

24AG401 THEORY OF MACHINES

UNIT V NOTES



INTRODUCTION

The high speed of engines and other machines is a common phenomenon now-a-days. It is, therefore, very essential that all the rotating and reciprocating parts should be completely balanced as far as possible. If these parts are not properly balanced, the dynamic forces are set up. These forces not only increase the loads on bearings and stresses in the various members, but also produce unpleasant and even dangerous vibrations.

Types of balancing

There are two basic types of balancing that can be carried out on your machinery – Static and Dynamic:

1. **Static balancing (knife edging)**

When we balance a section of machinery statically, its centre of gravity is located in the axis of its rotation. This process implies that it will remain stationary at the horizontal axis, without applying a braking force. The unbalance will be present even when the rotor is not spinning.

Low friction bearings are used to settle the component so that the heaviest part is at the bottom. We then remove material from the lower portion (heavy side). Or add on top (light side) until it rotates on a true axis. We repeat this process until the point that is heavy disappears and the rotor no longer rotates without assistance.

2. **Dynamic balancing (single and multiplane)**

Dynamic balancing involves the adjustment of an object's balance by adding or removing weight. Firstly, we have to determine the components unbalance. We calculate this whilst it's rotating at a predetermined speed. The information from this process gives insight into the amount of weight required to counterbalance areas that are either too light or too heavy.

Reducing vibration through dynamic balancing also ensures the machine is running smoothly, with reduced noise. This inevitably prevents premature system failure. A component is only in balance once the rotation does not produce any centrifugal force or couple unbalance as a result.

Balancing of Rotating Masses

whenever a certain mass is attached to a rotating shaft, it exerts some centrifugal force, whose effect is to bend the shaft and to produce vibrations in it. In order to prevent the effect of centrifugal force, another mass is attached to the opposite side of the shaft, at such a position so as to balance the effect of the centrifugal force of the first mass. This is done in such a way that the centrifugal force of both the masses are made to be equal and opposite. The process of providing the second mass in order to counteract the effect of the centrifugal force of the first mass, is called balancing of rotating masses. The following cases are important from the subject point of view:

1. Balancing of a single rotating mass by a single mass rotating in the same plane.
2. Balancing of a single rotating mass by two masses rotating in different planes.
3. Balancing of different masses rotating in the same plane.
4. Balancing of different masses rotating in different planes.

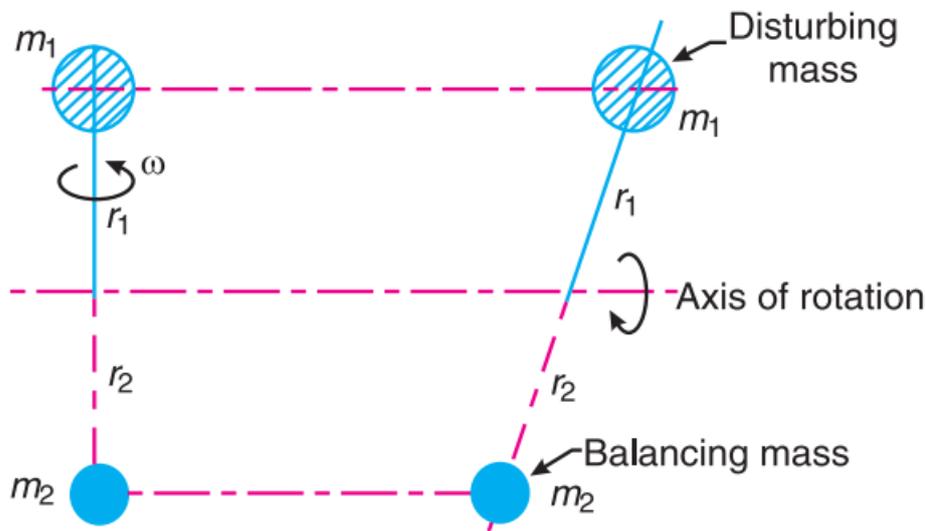
Balancing of a Single Rotating Mass By a Single Mass Rotating in the Same Plane

Consider a disturbing mass m_1 attached to a shaft rotating at ω rad/s as shown in Fig. Let r_1 be the radius of rotation of the mass m_1 (i.e. distance between the axis of rotation of the shaft and the centre of gravity of the mass m_1). We know that the centrifugal force exerted by the mass m_1 on the shaft,

$$F_{C1} = m_1 \cdot \omega^2 \cdot r_1$$

This centrifugal force acts radially outwards and thus produces bending moment on the shaft. In order to counteract the effect of this force, a balancing mass (m_2) may be attached in the same plane of rotation as that of disturbing mass (m_1)

such that the centrifugal forces due to the two masses are equal and opposite.



Let r_2 = Radius of rotation of the balancing mass m_2 (i.e. distance between the axis of rotation of the shaft and the centre of gravity of mass m_2).

