3.1 INTRODUCTION

A transformation is an operation which converts a mathematical expression to a different but equivalent form. The well known transformation logarithms reduce multiplication and division to a simpler process of addition subtraction.

The Laplace transform is a powerful mathematical technique which solves linear equations with given initial conditions by using algebra methods. The Laplace transform can also be used to solve systems of differential equations, Partial differential equations and integral equations. In this chapter, we will discuss about the definition, properties of Laplace transform and derive the transforms of some functions which usually occur in the solution of linear differential equations.

3.1(a) LAPLACE TRANSFORM

Let $f(t)$ be a function of t defined for all $t \geq 0$ .then the Laplace transform of $f(t)$, denoted by $L[\,f(t)]$ is defined by

$$L[\,f(t)] = \int_0^\infty e^{-st} f(t)dt$$

Provided that the integral exists, “s” is a parameter which may be real or complex. Clearly $L[\,f(t)]$ is a function of s and is briefly written as $F(s)$ $(i.e.)$ $L[\,f(t)] = F(s)$

Piecewise continuous function

A function $f(t)$ is said to be piecewise continuous is an interval $a \leq t \leq b$, if the interval can be sub divided into a finite number of intervals in each of which the function is continuous and has finite right and left hand limits.

Exponential order

A function $f(t)$ is said to be exponential order if $\lim_{t\to\infty} e^{-st} f(t)$ is a finite quantity, where $s > 0$(exists).

Example: 1 .Show that the function $f(t) = e^{t^3}$ is not of exponential order.

Solution:

$$\lim_{t\to\infty} e^{-st}\, e^{t^3} = \lim_{t\to\infty} e^{-st+t^3} = \lim_{t\to\infty} e^{t^3-st}$$

$$= e^{\infty} = \infty, \text{ not a finite quantity.}$$

Hence $f(t) = e^{t^3}$ is not of exponential order.

Sufficient conditions for the existence of the Laplace transform

The Laplace transform of $f(t)$ exists if

i) $f(t)$ is piecewise continuous in the interval $a \leq t \leq b$

ii) $f(t)$ is of exponential order.

Note: The above conditions are only sufficient conditions and not a necessary condition.

Example: 2. Prove that Laplace transform of e^{t^2} does not exist.

Solution:

$$\lim_{t\to\infty} e^{-st}\, e^{t^2} = \lim_{t\to\infty} e^{-st+t^2} = \lim_{t\to\infty} e^{t^2-st}$$

$$= e^{\infty} = \infty \text{ ,not a finite quantity.}$$

$\therefore e^{t^2}$ is not of exponential order.

Hence Laplace transform of e^{t^2} does not exist.

3.1(b) PROPERTIES OF LAPLACE TRANSFORM

Property: 1 Linear property

$\boldsymbol{L[af(t) \pm bg(t)] = aL[f(t)] \pm bL[g(t)]}$ **, where a and b are constants.**

Proof:

$$L[af(t) \pm bg(t)] = \int_0^\infty [af(t) \pm bg(t)]\, e^{-st} dt$$

$$= a\int_0^\infty f(t) e^{-st} dt \pm b\int_0^\infty g(t)\, e^{-st} dt$$

$$\boxed{L[af(t) \pm bg(t)] = a\, L[f(t)] \pm b\, L[g(t)]}$$

Property: 2 Change of scale property.

