Capacitance:

Capacitor have two or more conductors carrying equal but opposite charges. All the flux lines leaving one conductor must terminate at the surface of other conductor. The conductors are referred as the plates of a capacitor. The plates are separated by free space or a dielectric.

Consider a two conductor capacitor. The conductors are maintained at a potential difference V given by

$$V = V_1 - V_2 = -\int_2^1 E \cdot dl$$

E → Electric field existing between the conductors and conductor 1 is assumed to carry a positive charge.(E field is always normal to the conducting surface)

The capacitance C of the capacitor as the ratio of the magnitude of the charge on one of the plates to the potential difference between them.

$$C = \frac{Q}{V} = \frac{\epsilon \oint E \cdot dS}{\int E \cdot dl}$$

The negative sign before $V = -\int E \cdot dl$ represent the drop of the absolute potential. Capacitance is measured in farads.

The capacitance C for any given two- conductor can be obtained by either of following methods.

1. Assuming Q and determining V in terms of Q (Gauss's Law) .
2. Assuming V and determining Q in terms of V (solving Laplace's equation).

Former methods involves the following method.

a. Choose a suitable coordinate system.
b. Let the two conducting plates carry charges +Q and -Q .
c. Determine E using coulomb's or Gauss's Law and find V from $V = -\int E \cdot dl$ .The negative sign is ignored because the absolute potential is taken into account.
d. Obtain C from $c = \frac{Q}{V}$

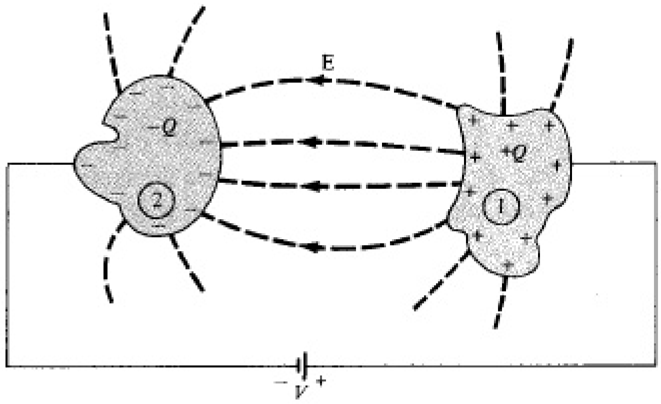

Fig: 8 A two – Conductor Capacitor

Parallel Plate Capacitor:

Consider a parallel plate capacitor .Each of the plates has an area S and are separated by a distance d. Assume plate 1 carry +Q charges and plate 2 carry –Q charges uniformly distributed on them.

$$\rho_s = \frac{Q}{S}$$

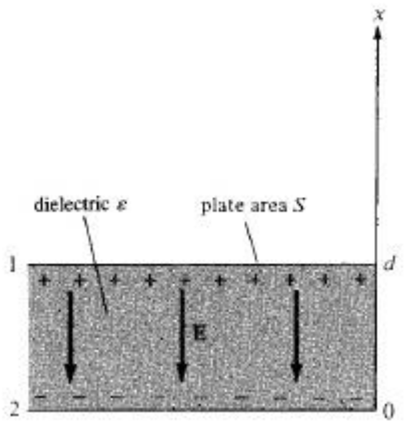

Fig 9a: Parallel – plate Capacitor

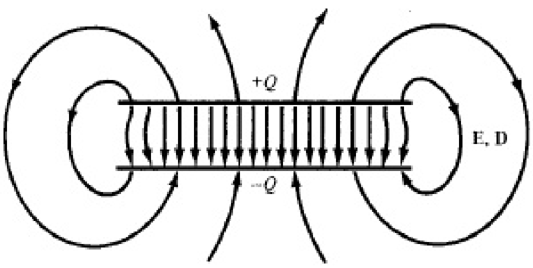

Fig 9b : Fringing effect of parallel plate capacitor.

In an ideal parallel plate capacitor the plate separation d (distance between the plates) is less than the dimension of the plate. Field distributed between them is uniform. If the space between the plates is filled with a homogeneous dielectric with permittivity ε and ignore the flux fringing at the edges of the plates

$$D = -\ \rho_s a_x$$

$$E = \frac{\rho_s}{\varepsilon}(-\ a_x)$$

$$= -\frac{Q}{\varepsilon S}\, a_x$$

Hence $V = -\int_2^1 E \,.dl = -\int_0^d \left[-\frac{Q}{\varepsilon S} a_x\right] .dx\, a_x = \frac{Qd}{\varepsilon S}$

For a parallel – plate capacitor Capacitance $C = \frac{Q}{V} = \frac{\varepsilon S}{d}$

The Resistance of a plate is $R = \frac{d}{\sigma S}$ ($R = \frac{\varepsilon}{\sigma C}$)

ε_r → Permittivity of the dielectric

$$\varepsilon_r = \frac{C}{C_0}$$

C → Capacitance of parallel plate with filled dielectric ,

C_0 → Capacitance of parallel plate with filled air

The energy stored in a Capacitor is $W_E = \frac{1}{2}\, CV^2 = \frac{1}{2} QV = \frac{Q^2}{2C}$

Electrostatic energy stored in a field is given by $W_E = \frac{1}{2} \int \varepsilon_0\, E^2\, dv$

Substitute C in the above equation

$$W_E = \frac{1}{2} \int_v \varepsilon \frac{Q^2}{\varepsilon^2 S^2}\, dv = \frac{\varepsilon Q^2 S d}{2\varepsilon^2 S^2}$$

$$= \frac{Q^2}{2}\left(\frac{d}{\varepsilon S}\right) = \frac{Q^2}{2C} = \frac{1}{2}\, QV$$

Coaxial Capacitor (Coaxial Cylindrical Capacitor)

Consider a coaxial cable of length L . Coaxial cable consists of two conductors inner conductor of radius ‘a’ and outer conductor of radius’ b’ .Let the space between the conductors be filled with homogeneous dielectric with permittivity ε . Assume that the inner and outer conductor carry +Q and

–Q charges and are uniformly distributed on them.