∴ Centrifugal force due to mass m_2 ,

$$F_{C2} = m_2 \cdot \omega^2 \cdot r_2$$

$$m_1 \cdot \omega^2 \cdot r_1 = m_2 \cdot \omega^2 \cdot r_2 \quad \text{or} \quad m_1 \cdot r_1 = m_2 \cdot r_2$$

Balancing of a Single Rotating Mass By Two Masses Rotating in Different Planes

The two forces are equal in magnitude and opposite in direction. But this type of arrangement for balancing gives rise to a couple which tends to rock the shaft in its bearings. Therefore in order to put the system in complete balance, two balancing masses are placed in two different planes, parallel to the plane of rotation of the disturbing mass, in such a way that they satisfy the following two conditions of equilibrium.

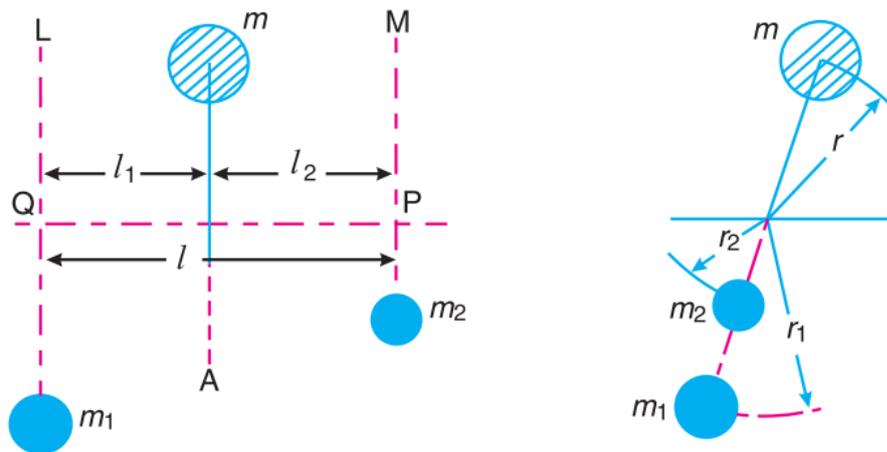
1. The net dynamic force acting on the shaft is equal to zero. This requires that the line of action of three centrifugal forces must be the same. In other words, the centre of the masses of the system must lie on the axis of rotation. This is the condition for static balancing.

2. The net couple due to the dynamic forces acting on the shaft is equal to zero. In other words, the algebraic sum of the moments about any point in the plane must be zero. The conditions (1) and (2) together give dynamic balancing.

The following two possibilities may arise while attaching the two balancing masses :

1. The plane of the disturbing mass may be in between the planes of the two balancing masses, and
2. The plane of the disturbing mass may lie on the left or right of the two planes containing the balancing masses.

Let l_1 = Distance between the planes A and L ,
 l_2 = Distance between the planes A and M , and
 l = Distance between the planes L and M .



We know that the centrifugal force exerted by the mass m in the plane A ,

$$F_C = m \cdot \omega^2 \cdot r$$

Similarly, the centrifugal force exerted by the mass m_1 in the plane L ,

$$F_{C1} = m_1 \cdot \omega^2 \cdot r_1$$

and, the centrifugal force exerted by the mass m_2 in the plane M ,

$$F_{C2} = m_2 \cdot \omega^2 \cdot r_2$$

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Since the net force acting on the shaft must be equal to zero, therefore the centrifugal force on the disturbing mass must be equal to the sum of the centrifugal forces on the balancing masses, therefore

$$F_C = F_{C1} + F_{C2} \quad \text{or} \quad m \cdot \omega^2 \cdot r = m_1 \cdot \omega^2 \cdot r_1 + m_2 \cdot \omega^2 \cdot r_2$$

$$\therefore m \cdot r = m_1 \cdot r_1 + m_2 \cdot r_2 \quad \dots (i)$$

Now in order to find the magnitude of balancing force in the plane L (or the dynamic force at the bearing Q of a shaft), take moments about P which is the point of intersection of the plane M and the axis of rotation. Therefore

$$F_{C1} \times l = F_C \times l_2 \quad \text{or} \quad m_1 \cdot \omega^2 \cdot r_1 \times l = m \cdot \omega^2 \cdot r \times l_2$$

$$\therefore m_1 \cdot r_1 \cdot l = m \cdot r \cdot l_2 \quad \text{or} \quad m_1 \cdot r_1 = m \cdot r \times \frac{l_2}{l} \quad \dots (ii)$$

