

Bus Impedance matrix building algorithm

Bus impedance matrix Z_{bus} of a power network can be obtained by inverting the bus admittance matrix Y_{bus} , which is easy to construct. However, when the order of matrix is large, direct inversion requires more core storage and enormous computer time. Therefore, inversion of Y_{bus} is prohibited for large size network. Bus impedance matrix can be constructed by adding the one after the other. Using impedance parameters, performance equations in bus frame of reference can be written as

$$E_{bus} = Z_{bus} I_{bus}$$

In the expanded form the above becomes

$$\begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_N \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1N} \\ Z_{21} & Z_{22} & \cdots & Z_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ Z_{N1} & Z_{N2} & \cdots & Z_{NN} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \end{bmatrix}$$

From this we can write

$$E_p = Z_{p1} I_1 + Z_{p2} I_2 + \dots + Z_{pq} I_q + \dots + Z_{pN} I_N$$

From the above, it can be noted that with $I_q = 1$ p.u. other bus currents set to zero, $E_p = Z_{pq}$. Thus Z_{pq} can be obtained by measuring E_p when 1 p.u. current is injected at bus q and leaving the other bus currents as zero. In fact, p and q can be varied from 1 to N . While making measurements all the buses except one, are open circuited. Hence, the bus impedance parameters are called open circuit impedances. The diagonal elements in Z_{bus} are known as driving point impedances, while the off-diagonal elements are called transfer impedances.

Symmetrical fault analysis through bus impedance matrix. Once the bus impedance matrix is constructed, symmetrical fault analysis can be carried out with a very few calculations. Bus voltages and currents in various elements can be computed quickly. When faults are to be simulated at different buses, this method is proved to be good. Symmetrical short circuit analysis essentially consists of determining the steady state solution of linear network with balanced sources. Since the short circuit currents are much larger compared to prefault currents the following assumptions are made while conducting short circuit study.

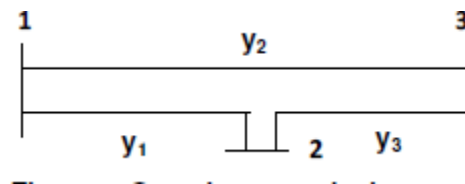
1. All the shunt parameters like loads, line charging admittances etc. are neglected.
2. All the transformer taps are at nominal position.
3. Prior to the fault, all the generators are assumed to operate at rated voltage of 1.0 p.u. with their emf's in phase. With these assumptions, in the pre-fault condition, there will not be any current flow in the network and all the bus voltages will be equal to 1.0 p.u.

The linear network that has to be solved comprises of

i) Transmission network ii) Generation system and iii) Fault

By properly combining the representations of the above three components, we can solve the short circuit problem.

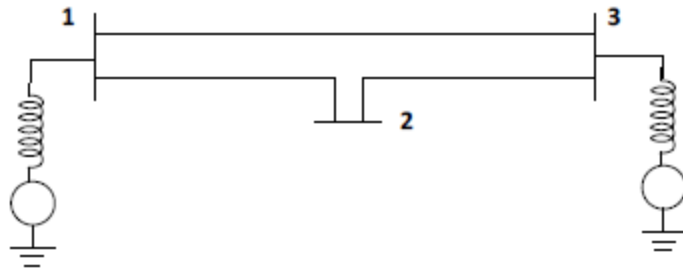
Consider the transmission network shown in Fig



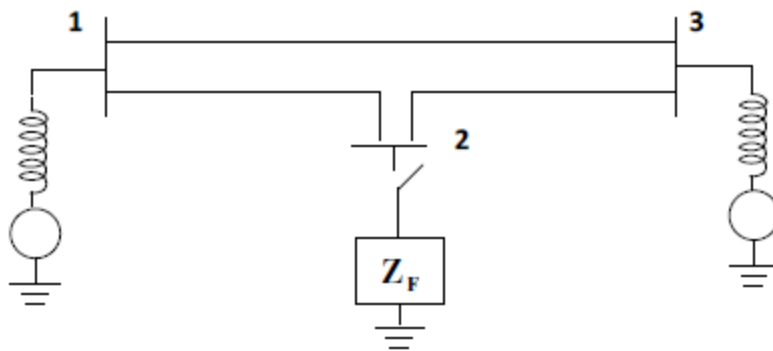
Taking the ground as the reference bus, the bus admittance matrix is obtained as

$$Y_{bus} = \begin{matrix} & \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} y_1 + y_2 & -y_1 & -y_2 \\ -y_1 & y_1 + y_3 & -y_3 \\ -y_2 & -y_3 & y_2 + y_3 \end{bmatrix} \end{matrix}$$

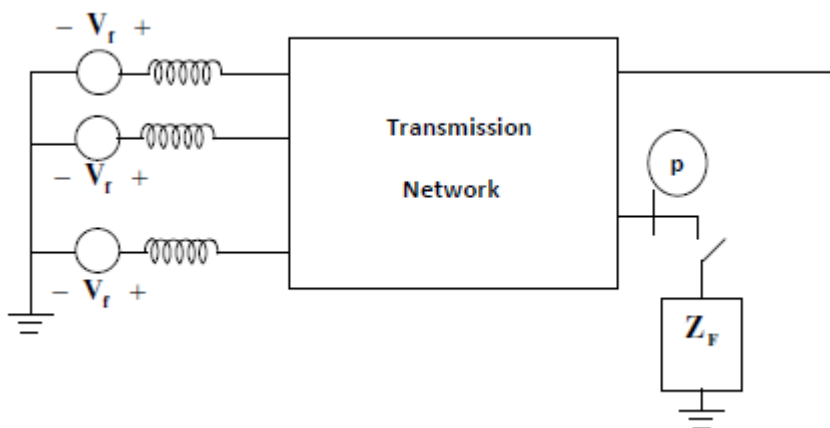
If we add all the columns (or rows) we get a column (or row) of all zero elements. Hence this Ybus matrix is singular and hence corresponding Zbus matrix of this transmission network does not exist. Thus, when all the shunt parameters are neglected, Zbus matrix will not exist for the transmission network. However, connection to ground is established at the generator buses, representing the generator as a constant voltage source behind appropriate reactance as shown in Fig.