If $\boldsymbol{L[f(t)] = F(s)}$, **then** $\boldsymbol{L[f(at)] = \frac{1}{a} F\left(\frac{s}{a}\right)}$; $\boldsymbol{a} > 0$

Proof:

Given $L[f(t)] = F(s)$

$$\therefore \int_0^\infty e^{-st} f(t)\, dt = F(s) \cdots\cdots (1)$$

By the definition of Laplace transform, we have

$$L[f(at)] = \int_0^\infty e^{-st} f(at)\, dt \cdots\cdots (2)$$

Put at= x $ie., t = \frac{x}{a} \Rightarrow dt = \frac{dx}{a}$

$$(2) \Rightarrow L[f(at)] = \int_0^\infty e^{\frac{-sx}{a}} f(x) \frac{dx}{a}$$

$$= \frac{1}{a}\int_0^\infty e^{\frac{-sx}{a}} f(x) dx$$

Replace x by t, $L[f(at)] = \frac{1}{a}\int_0^\infty e^{\frac{-st}{a}} f(t) dt$

$$\boxed{\boldsymbol{L[f(at)] = \frac{1}{a} F\left(\frac{s}{a}\right)} \;;\; \boldsymbol{a} > 0}$$

Property: 3 First shifting property.

If $L[f(t)] = F(s)$, then i) $\quad L[e^{-at}f(t)] = F(s+a)$

ii) $\quad L[e^{at}f(t)] = F(s-a)$

Proof:

(i) $L[e^{-at}f(t)] = F(s+a)$

Given $L[f(t)] = F(s)$

$$\therefore \int_0^{\infty} e^{-st} f(t)\, dt = F(s) \cdots (1)$$

By the definition of Laplace transform, we have

$$L[e^{-at}f(at)] = \int_0^{\infty} e^{-st}\, e^{-at} f(t)\, dt$$

$$= \int_0^{\infty} e^{-(s+a)t} f(t)\, dt$$

$$= F(s+a) \quad \text{by (1)}$$

(ii) $L[e^{at}f(at)] = \int_0^{\infty} e^{-st}\, e^{at} f(t)\, dt$

$$= \int_0^{\infty} e^{-(s-a)t} f(t)\, dt$$

$$= F(s-a) \quad \text{by (1)}$$

Property: 4 Laplace transforms of derivatives $L[f'(t)] = sL[f(t)] - f(0)$

Proof:

$$L[f'(t)] = \int_0^{\infty} e^{-st} f'(t)\, dt = \int_0^{\infty} u\,dv$$

$$= [uv]_0^{\infty} - \int u\,dv$$

$$= [e^{-st} f(t)]_0^{\infty} - \int_0^{\infty} f(t)\,(-s)e^{-st}\, dt$$

$$= 0 - f(0) + sL[f(t)]$$

$$= sL[f(t)] - f(0)$$

$$L[f'(t)] = sL[f(t)] - f(0)$$

$u = e^{-st}$
$\therefore du = -se^{-st}dt$
$dv = f'(t)dt$
$\therefore v = \int f'(t)dt$
$= f(t)$

Property: 5 Laplace transform of derivative of order n

$$L[f^n(t)] = s^n L[f(t)] - s^{n-1} f(0) - s^{n-2} f'(0) \cdots - s^{n-3} f''(0) - \cdots f^{n-1}(0)$$

Proof:

We know that $L[f'(t)] = sL[f(t)] - f(0) \cdots\cdots (1)$

$$L[f^n(t)] = L[[f'(t)]']$$

$$= sL[f'(t)] - f'(0)$$

$$= s[sL[f(t)] - f(0)] - f'(0)$$

$$= s^2 L[f(t)] - sf(0) - f'(0)$$

Similarly, $L[f'''(t)] = s^3 L[f(t)] - s^2 f(0) - sf'(0) - f''(0)$

In general, $\boldsymbol{L[f^n(t)] = s^n L[f(t)] - s^{n-1}\ f(0) - s^{n-2} f'(0) \cdots - s^{n-3}\ f''(0) - \cdots f^{n-1}\ (0)}$

Laplace transform of integrals

Theorem: 1 If $\boldsymbol{L[f(t)] = F(s)}$**, then** $\boldsymbol{L\left[\int_0^t f(t)dt\right] = \frac{F(s)}{s}}$

Proof:

Let $g(t) = \int_0^t f(t)dt$

$\therefore g'(t) = f(t)$

And $g(0) = \int_0^0 f(t)dt = 0$

Now $L[g'(t)] = L[f(t)]$

$sL[g(t)] - g(0) = L[f(t)]$

$sL[g(t)] = L[f(t)] \quad \therefore g(0) = 0$

$L[g(t)] = \frac{L[f(t)]}{s}$

$\therefore \boldsymbol{L\left[\int_0^t f(t)dt\right] = \frac{F(s)}{s}}$

Theorem: 2 If $\boldsymbol{L[f(t)] = F(s)}$**, then** $\boldsymbol{L[tf(t)] = -\frac{d}{ds}F(s)}$

Proof:

Given $L[f(t)] = F(s)$

$\therefore \int_0^\infty e^{-st}\ f(t)\ dt = F(s) \cdots\cdots (1)$

Differentiating (1) with respect to s, we get

$\frac{d}{ds}\int_0^\infty e^{-st}\ f(t)\ dt = \frac{d}{ds}F(s)$

$\int_0^\infty \frac{\partial}{\partial s}(e^{-st})f(t)\ dt = \frac{d}{ds}F(s)$

$\int_0^\infty (-t)e^{-st}\ f(t)\ dt = \frac{d}{ds}F(s)$

$-\int_0^\infty e^{-st}\ f(t)\ dt = \frac{\boldsymbol{d}}{\boldsymbol{ds}}F(s)$

$-L[tf(t)] = \frac{d}{ds}F(s)$

$\therefore \boldsymbol{L[tf(t)] = -\frac{d}{ds}F(s)}$

Note: In general $\boldsymbol{L[t^n f(t)] = (-1)^n \frac{d^n}{ds^n}F(s)}$

Example: If $\boldsymbol{L[f(t)] = \frac{s^2-s+1}{(2s+1)^2(s-1)}}$ **then find** $\boldsymbol{L[f(2t)]}$**.**

Solution:

Given $L[f(t)] = \frac{s^2-s+1}{(2s+1)^2(s-1)} = F(s)$

$$L[f(2t)] = \frac{1}{2} F\left(\frac{s}{2}\right)$$

$$= \frac{1}{2} \frac{\left(\frac{s}{2}\right)^2 - \frac{s}{2} + 1}{\left(2\frac{s}{2}+1\right)^2 \left(\frac{s}{2}-1\right)}$$

$$= \frac{1}{2} \frac{\left[\frac{s^2}{4} - \frac{s}{2} + 4\right]}{(s+1)^2 \left(\frac{s-2}{2}\right)}$$

$$= \frac{s^2 - 2s + 1}{4(s+1)^2(s-2)}$$

Laplace transform of some Standard functions

Result: 1 Prove that $L[t^n] = \frac{\Gamma(n+1)}{s^{n+1}}$

Proof:

We know that $L[f(t)] = \int_0^\infty e^{-st} f(t)\, dt$

$$L[t^n] = \int_0^\infty e^{-st} t^n\, dt$$

$$L[t^n] = \int_0^\infty e^{-u} \left(\frac{u}{s}\right)^n \frac{du}{s}$$

$$= \int_0^\infty e^{-u} \frac{u^n}{s^{n+1}}\, du$$

$$= \frac{1}{s^{n+1}} \int_0^\infty e^{-u} u^n\, du$$

$\therefore \boldsymbol{L[t^n] = \frac{\Gamma(n+1)}{s^{n+1}}}$ $\qquad \because \int_0^\infty e^{-u} u^n\, du$

Let $st = u \cdots\cdots (1)$

$t = \frac{u}{s}$

$dt = \frac{du}{s}$

When $t \to 0 (1) => u \to 0$

,

$t \to \infty, (1) => u \to \infty$

Note: If n is an integer, then $\Gamma(n+1) = n!$

$\therefore L[t^n] = \frac{n!}{s^{n+1}}$ if n is an integer

If $n = 0$, then $L[1] = \frac{1}{s}$

If $n = 1$, then $L[t] = \frac{1}{s^2}$

Similarly $L[t^2] = \frac{2!}{s^3}$

$L[t^3] = \frac{3!}{s^4}$

Result: 2 Prove that $L(e^{at}) = \frac{1}{s-a}$, $s > a$

Proof:

