Poynting Vector and Flow of Power:

Poynting vector and Poynting theorem:

For an electromagnetic [EM] waves an energy can be transported from transmitter to receiver. The energy stored in anelectric field and magnetic field is transmitted at a certain rate of energy flow which can be calculated by Poynting theorem.

E- Electric field in V/m.

H- Magnetic field in A/m.

The product of E and H gives power density which is expressed in watt per unit area [watt/m^2].

$$P = EH\ w/m^2$$

E and H are vectors. The power density may carry out either dot product or cross product. The result of dot product is always a scalar quantity. But power flow is a vector quantity. The power radiated from antenna has a particular direction. The power density must be calculated by using a cross product of $\overline{E}$ and $\overline{H}$.

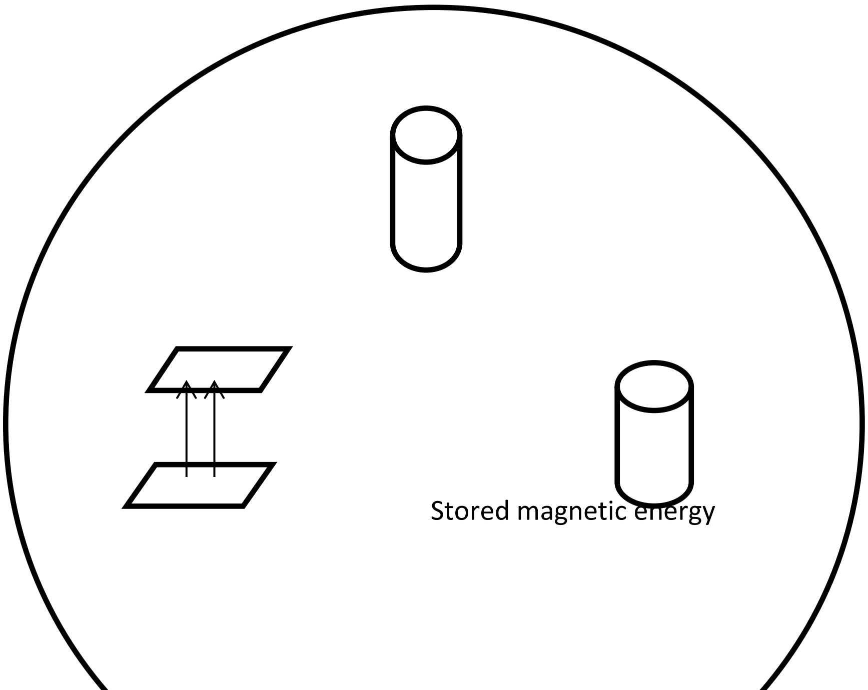
The power density is given by,

$\overline{P} = \overline{E} \times \overline{H}$ …….. (1)

Where $\overline{P}$- Poynting vector named after an English physicist John N. Poynting.

$\overline{P}$ is the instantaneous power density vector associated with the electromagnetic (EM) field at a given point. The direction of P indicates the power flow [instantaneous] at the point. To geta net power flowing out of any surface$\overline{P}$isintegrated over same closed surface.

Fig: Power balance representation in electromagnetic fields.



The Poynting theorem is based on law of conversation of energy in electromagnetism.

Poynting Theorem:

The vector product $P = \overline{E} \times \overline{H}$ at any point is a measure of the rule of energy flow per unit area at that point. The direction of flow is perpendicular to E and H in the direction of vector.

The net power flowing out of a given volume V is equal to the time rate of decrease in the energy stored within volume V minus the ohmic power dissipated.

E=Ex ax

H=Hy ay

P=E × H

=Ex ax × Hy ay

=Ex Hy ax × ay=Ex Hy az

P=Pz az…….. (2)

In the above equation E, H and P are mutually perpendicular to each other.

The electric field propagates in free space is given by,

$$E = [E_m \cos(\omega t - \beta z)]a$$

In the medium the ratio of magnitudes of E and H is called the intrinsic impedance η.

$$\eta = \eta_{0} = \frac{E_m}{H_m} = 120\pi = 377\Omega$$

In the free space, electromagnetic wave travels at a speed of light.

$E = [E_m \cos(\omega t - \beta z)]ax$……. (3)

$H = \left[\frac{E_m}{\eta_0}\cos(\omega t - \beta z)\right] ay$........ (4)

According to Poynting theorem,

$$P = E \times H$$

$$P = [E_m \cos(\omega t - \beta z)]ax \times \left[\frac{E_m}{\eta_0}\cos(\omega t - \beta z)\right] ay$$

$P = \frac{E_{m^2}}{\eta_0} cos^2(\omega t - \beta z)az \quad W/m^2$........ (5)

The above equation represents the power density in $Watt/m^2$ .The power passing through a particular area is given by,

$Power = Power\ density \times Area$ .... (6)

Average Power Density (Pavg):

To find the average power density integrate power density in z-direction over one cycle and divide by the period T, of one cycle.

$P_{avg} = \frac{1}{T}\int_0^T \frac{E_{m^2}}{\eta_0} cos^2(\omega t - \beta z).dt$......... (7)

$$= \frac{E_{m^2}}{T\,\eta_0}\int_0^T \frac{1 + cos2(\omega t - \beta z)}{2}.dt$$

$$= \frac{E_{m^2}}{T\,\eta_0}\left[\frac{t}{2} + \frac{sin2(\omega t - \beta z)}{2}\right]_0^T$$

$$= \frac{E_{m^2}}{T\,\eta_0}\left\{\left[\frac{T}{2} + \frac{sin2(\omega t - \beta z)}{4\omega}\right] - \left[\frac{0}{2} + \frac{sin2(0 - \beta z)}{4\omega}\right]\right.$$

$= \frac{E_{m^2}}{T\,\eta_0}\left[\frac{T}{2} + \frac{\sin(2\omega T - 2\beta z)}{4\omega} - \frac{\sin(-2\beta z)}{4\omega}\right]$ .... (8)

Sub $\omega T = 2\pi$

$$= \frac{E_{m^2}}{T\,\eta_0}\left[\frac{T}{2} + \frac{\sin(2 \times 2\pi - 2\beta z)}{4\omega} - \frac{\sin(-2\beta z)}{4\omega}\right]$$

$= \frac{E_{m^2}}{T\,\eta_0}\left[\frac{T}{2} + \frac{\sin[4\pi - 2\beta z]}{4\omega} + \frac{sin2\beta z}{4\omega}\right]\{\sin(-\theta) = -sin\theta\}$

$$P_{avg} = \frac{E_{m^2}}{T\ \eta_0}[\frac{T}{2} + \frac{\sin(-2\beta z)}{4\omega} + \frac{sin2\beta z}{4\omega}]$$

$$P_{avg} = \frac{Em^2}{T\eta}[\frac{T}{2} - \frac{\sin 2\beta z}{4\omega} + \frac{\sin 2\beta z}{4\omega}]$$

$$P_{avg} = \frac{Em^2}{T\eta} \times \frac{T}{2} = \frac{Em^2}{2\eta}$$

$$\eta = \eta$$

$$P_{avg} = \frac{Em^2}{2\eta} \qquad (9)$$

Hence the average power is

$$P_{avg} = \frac{1}{2}\frac{Em^2}{\eta}\ \mathrm{W}/m^2$$