If the generator reactances are included with the transmission network, Zbus matrix of the combined network can be obtained. As stated earlier, there is no current flow in the network in the pre-fault condition and all the bus voltages will be 1.0 p.u. Consider the network shown in Fig. Symmetrical fault occurring at bus 2 can be simulated by closing the switch shown in Fig. Here Z_f is the fault impedance.



When the fault is simulated, there will be currents in different elements and the bus voltages will be different from 1.0 p.u. These changes have occurred because of i) Generator voltages and ii) Fault current. Any general power system with a number of generators and N number of buses subjected to symmetrical fault at pth bus will be represented as shown in Fig.



In the faulted system there are two types of sources:

1. Current injection at the faulted bus
2. Generated voltage sources.

The bus voltages in the faulted system can be obtained using Superposition Theorem.

Bus voltages due to current injection:

Make all the generator voltages to zero. Then we have Generator-Transmission system without voltage sources. Such network has transmission parameters and generator reactances between generator buses and the ground. Let Z_{bus} be the bus impedance matrix of such Generator-Transmission network. Then the bus voltages due to the current injection will be given by

$$V_{bus} = Z_{bus} I_{bus} (F)$$

where $I_{bus} (F)$ is the bus current vector having only one non-zero element.

Thus when the fault is at the pth bus

$$I_{bus} (F) = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ I_p (F) \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

Here $I_p (F)$ is the faulted bus current

Thus, bus voltages due to current injection will be

$$V_{bus} = \begin{bmatrix} Z_{11} & Z_{12} & \dots & Z_{1p} & \dots & Z_{1N} \\ Z_{21} & Z_{22} & \dots & Z_{2p} & \dots & Z_{2N} \\ \vdots & \vdots & & \vdots & & \vdots \\ Z_{p1} & Z_{p2} & \dots & Z_{pp} & \dots & Z_{pN} \\ \vdots & \vdots & & \vdots & & \vdots \\ Z_{N1} & Z_{N2} & \dots & Z_{Np} & \dots & Z_{NN} \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ \vdots \\ I_p (F) \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} Z_{1p} \\ Z_{2p} \\ \vdots \\ Z_{pp} \\ \vdots \\ Z_{Np} \end{bmatrix} I_p (F)$$

Bus voltages due to generator voltages

Make the fault current to be zero. Since there is no shunt element, there will be no current flow and all the bus voltages are equal to V_0 , the pre-fault voltage which will be normally equal to 1.0 p.u. Thus, bus voltages due to generator voltages will be

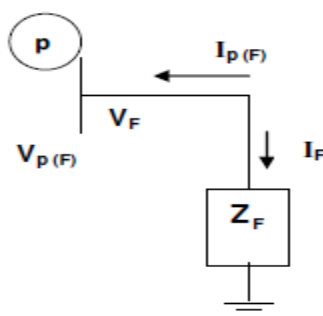
$$V_{\text{bus}} = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \\ \vdots \\ 1 \end{bmatrix} V_0$$

Thus for the faulted system, wherein both the current injection and generator sources are simultaneously present, the bus voltages can be obtained by adding the voltages. Therefore, for the faulted system the bus voltages are

$$V_{\text{bus (F)}} = \begin{bmatrix} V_{1(F)} \\ V_{2(F)} \\ \vdots \\ V_{p(F)} \\ \vdots \\ V_{N(F)} \end{bmatrix} = \begin{bmatrix} Z_{1p} \\ Z_{2p} \\ \vdots \\ Z_{pp} \\ \vdots \\ Z_{Np} \end{bmatrix} I_{p(F)} + \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \\ \vdots \\ 1 \end{bmatrix} V_0$$

To calculate $V_{\text{bus (F)}}$ we need the faulted bus current $I_{p(F)}$ which can be, determined as discussed below.

The fault can be described as shown in Fig.



It is clear that $V_F = Z_F I_F$, $V_{p(F)} = V_F$ and $I_{p(F)} = -I_F$ (3.40)

Therefore $V_p (F) = - Z_F I_p (F) .$

The pth equation extracted from eqn gives $V_p (F) = Z_{pp} I_p (F) + V_0$

$$V_p (F) = - Z_F I_p (F)$$

The pth equation extracted from eqn. (3.39) gives

$$V_p (F) = Z_{pp} I_p (F) + V_0$$

Substituting eqn. in the above, we get

$$- Z_F I_p (F) = Z_{pp} I_p (F) + V_0$$

Thus the faulted bus current $I_p (F)$ is given by

$$I_p (F) = - \frac{V_0}{Z_{pp} + Z_F}$$

Substituting the above in eqn. (3.41), the faulted bus voltage $V_p (F)$ is

$$V_p (F) = \frac{Z_F}{Z_{pp} + Z_F} V_0$$

Finally, voltages at other buses at faulted condition are to be obtained. The ith equation extracted from eqn. (3.39) gives

$$V_i (F) = Z_{ip} I_p (F) + V_0$$

Substituting eqn. (3.43) in the above, we get

$$V_i (F) = V_0 - \frac{Z_{ip}}{Z_{pp} + Z_F} V_0 \quad \begin{array}{l} i = 1, 2, \dots, N \\ i \neq p \end{array}$$

Knowing all the bus voltages, current flowing through the various network elements can be computed as $i_{km} (F) = (V_k (F) - V_m (F)) y_{km}$ where y_{km} is the admittance of element k-m.