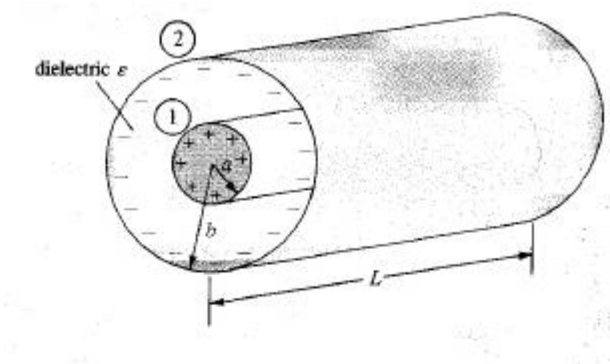

Fig 10: Coaxial Capacitor

The radius of the cylinder is $a < \rho < b$.

The total Charge $Q = \varepsilon \oint E \,.ds = \varepsilon E_\rho \; 2\pi\rho L$

Hence $$E = \frac{Q}{2\pi\varepsilon\rho L} \; a_\rho$$

Neglecting flux fringing at the cylinder ends,

$$V = -\int_2^1 E \,.dl = -\int_b^a \left[\frac{Q}{2\pi\varepsilon\rho L} \; a_\rho\right] .d\rho \; a_\rho$$

$$= \frac{Q}{2\pi\varepsilon L} \; ln\frac{b}{a}$$

Thus the capacitance of a coaxial cylinder is $C = \frac{Q}{V} = \frac{2\pi\varepsilon L}{ln\frac{b}{a}}$

$$R = \frac{ln\frac{b}{a}}{2\pi\sigma L} \qquad \left(R = \frac{\varepsilon}{\sigma C} \right)$$

Spherical Capacitor:

Two concentric spherical conductor of inner radius ‘a’ and outer radius ‘b’ separated by a dielectric medium with permittivity ε . Assume charges +Q and –Q on inner and outer spheres respectively. By apply Gauss’s law to an Gaussian spherical surface of radius r (a< r < b)

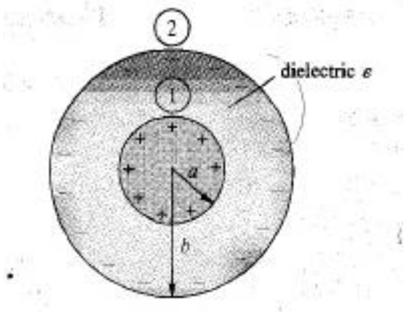

Fig 11: Spherical Capacitor

$$Q = \varepsilon \oint E \,.ds = \varepsilon E_r \, 4\pi r^2$$

$$E = \frac{Q}{4\pi\varepsilon r^2} a_r$$

The potential difference between the conductor is

$$V = -\int_2^1 E\,.dl = -\int_b^a \left[\frac{Q}{4\pi\varepsilon r^2}\, a_r\right] dr\,.a_r$$

$$= \frac{Q}{4\pi\varepsilon}\left[\frac{1}{a} - \frac{1}{b}\right]$$

Thus the Capacitance of spherical capacitor is

$$C = \frac{Q}{V} = \frac{4\pi\varepsilon}{\frac{1}{a} - \frac{1}{b}}$$

$$R = \frac{\frac{1}{a} - \frac{1}{b}}{4\pi\sigma} \qquad \left(R = \frac{\varepsilon}{\sigma C} \right)$$

By letting b→∞, C = $4\pi\varepsilon a$(Spherical conductor is at a large distance from other conducting bodies, isolated sphere)

$$R = \frac{1}{4\pi\sigma a}$$

If two capacitor with capacitance C1 and C2 are connected in series , the total capacitance is

$$\frac{1}{C} = \frac{1}{C1} + \frac{1}{C2}$$

$$C = \frac{C1\, C2}{c1 + C2}$$

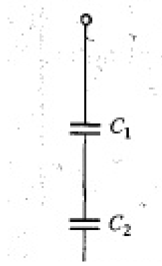

Fig 12: Capacitor in series

If two capacitor connected in parallel the total capacitance is $C = C1 + C2$

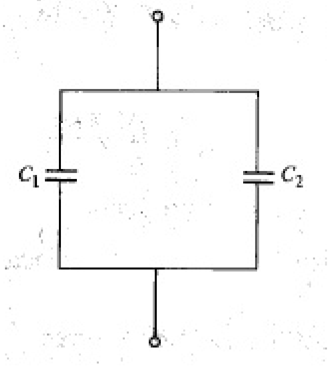

Fig 13: Capacitor in parallel