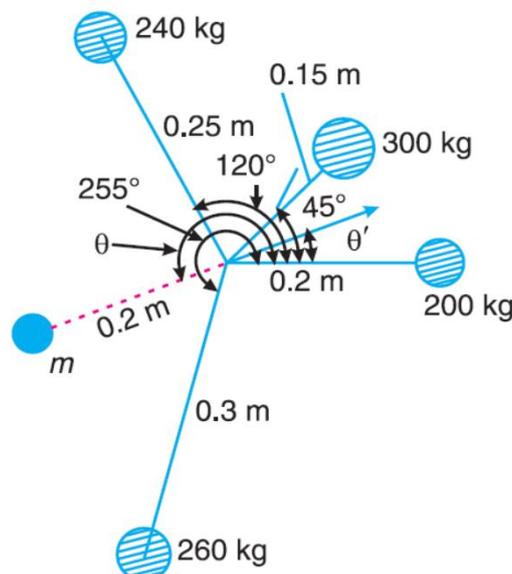
Similarly, in order to find the balancing force in plane M (or the dynamic force at the bearing P of a shaft), take moments about Q which is the point of intersection of the plane L and the axis of rotation. Therefore

$$F_{C2} \times l = F_C \times l_1 \quad \text{or} \quad m_2 \cdot \omega^2 \cdot r_2 \times l = m \cdot \omega^2 \cdot r \times l_1$$

$$\therefore m_2 \cdot r_2 \cdot l = m \cdot r \cdot l_1 \quad \text{or} \quad m_2 \cdot r_2 = m \cdot r \times \frac{l_1}{l} \quad \dots (iii)$$

Four masses m_1 , m_2 , m_3 and m_4 are 200 kg, 300 kg, 240 kg and 260 kg respectively. The corresponding radii of rotation are 0.2 m, 0.15 m, 0.25 m and 0.3 m respectively and the angles between successive masses are 45° , 75° and 135° . Find the position and magnitude of the balance mass required, if its radius of rotation is 0.2 m

Given : $m_1 = 200 \text{ kg}$; $m_2 = 300 \text{ kg}$; $m_3 = 240 \text{ kg}$; $m_4 = 260 \text{ kg}$; $r_1 = 0.2 \text{ m}$;
 $r_2 = 0.15 \text{ m}$; $r_3 = 0.25 \text{ m}$; $r_4 = 0.3 \text{ m}$; $\theta_1 = 0^\circ$; $\theta_2 = 45^\circ$; $\theta_3 = 45^\circ + 75^\circ = 120^\circ$; $\theta_4 = 45^\circ + 75^\circ + 135^\circ = 255^\circ$; $r = 0.2 \text{ m}$



$$m_1 \cdot r_1 = 200 \times 0.2 = 40 \text{ kg-m}$$

$$m_2 \cdot r_2 = 300 \times 0.15 = 45 \text{ kg-m}$$

$$m_3 \cdot r_3 = 240 \times 0.25 = 60 \text{ kg-m}$$

$$m_4 \cdot r_4 = 260 \times 0.3 = 78 \text{ kg-m}$$

Analytical method:

Resolving $m_1 \cdot r_1$, $m_2 \cdot r_2$, $m_3 \cdot r_3$ and $m_4 \cdot r_4$ horizontally,

$$\begin{aligned} \Sigma H &= m_1 \cdot r_1 \cos \theta_1 + m_2 \cdot r_2 \cos \theta_2 + m_3 \cdot r_3 \cos \theta_3 + m_4 \cdot r_4 \cos \theta_4 \\ &= 40 \cos 0^\circ + 45 \cos 45^\circ + 60 \cos 120^\circ + 78 \cos 255^\circ \\ &= 40 + 31.8 - 30 - 20.2 = 21.6 \text{ kg-m} \end{aligned}$$

Now resolving vertically,

$$\begin{aligned} \Sigma V &= m_1 \cdot r_1 \sin \theta_1 + m_2 \cdot r_2 \sin \theta_2 + m_3 \cdot r_3 \sin \theta_3 + m_4 \cdot r_4 \sin \theta_4 \\ &= 40 \sin 0^\circ + 45 \sin 45^\circ + 60 \sin 120^\circ + 78 \sin 255^\circ \\ &= 0 + 31.8 + 52 - 75.3 = 8.5 \text{ kg-m} \end{aligned}$$

$$\therefore \text{Resultant, } R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = \sqrt{(21.6)^2 + (8.5)^2} = 23.2 \text{ kg-m}$$

We know that

$$m \cdot r = R = 23.2 \quad \text{or} \quad m = 23.2 / r = 23.2 / 0.2 = 116 \text{ kg}$$

and

$$\tan \theta' = \Sigma V / \Sigma H = 8.5 / 21.6 = 0.3935 \quad \text{or} \quad \theta' = 21.48^\circ$$

Since θ' is the angle of the resultant R from the horizontal mass of 200 kg, therefore the angle of the balancing mass from the horizontal mass of 200 kg,

$$\theta = 180^\circ + 21.48^\circ = 201.48^\circ$$