When the fault is direct, $Z_F = 0$ and hence

$$I_{p(F)} = - \frac{V_0}{Z_{pp}}$$

$$V_{p(F)} = 0 \text{ and}$$

$$V_{i(F)} = V_0 - \frac{Z_{ip}}{Z_{pp}} V_0 \quad \begin{array}{l} i = 1, 2, \dots, N \\ i \neq p \end{array}$$

It is to be noted that when the fault occurs at the pth bus, only the pth column of Zbus matrix (and not the entire Zbus matrix) is required for further calculations.

The following are the various steps for conducting symmetrical short circuit analysis.

- Step 1** Read
- i) Transmission line data
 - ii) Generator reactances data
 - iii) Faulted bus number p and
 - iv) Fault impedance Z_F .

Step 2 Construct the bus impedance matrix of the transmission network including the generator reactances.

Step 3 Compute $I_{p(F)} = - \frac{V_0}{Z_{pp} + Z_F}$

Step 4 Compute $V_{p(F)} = \frac{Z_F}{Z_{pp} + Z_F} V_0$

Step 5 Compute $V_{i(F)} = V_0 - \frac{Z_{ip}}{Z_{pp} + Z_F} V_0 \quad \begin{array}{l} i = 1, 2, \dots, N \\ i \neq p \end{array}$

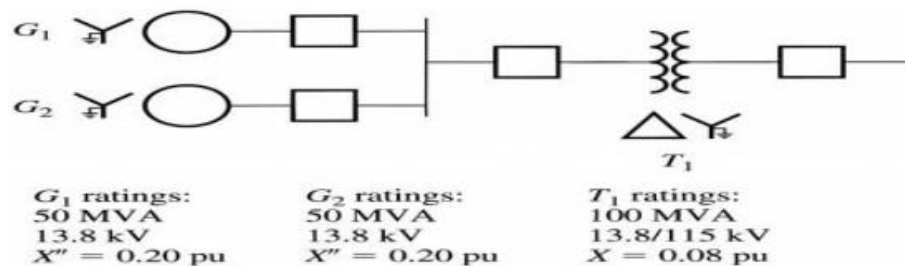
Step 6 Calculate the element currents from $i_{km(F)} = (V_{k(F)} - V_{n(F)}) Y_{km}$

Symmetrical fault analysis through bus impedance matrix

Problems:

1. Two generators are connected in parallel to the low voltage side of a transformer. Generators G1 and G2 are each rated at 50 MVA, 13.8 kV, with a subtransient resistance of 0.2 pu. Transformer T1 is rated at 100 MVA, 13.8/115 kV with a series reactance of 0.08 pu and negligible resistance.

Assume that initially the voltage on the high side of the transformer is 120 kV, that the transformer is unloaded, and that there are no circulating currents between the generators. Calculate the subtransient fault current that will flow if a 3 phase fault occurs at the high-voltage side of transformer



Solution:

Let choose the per-unit base values for this power system to be 100 MVA and 115 kV at the high-voltage side and 13.8 kV at the low-voltage side of the transformer. The subtransient reactance of the two generators to the system base is

$$X_{pu,new} = X_{pu,old} \times \left(\frac{kV_{b,old}}{kV_{b,new}} \right)^2 \times \left(\frac{MVA_{b,new}}{MVA_{b,old}} \right)$$

$$X_1'' = X_2'' = 0.2 \times \left(\frac{13,800}{13,800} \right)^2 \times \left(\frac{100,000}{50,000} \right) = j0.4 p.u$$

The reactance of the transformer is already given on the system base, it will not change

$$X_T = 0.08 p. u$$

The per-unit voltage on the high-voltage side of the transformer is

$$V_{pu} = \frac{\text{Actual value}}{\text{Base value}} = \frac{120,000}{115,000} = j1.044 p. u$$

Since there is no load on the system, the voltage at the terminals of each generator, and the internal generated voltage of each generator must also be 1.044 pu. To find this voltage, we must convert first the per-unit impedances to admittances, and the voltage

sources to equivalent current sources. The Thevenin impedance of each generator is $Z_{Th} = j0.4$, so the short circuit current of each generator is

$$I_{su} = \frac{V_{oc}}{Z_{th}} = \frac{1.044 \angle 0^\circ}{j0.4} = 2.61 \angle 90^\circ$$