We know that $L[f(t)] = \int_0^\infty e^{-st} f(t)\, dt$

$$\therefore L(e^{at}) = \int_0^\infty e^{-st} e^{at}\, dt$$

$$= \int_0^\infty e^{-t(s-a)} f(t)\, dt$$

$$= \left[\frac{e^{-t(s-a)}}{-(s-a)}\right]_0^\infty$$

$$= -\left[0 - \left(\frac{1}{s-a}\right)\right]$$

$$\therefore L(e^{at}) = \frac{1}{s-a}$$

Result: 3 Prove that $L(e^{-at}) = \frac{1}{s+a}$, $s > a$

Proof:

We know that $L[f(t)] = \int_0^\infty e^{-st} f(t)\, dt$

$$\therefore L(e^{-at}) = \int_0^\infty e^{-st} e^{-at}\ dt$$

$$= \int_0^\infty e^{-t(s+a)} f(t)\, dt$$

$$= \left[\frac{e^{-t(s+a)}}{-(s+a)}\right]_0^\infty$$

$$= -\left[0 - \left(\frac{1}{s+a}\right)\right]$$

$$\therefore L(e^{at}) = \frac{1}{s+a}$$

Result: 4 Prove that $L[sinat] = \frac{a}{s^2+a^2}$

Proof:

We know that $L[f(t)] = \int_0^\infty e^{-st} f(t)\, dt$

$$L[sinat] = \int_0^\infty e^{-st}\ sinat\ dt$$

$\therefore L[sinat] = \frac{a}{s^2+a^2}, s > |a|$ $\left[\because \int_0^\infty e^{-at}\ sinbt\ dt = \frac{b}{a^2+b^2}\right]$

Result: 5 Prove that $L[cosat] = \frac{s}{s^2+a^2}$

Proof:

We know that $L[f(t)] = \int_0^\infty e^{-st} f(t)\, dt$

$$L[cosat] = \int_0^\infty e^{-st}\ cosat\ dt$$

$\therefore L[cosat] = \frac{s}{s^2+a^2}, s > |a|$ $\because \int_0^\infty e^{-at}\ cosbt\ dt = \frac{a}{a^2+b^2}$

Result: 6 Prove that $L[sinhat] = \frac{a}{s^2-a^2}$, $s > |a|$

Proof:

We have $L[sinhat] = L\left[\frac{e^{at}-e^{-at}}{2}\right]$

$$= \frac{1}{2}[L(e^{at}) - L(e^{-at})]$$

$$= \frac{1}{2}\left[\frac{1}{s-a} - \frac{1}{s+a}\right]$$

$$= \frac{1}{2}\left[\frac{s+a-s+a}{s^2-a^2}\right]$$

$$= \frac{1}{2}\left[\frac{2a}{s^2-a^2}\right]$$

$$\therefore L[sinhat] = \frac{a}{s^2-a^2} \quad , s > |a|$$

Result: 7 Prove that $L[coshat] = \frac{s}{s^2-a^2} \quad , s > |a|$

Proof:

We have $L[coshat] = L\left[\frac{e^{at}+e^{-at}}{2}\right]$

$$= \frac{1}{2}[L(e^{at}) + L(e^{-at})]$$

$$= \frac{1}{2}\left[\frac{1}{s-a} + \frac{1}{s+a}\right]$$

$$= \frac{1}{2}\left[\frac{s+a+s-a}{s^2-a^2}\right]$$

$$= \frac{1}{2}\left[\frac{2s}{s^2-a^2}\right]$$

$$\therefore L[coshat] = \frac{s}{s^2-a^2} \quad , s > |a|$$

Example: Find $L\left[t^{\frac{1}{2}}\right]$

Solution:

We have $L[t^n] = \frac{\Gamma(n+1)}{s^{n+1}}$

Put $n = \frac{1}{2}$

$$\therefore L\left[t^{\frac{1}{2}}\right] = \frac{\Gamma\left(\frac{1}{2}+1\right)}{s^{\frac{1}{2}+1}} \qquad \because \Gamma(n+1) = n\Gamma n$$

$$= \frac{\frac{1}{2}\Gamma\left(\frac{1}{2}\right)}{s^{\frac{1}{2}+1}} \qquad \because \Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$$

$$= \frac{\sqrt{\pi}}{2s^{\frac{3}{2}}}$$

$$\therefore L\left[t^{\frac{1}{2}}\right] = \frac{\sqrt{\pi}}{2s\sqrt{s}}$$

Example: Find the Laplace transform of $t^{-\frac{1}{2}}$ or $\frac{1}{\sqrt{t}}$

Solution:

We have $L[t^n] = \frac{\Gamma(n+1)}{s^{n+1}}$

Put $n = -\frac{1}{2}$

$$\therefore L\left[t^{-\frac{1}{2}}\right] = \frac{\Gamma\left(-\frac{1}{2}+1\right)}{s^{-\frac{1}{2}+1}} \qquad \because \Gamma(n+1) = n\Gamma n$$

$= \frac{\Gamma\left(\frac{1}{2}\right)}{s^{\frac{1}{2}}}$ $\quad\quad\quad \because \Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$

$= \frac{\sqrt{\pi}}{\sqrt{s}}$

$\therefore L\left[\frac{1}{\sqrt{t}}\right] = \sqrt{\frac{\pi}{s}}$

FORMULA

$L[f(t)] = F(s)$	$L[f(t)] = F(s)$
$L[1] = \frac{1}{s}$ $L[t] = \frac{1}{s^2}$ $L[t^n] = \frac{\Gamma(n+1)}{s^{n+1}}$ **if n is not an integer** $L[t^n] = \frac{n!}{s^{n+1}}$ **if n is an integer** $L(e^{at}) = \frac{1}{s-a}$ $L(e^{at}) = \frac{1}{s+a}$	$L[sinat] = \frac{a}{s^2 + a^2}$ $L[cosat] = \frac{s}{s^2 + a^2}$ $L[coshat] = \frac{s}{s^2 - a^2}$ $L[sinhat] = \frac{a}{s^2 - a^2}$

Problems using Linear property

Example: **Find the Laplace transform for the following**

i.	$3t^2 + 2t + 1$	**v.**	$sin\sqrt{2}\, t$	**ix.**	$sin^2 t$
ii.	$(t+2)^3$	**vi.**	$sin(at+b)$	**x.**	$cos^2 2t$
iii.	a^t	**vii.**	$cos^3 2t$	**xi.**	$cos5tcos4t$
iv.	e^{2t+3}	**viii.**	$sin^3 t$		

Solution:

(i) Given $f(t) = 3t^2 + 2t + 1$

$$L[f(t)] = L[3t^2 + 2t + 1]$$
$$= L[3t^2] + L[2t] + L[1]$$
$$= L[3t^2] + L[2t] + L[1]$$
$$= 3L[t^2] + 2L[t] + L[1]$$
$$= 3\frac{2}{s^3} + 2\frac{1}{s^2} + \frac{1}{s}$$

