## 2.3 ANALYSIS OF CT SYSTEMS

### IMPULSE RESPONSE

### LTI SYSTEM WITH AND WITHOUT MEMORY

Convolution is given by $y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$

Convolution is commutative $y(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau$

A CT system is memoryless if present output depends on present input. Above condition is true only for $h(\tau) = k\delta(\tau)$ and such a memory system has the form

$$y(t) = \int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau$$

$$y(t) = \int_{-\infty}^{\infty} x(t-\tau)k\delta(\tau)d\tau$$

$$= kx(t) * \delta(t) \text{ ----- } (1)$$

We know that, $x(t) * \delta(t) = x(t)$

$$(1) \rightarrow \; y(t) = kx(t)$$

K is the constant

The system is memory less or static if

$$h(t) = \delta(t) - - - - (2)$$

If (2) is not satisfied system is dynamic system

Hence the output is equal to the input, this system becomes an identity system.

## INVERTIBILITY OF LTI SYSTEM

Consider a CT LTI system with impulse response h(t). This system is invertible, if it has an LTI inverse system.

We know, for an identity system

$$x(t) * \delta(t) = x(t)$$

The impulse response of an inverse system should satisfy the condition

$$h(t) * h_1(t) = \delta(t)$$

## CAUSALITY FOR LTI SYSTEM

For a system to be causal, the output of the system must depend on present and past inputs only.

$$\therefore h(t) = 0 \; for \; t < 0$$

The convolution integral becomes

$$y(t) = \int_{\infty}^{t} x(\tau)h(t-\tau)d\tau$$

$$y(t) = \int_{0}^{\infty} h(\tau)x(t-\tau)d\tau$$

## STABILITY OF LTI SYSTEM

For a LTI system to be stable

$$\int_{-\infty}^{\infty} |h(\tau)| d\tau < \infty$$

**FOURIER METHOD FOR ANALYSIS**

**FREQUENCY RESPONSE OF LTI SYSTEM**

Consider a LTI system

$$y(t) = x(t) * h(t)$$

Take Fourier transform on both sides

$$Y(j\omega) = X(j\omega).\,H(j\omega) -----(1)$$

Convolution in time domain gives multiplication in frequency domain.

Where, $H(j\omega)\ is\ the\ frequency\ response\ of\ the\ system$

$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)}$$

Magnitude of equation (1) is

$$|Y(j\omega)| = |X(j\omega)|.\,|H(j\omega)|$$

$$|H(j\omega)|\ is\ the\ gain\ of\ the\ system$$

phase of equation (1) is

$$\angle Y(j\omega) = \angle X(j\omega).\angle H(j\omega)$$

$$\angle H(j\omega)\ is\ the\ phase\ shift\ of\ the\ system$$

**ANALYSIS AND CHARACTERIZATION OF LTI SYSTEM USING LAPLACE TRANSFORM**

Output of the system is given by

$$y(t) = x(t) * h(t)$$

Taking Laplace Transform on both sides.

$$Y(S) = X(S).H(S)$$

Where, $H(S)$ *is the system function or transfer function*

If $S = j\omega$, then $H(S)$ is frequency response of LTI system.

**CAUSALITY:**

For causal LTI system

$$h(t) = 0 \; for \; t < 0$$

1. ROC for a causal system is in the right half plane.
2. ROC for causal system with rational system is right half plane to right of right most pole.

**STABILITY:**

An LTI system is stable if and only if ROC of its system function $H(S)$ includes $j\omega$ axis $.ie, Re(S) = 0$

A causal system with rational system function is stable if and only if all poles(X) of $H(S)$ lie in left half of S-plane.