Then the node equation for voltage V_1

$$V_1 - j2.5 + V_1 - j2.5 + V_1 - j12.5 = 2.61 \angle -90^\circ + 2.61 \angle -90^\circ$$

$$V_1 = \frac{5.22 \angle 90^\circ}{-j17.5} = 0.298 \angle 0^\circ$$

Therefore, the subtransient current in the fault is

$$I_F = V_1 - j12.5 = 3.729 \angle -90^\circ \text{ p.u.}$$

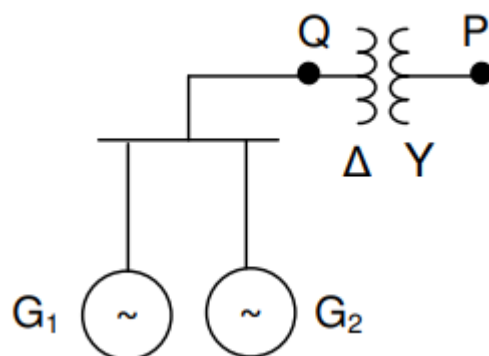
Since the base current at the high-voltage side of the transformer is

$$I_{base} = \frac{S_{3\phi,base}}{\sqrt{3}V_{LL,base}} = \frac{100,000,000}{\sqrt{3}115,000} = 502 \text{ A}$$

the subtransient fault current will be

$$I_F = I_F, \text{p.u.} \times I_{base} = 3.729 \times 502 = 1872 \text{ A}$$

2. Two synchronous generators are connected in parallel at the low voltage side of a three-phase -Y transformer as shown in Fig. 3.2. Machine 1 is rated 50 MVA, 13.8 kV. Machine 2 is rated 25 MVA, 13.8 kV. Each generator has subtransient reactance, transient reactance and direct axis synchronous reactance of 25%, 40% and 100% respectively. The transformer is rated 75 MVA, 13.8/69Y with a reactance of 10%. Before the fault occurs, the voltage on high voltage side of the transformer is 66 kV. The transformer is unloaded and there is no circulating current between the generators.



(a) Find the current supplied by the generators.

(b) A three-phase short circuit occurs at P. Determine the subtransient, transient and steady state short circuit current in each generator.

(c) A three-phase short circuit occurs at Q. Determine the subtransient, transient and steady state short circuit current in each generator.

Select a base of 75 MVA and 69 kV in the high tension circuit.

Summarize the results in a tabular form.

Solution:

Base voltage at the low tension circuit = 13.8 kV

Prefault voltage at the LV side = $\frac{13.8}{69} \times 66 = 13.2$ kV

Base current at the LV side = $\frac{75 \times 10^3}{\sqrt{3} \times 13.8} = 3137.77$ amp.

On the selected base

Generator 1: $X_d'' = 0.25 \times \frac{75}{50} = 0.375$ p.u. $X_d' = 0.4 \times \frac{75}{50} = 0.6$ p.u.

$X_d = 1.0 \times \frac{75}{50} = 1.5$ p.u. $E_{g1} = \frac{13.2}{13.8} = 0.9565$ p.u.

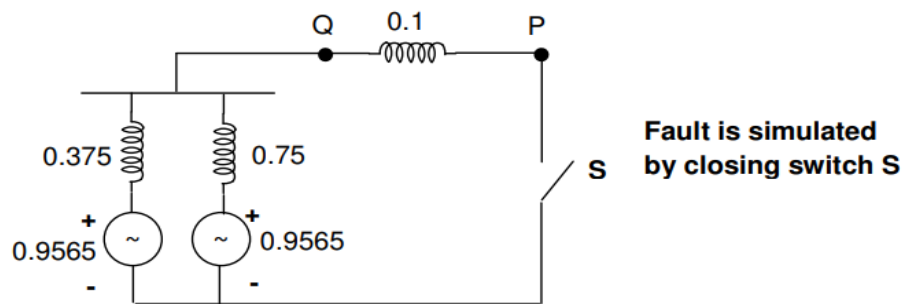
Generator 2: $X_d'' = 0.25 \times \frac{75}{25} = 0.75$ p.u. $X_d' = 0.4 \times \frac{75}{25} = 1.2$ p.u.

$X_d = 1.0 \times \frac{75}{25} = 3.0$ p.u. $E_{g2} = \frac{13.2}{13.8} = 0.9565$ p.u.

Transformer: $X = 0.1$ p.u.

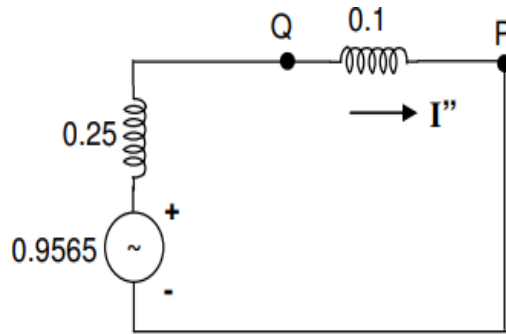
(a) Transformer is unloaded. Therefore, $I_{g1} = I_{g2} = 0$

(b) per unit subtransient reactance diagram is shown in Fig.



Using Thevenin's equivalent above reactance diagram for the faulted condition can be reduced as shown in Fig.

$$\frac{0.375 \times 0.75}{0.375 + 0.75} = 0.25 \text{ p.u.}$$



Subtransient current $I'' = \frac{0.9565}{j0.35} = -j 2.7329$ p.u.

Voltage at Q = $j 0.1 \times (-j 2.7329) = 0.27329$ p.u.

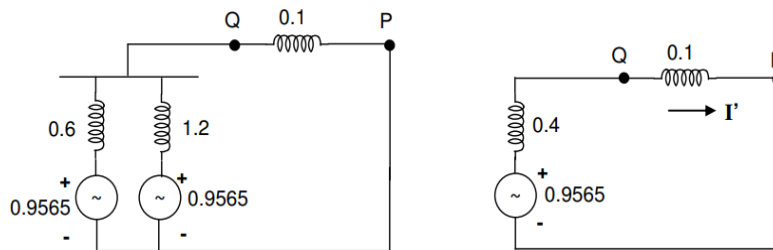
Current supplied by generator 1 = $\frac{0.9565 - 0.27329}{j0.375} = -j 1.8219$ p.u.

