K-Map.

$F(a,b,c) = \sum m(1, 2, 3, 4, 5, 6)$



$F = A'C + BC' + AB'$

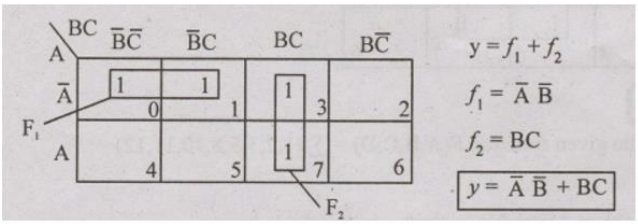
Reduce the function using K-map F(A,b) = Σ(0,2,3)



Reduce the function y(A,B,C) = Σ (0,1,3,6,7)

Solution

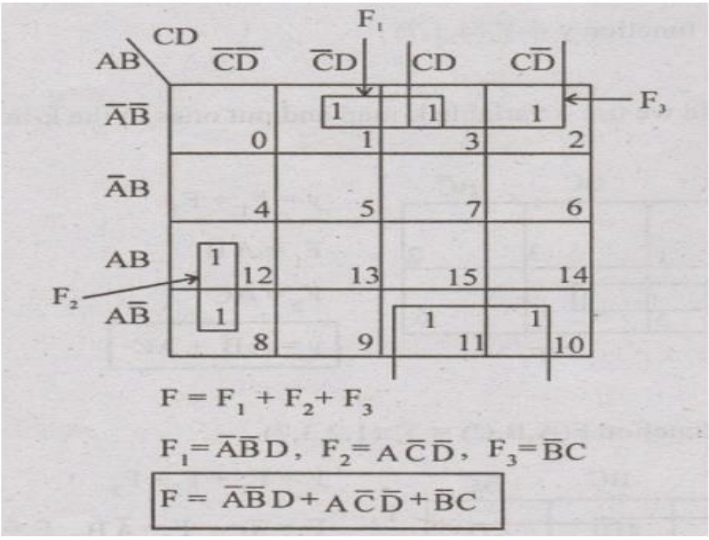
Here the 3 variable as in the function so we draw 3 variable k-map and fill ones in the given minterms



Reduce the given function F(A,B,C,D) = Σ (1,2,3,5,8,10,11,12)

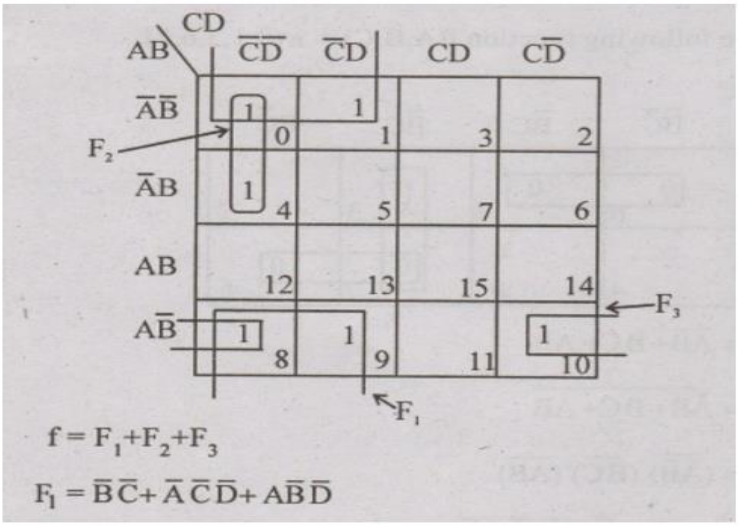
Solution:

In this problem 4 variable is given use 4 variable k-map



Reduce the following using karnaugh map f (A, B, C, D) = Σ_m (0,1,4,8,9,10)