$$\therefore L[3t^2 + 2t + 1] = \frac{6}{s^3} + \frac{2}{s^2} + \frac{1}{s}$$

(ii) Given $f(t) = (t+2)^3 = t^3 + 3t^2(2) + 3t2^2 + 2^3$

$$L[f(t)] = L[t^3 + 3t^2(2) + 3t2^2 + 2^3]$$

$$= L[t^3] + L[6t^2] + L[12t] + L[8]$$

$$= L[t^3] + 6L[t^2] + 12L[t] + 8L[1]$$

$$= \frac{6}{s^4} + \frac{12}{s^3} + \frac{12}{s^2} + \frac{12}{s}$$

(iii) Given $f(t) = a^t$

$$L[f(t)] = L[a^t] = L[e^{tloga}]$$

$$L[a^t] = \frac{1}{s-loga}$$

(iv)Given $f(t) = e^{2t+3}$

$$L[f(t)] = L[e^{2t+3}] = L[e^{2t}.e^3]$$

$$= e^3 L[e^{2t}]$$

$$= e^3\left[\frac{1}{s-2}\right]$$

$$\therefore L[e^{2t+3}] = e^3\left[\frac{1}{s-2}\right]$$

(v) $L\left[sin\sqrt{2}t\right] = \frac{\sqrt{2}}{s^2+2}$

(vi)Given $f(t) = \sin(at+b) = sinatcosb + cosatsinb$

$$L[f(t)] = L[\sin(at+b)]$$

$$= L[sinatcosb + cosatsinb]$$

$$= cosb\, L[\text{sin}at] + sinb\, L[\text{cos}at]$$

$$L[\sin(at+b)] = cosb\frac{s}{s^2+a^2} + sinb\,\frac{s}{s^2+a^2}$$

(vii) Given $f(t) = \cos^3 2t = \frac{1}{4}[3cos2t + cos6t]$

$$\because\ \cos^3\theta = \frac{3cos\theta + cos3\theta}{4}$$

$$L[f(t)] = \frac{1}{4}L[3cos2t + cos6t]$$

$$= \frac{1}{4}[3L(cos2t) + L(cos6t)]$$

$$= \frac{1}{4}\left[3\frac{s}{s^2+4} + \frac{s}{s^2+36}\right]$$

$$L[\cos^3 2t] = \frac{1}{4}\left[3\frac{s}{s^2+4} + \frac{s}{s^2+36}\right]$$

(viii) Given $f(t) = \sin^3 t = \frac{1}{4}[3sint - sin3t]$

$$L[f(t)] = \frac{1}{4}L[3sint - sin3t]$$

$$= \frac{1}{4}[3L(sint) - L(sin3t)]$$

$$= \frac{1}{4}\left[3\frac{1}{s^2+1} - \frac{3}{s^2+9}\right]$$

$$L[\sin^3 t] = \frac{3}{4}\left[\frac{1}{s^2+1} - \frac{1}{s^2+9}\right]$$

(ix) Given $f(t) = \sin^2 t = \frac{1-cos2t}{2}$

$$L[f(t)] = L\left[\frac{1-cos2t}{2}\right]$$

$$= \frac{1}{2}[L(1) - L(cos2t)]$$

$$= \frac{1}{2}\left[\frac{1}{s} - \frac{s}{s^2+4}\right]$$

$$L[\cos^2 2t] = \frac{1}{2}\left[\frac{1}{s} - \frac{s}{s^2+4}\right]$$

(x) Given $f(t) = \cos^2 2t = \frac{1+cos4t}{2}$

$$L[f(t)] = L\left[\frac{1+cos4t}{2}\right]$$

$$= \frac{1}{2}[L(1) + L(cos4t)]$$

$$= \frac{1}{2}\left[\frac{1}{s} + \frac{s}{s^2+16}\right]$$

$$L[\cos^2 2t] = \frac{1}{2}\left[\frac{1}{s} + \frac{s}{s^2+16}\right]$$

(xi) Given $f(t) = cos5tcos4t$

$$L[f(t)] = L[cos5tcos4t]$$

$$= \frac{1}{2}[L(cos9t) + L(cost)]$$

$$= \frac{1}{2}\left[\frac{s}{s^2+81} + \frac{s}{s^2+1}\right]$$

Problems using First Shifting theorem

$$L[e^{-at}f(t)] = L[f(t)]_{s\to s+a}$$

$$L[e^{at}f(t)] = L[f(t)]_{s\to s-a}$$

Example: Find the Laplace transform for the following:

i. te^{-3t}	vii. $t^2 2^t$
ii. t^3e^{2t}	viii. $t^3 2^{-t}$
iii. $e^{4t}sin2t$	ix. $e^{-2t}sin3tcos2t$
iv. $e^{-5t}cos3t$	x. $e^{-3t}cos4tcos2t$
v. $sinh2tcos3t$	xi. $e^{4t}cos3tsin2t$

vi. cosh3tsin2t	

(i) te^{-3t}

$$L[te^{-3t}] = L[t]_{s\to s+3}$$

$$= \left(\frac{1}{s^2}\right)_{s\to s+3} \qquad \because L(t) = \frac{1}{s^2}$$

$$\therefore L[te^{-3t}] = \frac{1}{(s+3)^2}$$

(ii) t^3e^{2t}

$$L[t^3e^{2t}] = L[t^3]_{s\to s-2}$$

$$= \left(\frac{3!}{s^4}\right)_{s\to s-2} \qquad \because L(t) = \frac{3!}{s^{3+1}}$$

$$\therefore L[t^3e^{2t}] = \frac{6}{(s-2)^4}$$

(iii) $e^{4t}sin2t$

$$L[e^{4t}sin2t] = L[sin2t]_{s\to s-4}$$

$$= \left(\frac{2}{s^2+2^2}\right)_{s\to s-4}$$

$$= \frac{2}{(s-4)^2+4}$$

$$= \frac{2}{s^2-8s+16+4}$$

$$\therefore L[e^{4t}sin2t] = \frac{2}{s^2-8s+20}$$

(iv) $L[e^{-5t}cos3t]$

$$L[e^{-5t}cos3t] = L[cos3t]_{s\to s+5}$$

$$= \left(\frac{s}{s^2+3^2}\right)_{s\to s+5}$$

$$= \frac{s+5}{(s+5)^2+9}$$

$$= \frac{s+5}{s^2+10s+25+9}$$

$$\therefore L[e^{-5t}cos3t] = \frac{s+5}{s^2+10s+34}$$

(v) $L[sinh2tcos3t]$

$$L[sinh2tcos3t] = L\left[\left(\frac{e^{2t}-e^{-2t}}{2}\right)cos3t\right]$$

$$= \frac{1}{2}[L(e^{2t}cos3t) - L(e^{-2t}cos3t)]$$

$$= \frac{1}{2}[L(cos3t)_{s\to s-2} - L(cos3t)_{s\to s+2}]$$

$$= \frac{1}{2}\left[\left(\frac{s}{s^2+3^2}\right)_{s\to s-2} - \left(\frac{s}{s^2+3^2}\right)_{s\to s+2}\right]$$

$$\therefore L[sinh2tcos3t] = \frac{1}{2}\left[\frac{s-2}{(s-2)^2+9} - \frac{s+2}{(s+2)^2+9}\right]$$

(vi) $\boldsymbol{L[cosh3tsin2t]}$

$$L[cosh3tsin2t] = L\left[\left(\frac{e^{3t}+e^{-3t}}{2}\right)sin2t\right]$$

$$= \frac{1}{2}[L(e^{3t}sin2t) + L(e^{-3t}sin2t)]$$

$$= \frac{1}{2}[L(sin2t)_{s\to s-3} + L(sin2t)_{s\to s+3}]$$

$$= \frac{1}{2}\left[\left(\frac{2}{s^2+2^2}\right)_{s\to s-3} + \left(\frac{2}{s^2+2^2}\right)_{s\to s+3}\right]$$

$$\therefore L[cosh3tsin2t] = \frac{1}{2}\left[\frac{2}{(s-3)^2+4} + \frac{2}{(s+3)^2+4}\right]$$