Subtransient current in machine 1 $|I_1''| = 5716.7$ A

Current supplied by generator 2 = $\frac{0.9565 - 0.27329}{j0.75} = -j 0.9109$ p.u.

Subtransient current in machine 2 $|I_2''| = 2858.3$ A

per unit transient reactance diagram is shown in Fig



Subtransient current $I' = \frac{0.9565}{j0.5} = -j 1.913$ p.u.

Voltage at Q = $j 0.1 \times (-j 1.913) = 0.1913$ p.u.

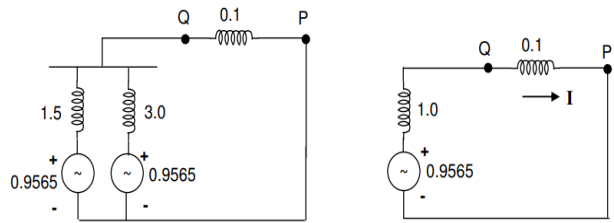
Current supplied by generator 1 = $\frac{0.9565 - 0.1913}{j0.6} = -j 1.275$ p.u.

Transient current in machine 1 $|I_1'| = 4001.7$ A

Current supplied by generator 2 = $\frac{0.9565 - 0.1913}{j1.2} = -j 0.6377$ p.u.

Transient current in machine 2 $|I_2'| = 2000.9$ A

per unit direct axis reactance diagram is shown in Fig.



$$\text{Steady state short circuit current } I = \frac{0.9565}{j1.1} = -j 0.8695 \text{ p.u.}$$

$$\text{Voltage at Q} = j 0.1 \times (-j 0.8695) = 0.08695 \text{ p.u.}$$

$$\text{Current supplied by generator 1} = \frac{0.9565 - 0.08695}{j1.5} = -j 0.5797 \text{ p.u.}$$

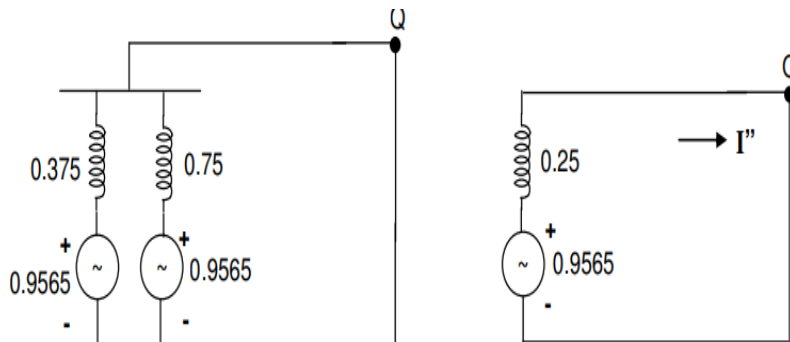
$$\text{Steady state short circuit current in machine 1 } |I_1| = 1819 \text{ A}$$

$$\text{Current supplied by generator 2} = \frac{0.9565 - 0.08695}{j3.0} = -j 0.2899 \text{ p.u.}$$

$$\text{Steady state short circuit current in machine 2 } |I_2| = 909.48 \text{ A}$$

(b) Fault occurs at point Q.

per unit subtransient reactance diagram is shown in Fig.



$$\text{Subtransient current } I'' = \frac{0.9565}{j0.25} = -j 3.826 \text{ p.u.}$$

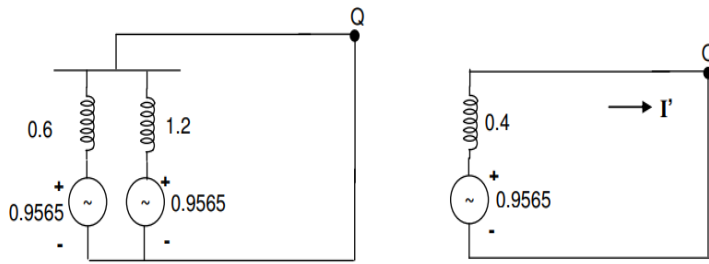
$$\text{Current supplied by generator 1} = \frac{0.9565 - 0}{j0.375} = -j 2.5507 \text{ p.u.}$$

$$\text{Subtransient current in machine 1 } |I_1''| = 8003.4 \text{ A}$$

$$\text{Current supplied by generator 2} = \frac{0.9565 - 0}{j0.75} = -j 1.2753 \text{ p.u.}$$

$$\text{Subtransient current in machine 2 } |I_2''| = 4001.7 \text{ A}$$

per unit transient reactance diagram is shown in Fig



Transient current $I' = \frac{0.9565}{j0.4} = -j 2.3913$ p.u.

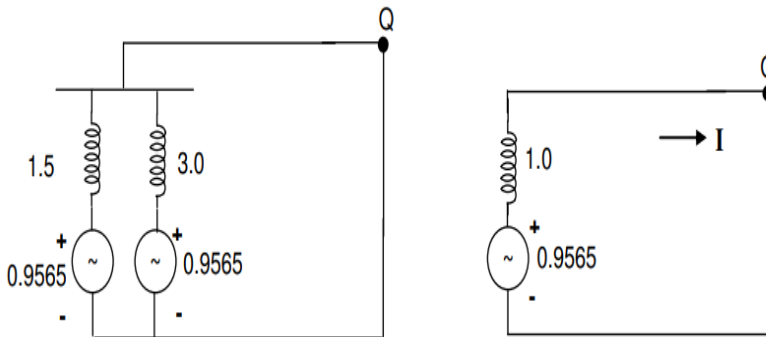
Current supplied by generator 1 = $\frac{0.9565 - 0}{j0.6} = -j 1.5942$ p.u.