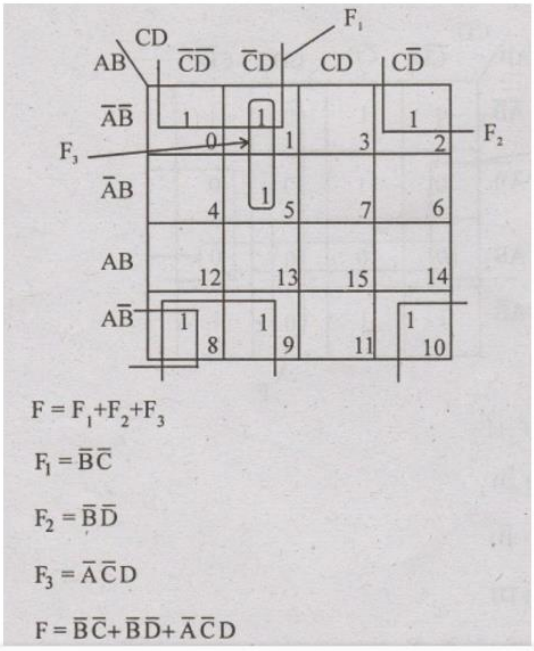
Solution:



Simplify the following in (i) SOP (ii) POS and implement in basic gates F(A, B, C, D) = E_m(0,1,2,5,8,9,10)

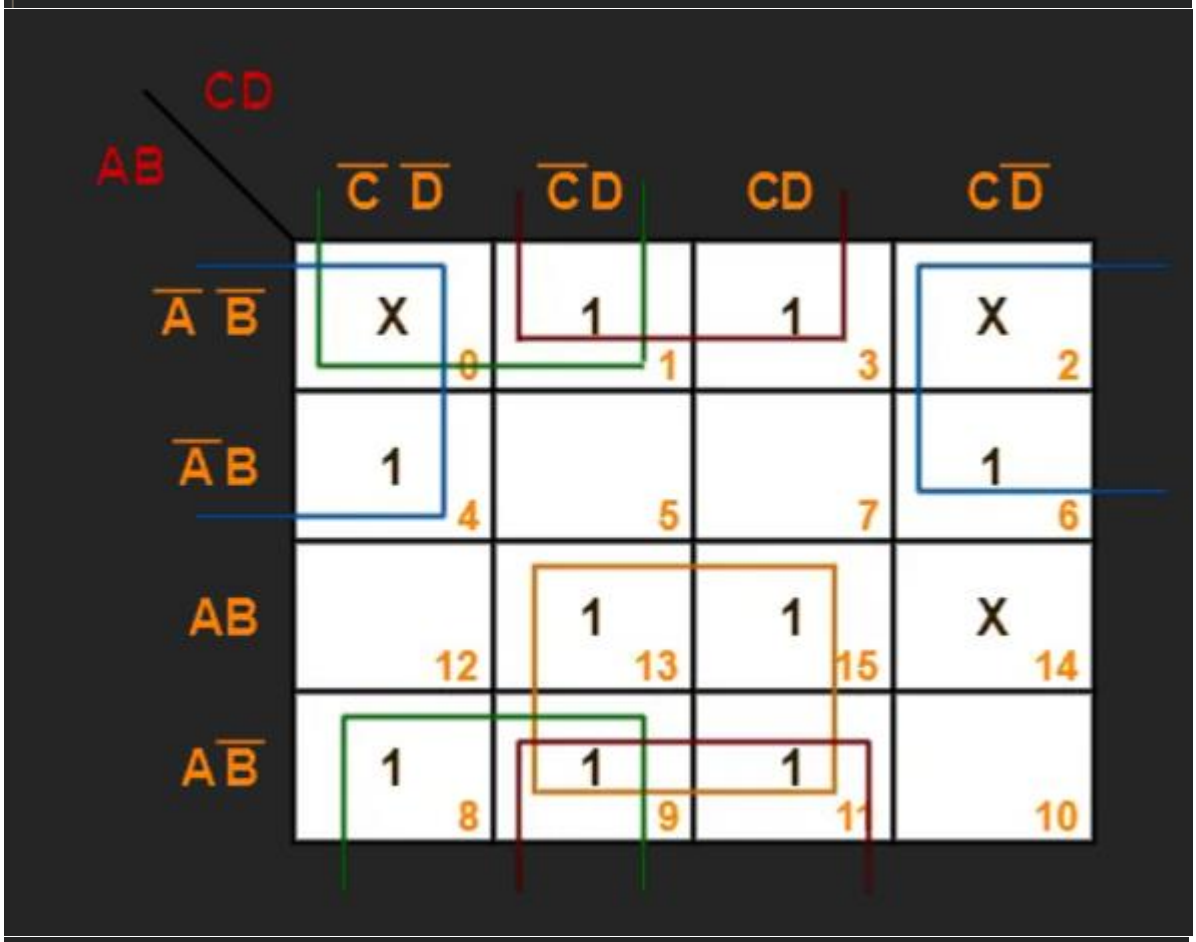
Solution:

(i) SOP put one's in the given numbers



Minimize the following boolean function-

$$F(A, B, C, D) = \Sigma m(1, 3, 4, 6, 8, 9, 11, 13, 15) + \Sigma d(0, 2, 14)$$

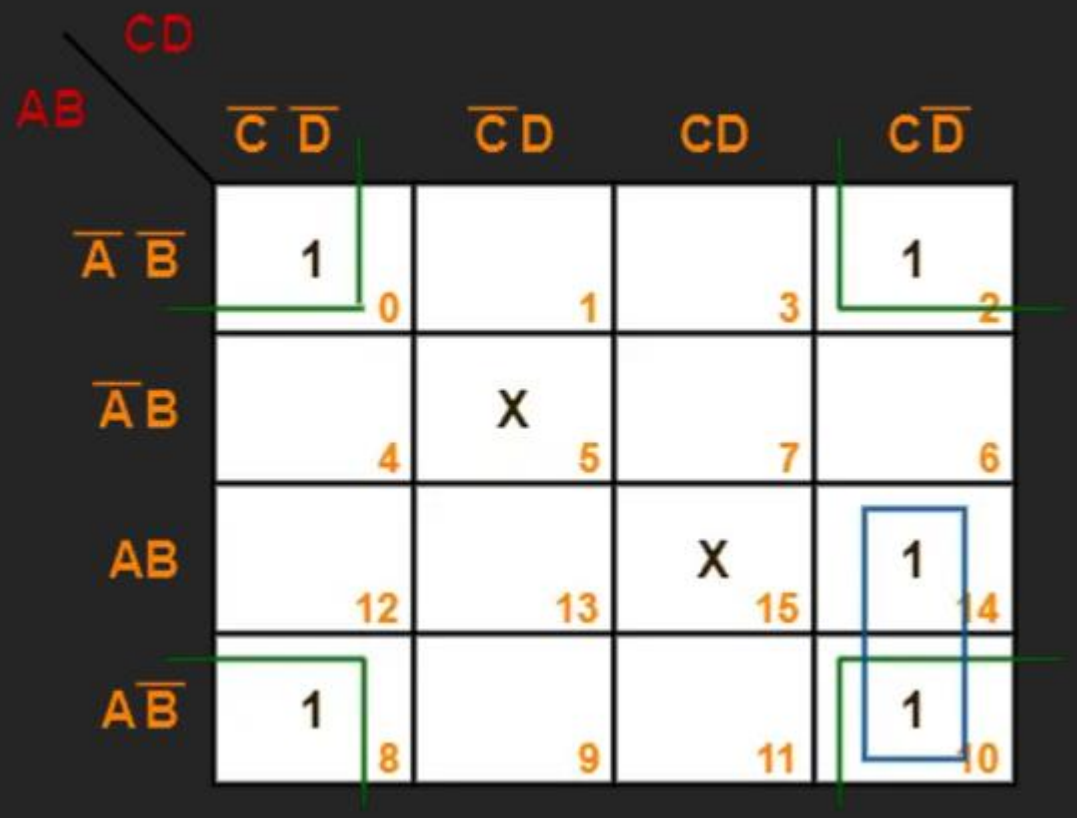


s, minimized boolean expression is-

F(A, B, C, D) = AD + B'D + B'C' + A'D'

Minimize the following boolean function-

$$F(A, B, C, D) = \Sigma m(0, 2, 8, 10, 14) + \Sigma d(5, 15)$$



s, minimized boolean expression is-

F(A, B, C, D) = ACD' + B'D'