(vii) $\boldsymbol{t^2 2^t}$

$$L[t^2 2^t] = L\left[t^2 e^{log2^t}\right]$$

$$= L[t^2 e^{tlog2}] = L[t^2]_{s\to s-log2}$$

$$= \left(\frac{2!}{s^3}\right)_{s\to s-log2}$$

$$= \frac{2}{(s-log2)^3}$$

$$\therefore L[t^2 2^t] = \frac{2}{(s-log2)^3}$$

(viii) $\boldsymbol{t^3 2^{-t}}$

$$L[t^3 2^{-t}] = L\left[t^3 e^{log2^{-t}}\right]$$

$$= L[t^3 e^{-tlog2}] = L[t^3]_{s\to s+log2}$$

$$= \left(\frac{3!}{s^4}\right)_{s\to s+log2}$$

$$= \frac{6}{(s+log2)^4}$$

$$\therefore L[t^3 2^{-t}] = \frac{6}{(s+log2)^4}$$

(ix) $\boldsymbol{L[e^{-2t}sin3tcos2t]}$

$$L[e^{-2t}sin3tcos2t] = L[sin3tcos2t]_{s\to s+2}$$

$$= \frac{1}{2}L[\sin(3t+2t) + \sin(3t-2t)]_{s\to s+2}$$

$$= \frac{1}{2}L[\sin 5t + \sin t]_{s\to s+2}$$

$$= \frac{1}{2}[\mathrm{L}(\sin 5t) + L(\sin t)]_{s\to s+2}$$

$$= \frac{1}{2}\left[\frac{5}{s^2+5^2} + \frac{1}{s^2+1^2}\right]_{s\to s+2}$$

$$= \frac{1}{2}\left[\frac{5}{(s+2)^2+25} + \frac{1}{(s+2)^2+1}\right]$$

$$\therefore L[e^{-2t} sin3tcos2t] == \frac{1}{2}\left[\frac{5}{(s+2)^2+25} + \frac{1}{(s+2)^2+1}\right]$$

(x) $L[e^{-3t}cos4tcos2t]$

$$L[e^{-3t}cos4tcos2t] = L[cos4tcos2t]_{s\to s+3}$$

$$= \frac{1}{2}L[\cos(4t+2t)+\cos(4t-2t)]_{s\to s+3}$$

$$= \frac{1}{2}L[\cos6\mathrm{t}+\cos2t]_{s\to s+3}$$

$$= \frac{1}{2}[\mathrm{L}(\cos6\,t)+L(\cos2\,t)]_{s\to s+3}$$

$$= \frac{1}{2}\left[\frac{s}{s^2+6^2}+\frac{s}{s^2+2^2}\right]_{s\to s+3}$$

$$= \frac{1}{2}\left[\frac{s+3}{(s+3)^2+36}+\frac{s+3}{(s+3)^2+4}\right]$$

$$\therefore L[e^{-3t}cos4tcos2t] = \frac{1}{2}\left[\frac{s+3}{(s+3)^2+36}+\frac{s+3}{(s+3)^2+4}\right]$$

(xi) $L[e^{4t}cos3tsin2t]$

$$L[e^{4t}cos3tsin2t] = L[cos3tsin2t]_{s\to s-4}$$

$$= \frac{1}{2}L[\sin(3t+2t)-\sin(3t-2t)]_{s\to s-4}$$

$$= \frac{1}{2}L[\sin 5t-\sin t]_{s\to s-4}$$

$$= \frac{1}{2}[\mathrm{L}(\sin 5t)-L(\sin t)]_{s\to s-4}$$

$$= \frac{1}{2}\left[\frac{5}{s^2+5^2}-\frac{1}{s^2+1^2}\right]_{s\to s-4}$$

$$= \frac{1}{2}\left[\frac{5}{(s-4)^2+25}+\frac{1}{(s-4)^2+1}\right]$$

$$\therefore L[e^{4t}cos3tsin2t] = \frac{1}{2}\left[\frac{5}{(s-4)^2+25}+\frac{1}{(s-4)^2+1}\right]$$