Transient current in machine 1 $|I_1'| = 5002.2$ A

Current supplied by generator 2 = $\frac{0.9565 - 0}{j1.2} = -j 0.7971$ p.u.

Subtransient current in machine 2 $|I_2'| = 2501.1$ A

per unit direct axis transient reactance diagram is shown in Fig.



Direct axis steady state short circuit current $I = \frac{0.9565}{j1.0} = -j 0.9565$ p.u.

Current supplied by generator 1 = $\frac{0.9565 - 0}{j1.5} = -j 0.6377$ p.u.

Steady state short current in machine 1 $|I_1| = 2000.9$ A

Current supplied by generator 2 = $\frac{0.9565 - 0}{j3.0} = -j 0.3188$ p.u.

Steady state short circuit current in machine 2 $|I_2| = 1000.4$ A

In the prefault condition, since the transformer is not loaded $I_{g1} = I_{g2} = 0$

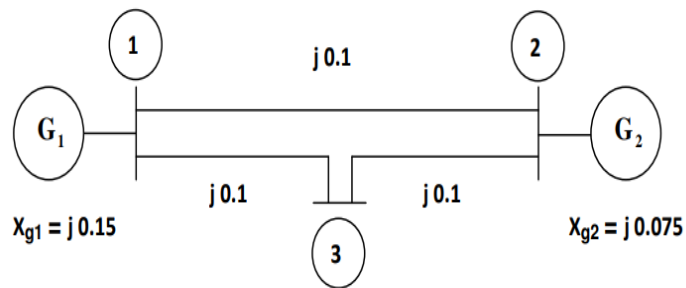
Fault occurs at the HV side of the transformer:

Subtransient		Transient		Steady state	
$ I_1 $	$ I_2 $	$ I_1 $	$ I_2 $	$ I_1 $	$ I_2 $
5717 A	2858 A	4002 A	2001 A	1819 A	909 A

Fault occurs at the LV side of the transformer i.e. at the generator terminals:

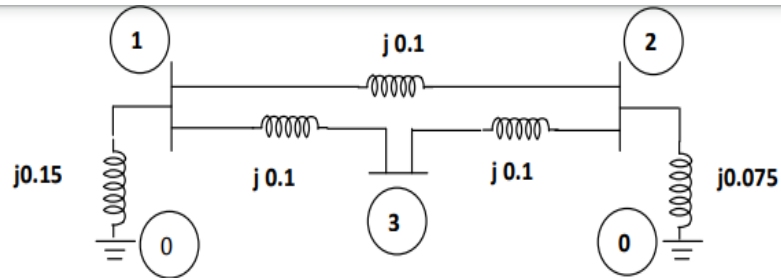
Subtransient		Transient		Steady state	
$ I_1 $	$ I_2 $	$ I_1 $	$ I_2 $	$ I_1 $	$ I_2 $
8003 A	4002 A	5002 A	2501 A	2001 A	1000 A

3. Consider the power system shown in Fig. The values marked are p.u. impedances. The p.u. reactances of the generator 1 and 2 are 0.15 and 0.075 respectively. Compute the bus impedance matrix of the generator – transmission network.



Solution:

The ground bus is numbered as 0 and it is taken as reference bus. The p.u. impedance diagram is shown in Fig.



When element 0-1 is included

$$Z_{bus} = j \begin{matrix} & 1 \\ 1 & [0.15] \end{matrix}; \text{ When element 0-2 is included } Z_{bus} = j \begin{matrix} & 1 & 2 \\ 1 & \begin{bmatrix} 0.15 & 0 \\ 0 & 0.075 \end{bmatrix} \\ 2 & \end{matrix}$$

Element 1-2 is added; it is a link between buses 1 and 2. With bus ℓ

$$Z_{bus} = j \begin{matrix} & 1 & 2 & \ell \\ 1 & \begin{bmatrix} 0.15 & 0 & 0.15 \\ 0 & 0.075 & -0.075 \\ 0.15 & -0.075 & 0.325 \end{bmatrix} \\ 2 & \\ \ell & \end{matrix}; \quad \text{Eliminating the } \ell^{\text{th}} \text{ bus}$$

$$Z_{bus} = j \begin{matrix} & 1 & 2 \\ 1 & \begin{bmatrix} 0.08077 & 0.034615 \\ 0.034615 & 0.05769 \end{bmatrix} \\ 2 & \end{matrix}$$

Add element 1-3. It is a branch from bus 1 and it creates bus 3.

$$Z_{bus} = j \begin{matrix} & 1 & 2 & 3 \\ 1 & \begin{bmatrix} 0.08077 & 0.034615 & 0.08077 \\ 0.034615 & 0.05769 & 0.034615 \\ 0.08077 & 0.034615 & 0.18077 \end{bmatrix} \\ 2 & \\ 3 & \end{matrix}$$

Finally add element 2-3. It is a link between buses 2 and 3. With bus ℓ

$$Z_{bus} = j \begin{matrix} & 1 & 2 & 3 & \ell \\ 1 & \begin{bmatrix} 0.08077 & 0.034615 & 0.08077 & -0.046155 \\ 0.034615 & 0.05769 & 0.034615 & 0.023075 \\ 0.08077 & 0.034615 & 0.18077 & -0.146155 \\ -0.046155 & 0.023075 & -0.146155 & 0.26923 \end{bmatrix} \\ 2 & \\ 3 & \\ \ell & \end{matrix}$$

$$Z_{bus} = j \begin{matrix} & 1 & 2 & 3 \\ 1 & \begin{bmatrix} 0.07286 & 0.03857 & 0.05571 \\ 0.03857 & 0.05571 & 0.04714 \\ 0.05571 & 0.04714 & 0.10143 \end{bmatrix} \\ 2 & \\ 3 & \end{matrix}$$

Symmetrical fault analysis through bus impedance matrix

4.

Consider the power system discussed in Example 3.2. The p.u. impedances are on a base of 50 MVA and 12 kV. Symmetrical short circuit occurs at bus 3 with zero fault impedance. Using Z_{bus} matrix determine the fault current, bus voltages and also the currents contributed by the generators.

Solution

As seen in example 3.2, Z_{bus} matrix of the transmission-generator network is

$$Z_{bus} = j \begin{matrix} & \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} 0.07286 & 0.03857 & 0.05571 \\ 0.03857 & 0.05571 & 0.04714 \\ 0.05571 & 0.04714 & 0.10143 \end{bmatrix} \end{matrix}$$

Faulted system is shown in Fig. 3.28

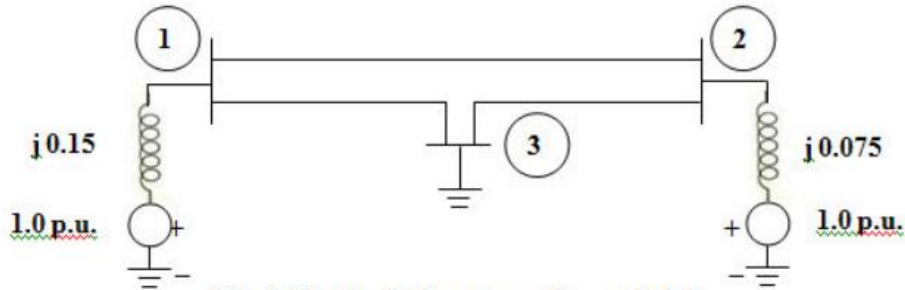


Fig. 3.28 Faulted system – Example 3.3

Faulted bus current $I_{3(F)} = - \frac{1}{j0.10143} = j9.8590 \text{ p.u.}$

Fault current $I_F = - I_{3(F)} = - j9.8590 \text{ p.u.}$

Base current = $\frac{50 \times 1000}{\sqrt{3} \times 12} = 2405.6 \text{ amp.}$

Fault current $I_F = -j 9.8590 \times 2405.6 = -j 23717 \text{ amp.}$

Since fault impedance is zero, $V_{3(F)} = V_F = 0$

Taking the pre-fault voltage $V_0 = 1.0 \text{ p.u.}$

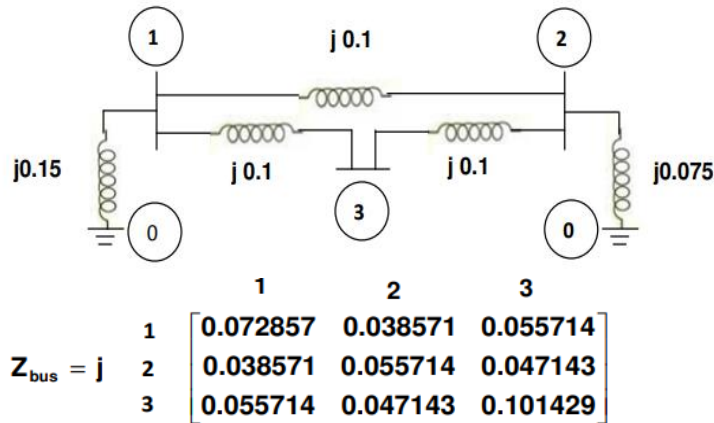
$$V_{1(F)} = 1.0 - \frac{Z_{13}}{Z_{33}} = 1.0 - \frac{0.05571}{0.10143} = 0.45075 \text{ p.u.} = \frac{12}{\sqrt{3}} \times 0.45075 = 3.1229 \text{ kV}$$

$$V_{2(F)} = 1.0 - \frac{Z_{23}}{Z_{33}} = 1.0 - \frac{0.04714}{0.10143} = 0.53525 \text{ p.u.} = \frac{12}{\sqrt{3}} \times 0.53525 = 3.7083 \text{ kV}$$

Current supplied by gen. 1, $I_{G1} = \frac{1.0 - 0.45075}{j0.15} = -j3.6617 \text{ p.u.} = -j8808.6 \text{ amp.}$

Current supplied by gen. 2, $I_{G2} = \frac{1.0 - 0.53525}{j0.075} = -j6.1967 \text{ p.u.} = -j14906.8 \text{ amp.}$

5. For the transmission-generator system shown in Fig.3.29, the bus impedance matrix is obtained as



Symmetrical three phase fault with fault impedance $j 0.052143$ p.u. occurs at bus 1. Find the p.u. currents in all the elements and mark them on the single line diagram.

Solution:

Fault occurs at bus 1 and we need the first column of ZBus, which is

$$j \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} \begin{bmatrix} 0.072857 \\ 0.038571 \\ 0.055714 \end{bmatrix} \text{ and } Z_F = j 0.052143$$

$$\text{Faulted bus current } I_{1(F)} = - \frac{1}{j0.072857 + j0.052143} = - \frac{1}{j0.125} = j 8$$

$$\text{Fault current } I_F = - I_{1(F)} = - j 8 \text{ p.u.}$$

$$V_1(F) = V_F = Z_F I_F = (j 0.052143) (- j 8) = 0.41714 \text{ p.u.}$$

$$V_2(F) = 1 - \frac{0.038571}{0.125} = 0.69143 \text{ p.u. } V_3(F) = 1 - \frac{0.055714}{0.125} = 0.55429 \text{ p.u.}$$

$$i_{2-1} = (0.69143 - 0.41714) / (j 0.1) = - j 2.7429 \text{ p.u.}$$

$$i_{3-1} = (0.55429 - 0.41714) / (j 0.1) = - j 1.3715 \text{ p.u.}$$

$$i_{2-3} = (0.69143 - 0.55429) / (j 0.1) = - j 1.3714 \text{ p.u.}$$

$$i_{G1} = (1 - 0.41714) / (j 0.15) = - j 3.8857 \text{ p.u.}$$

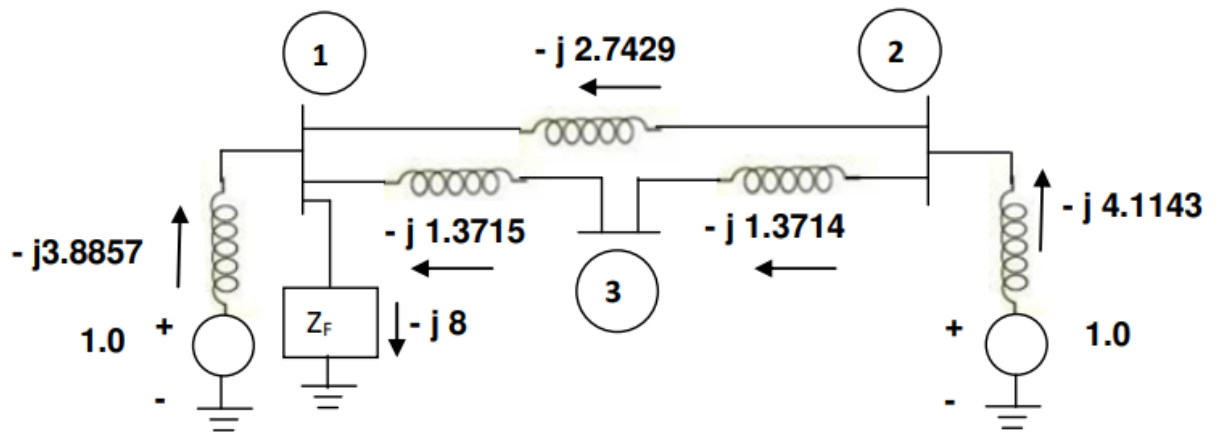
$$i_{G2} = (1 - 0.69143) / (j 0.075) = - j 4.1143 \text{ p.u.}$$

$$V_1(F) = 0.41714 \text{ p.u.}$$

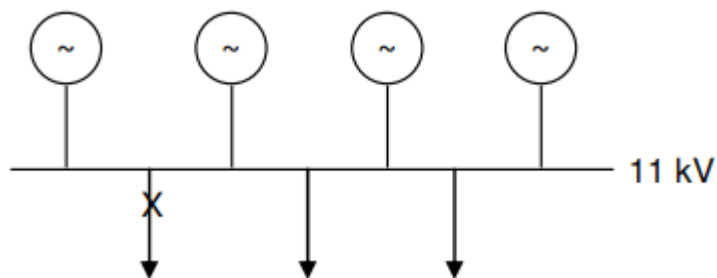
$$V_2(F) = 0.69143 \text{ p.u.}$$

$$V_3(F) = 0.55429 \text{ p.u.}$$

Currents are marked in Fig.

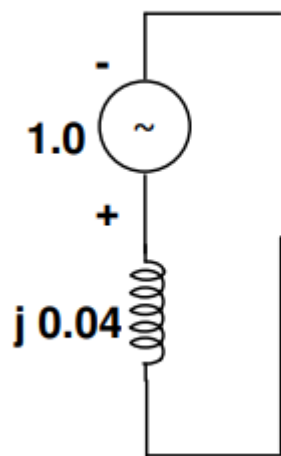
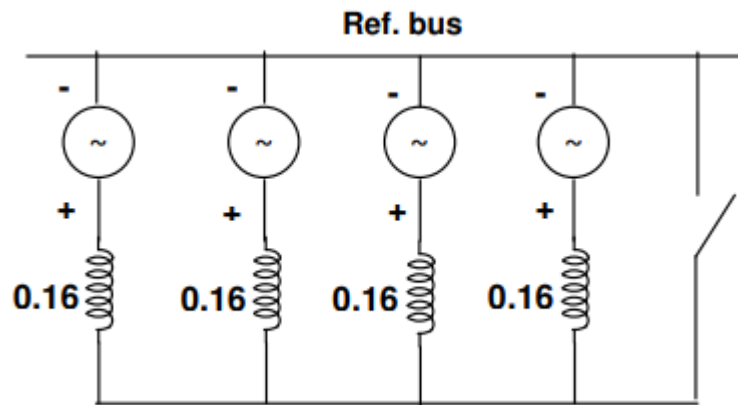


6. Fig. shows four identical alternators in parallel. Each machine is rated for 25 MVA, 11 kV and has a subtransient reactance of 16 % on its rating. Compute the short circuit MVA when a three phase fault occurs at one of the outgoing feeders.



Solution

Fault is simulated by closing the switch shown in the p.u. reactance diagram shown in fig and Its Thevenin's equivalent is also shown in next Fig.



Fault current $|I_F| = \frac{1}{0.04} = 25$ p.u.

Short circuit MVA = prefault voltage in p.u. x fault current in p.u. x Base MVA

= 1.0 x 25 x 25

= 625