NOISE MODELS IN MEDICAL IMAGES

The statistical properties of three common types of noise found in medical imaging (Gaussian, Poisson, and Rician) and derive the relationship for each of them. Whenever an image contains different types of uncorre-lated noise, the overall noise variance σ_N^2 can be expressed by summing up the various noise contributions

$$\sigma_N^2 = \sigma_1^2 + \sigma_2^2 + \ldots + \sigma_n^2 \qquad \text{---------} (1)$$

1. GAUSSIAN NOISE

This most common type of noise results from the contribu-tions of many independent signals. This is a consequence of the central limit theorem which states that the sum of many random variables with various PDFs results in a signal with a Gaussian PDF. For example, the reading noise from a CCD de-tector is generated by the thermal fluctuations in many intercon-nected electronics components and, thus, has a Gaussian PDF. Gaussian noise is such that σ_N^2 is a constant.

2. POISSON NOISE

Poisson noise prevails in situations where an image is cre-ated by the accumulation of photons over a detector. Typical examples are found in standard X-ray films, CCD cameras, and infrared photometers. We focus our attention on images saved with linear or logarithmic intensity scalings.

Linear Intensity Scaling:

The following analysis as-sumes a pixel intensity corresponding to the number of monochromatic photons captured in a given amount of time.

For a Poisson process of mean X (where X represents the number of captured photons), the expectation value for the vari-ance is

$$\sigma_X^2 = X. \qquad \text{-------------} \ (2)$$

Because a recorded image is usually linearly rescaled to ac-commodate a given range in grey scales, the relation between the intensity X reaching the detector and the recorded intensity

I is

$$I = \gamma X + \delta \qquad \text{----------} \ (3)$$

where γ and δ are constants. The corresponding variance σ_I^2 varies linearly with the intensity I

$$\sigma_I^2 = \gamma I - \delta\gamma. \qquad \text{------------} \ (4)$$

Logarithmic Intensity Scaling:

The noise statistics are al-tered on photographic plates, such as those used for X-rays, where the stored information is the optical density. An X-ray film being a *negative* recorder, an increase in light exposure causes the developed film to become darker. The degree of dark-ness of the film is quantified by the optical density D which is measured in this work by a scanner. Within the operational intensity range of a given film, the relation can be expressed as

$$D = D_m - \log\left(\frac{X}{X_M}\right) \qquad \text{------------} \ (5)$$

The minimum optical density D_m (or maximum transmittance) is found in the bright bone regions and the maximum optical density (or minimum transmittance) is found in the dark background regions. Taking into account image rescaling, the relation between the optical density and the scanned intensity, I, becomes

$$I = \gamma D + \delta \qquad \text{------------ (6)}$$

and the corresponding variance σ_I^2 varies exponentially with the intensity I

$$\ln\left(\sigma_I^2\right) = \left[\ln\left(\frac{\alpha^2\gamma^2}{X_M}\right) \quad \frac{I_m}{\alpha\gamma}\right] \quad \frac{I}{\alpha\gamma} \qquad \text{----------- (7)}$$

with

$$\alpha \ = \ln_{10}(\boldsymbol{e}). \qquad \text{----------(8)}$$

3. RICIAN NOISE:

The noise in MR images has a Rician PDF . The signals are acquired in quadrature. Each signal produces an image. The signals are acquired in quadrature. Each signal pro-duces an image X that is degraded by a zero-mean Gaussian noise of standard deviation σ_0 (which we define as the noise level). The two images are then combined into a magnitude

image I and the Gaussian noise PDF is transformed into a Ri-cian noise PDF. The expectation values for the mean magnitude and the variance are

$$I = \sigma_0\sqrt{\frac{\pi}{2}}\exp\left(-\frac{X^2}{4\sigma_0^2}\right)$$
$$\left[\left(1+\frac{X^2}{2\sigma_0^2}\right)I_0\left(\frac{X^2}{4\sigma_0^2}\right)+\frac{X^2}{2\sigma_0^2}I_1\left(\frac{X^2}{4\sigma_0^2}\right)\right] \qquad \text{--------- (9)}$$

$$\sigma_I^2 = X^2 + 2\sigma_0^2 - \frac{\pi\sigma_0^2}{2}\exp\left(-\frac{X^2}{2\sigma_0^2}\right)$$
$$\left[\left(1+\frac{X^2}{2\sigma_0^2}\right) \quad \left(\frac{X^2}{4\sigma_0^2}\right) \quad \frac{X^2}{2\sigma_0^2}I_1\left(\frac{X^2}{4\sigma_0^2}\right)\right]^2 \qquad \text{--------- (10)}$$

where I_0 and I_1 are modified Bessel functions of the first kind.

When $X \gg \sigma_0$, the Rician PDF approaches a Gaussian PDF
with $I \cong$ and $\sigma_I \cong \sigma_0$. When $X \ll \sigma_0$, the Rician PDF approaches the Rayleigh PDF which does not depend on X. The expectation values for the mean magnitude and the variance of a Rayleigh PDF are

$$I = \sigma_0\sqrt{\frac{\pi}{2}} \qquad \text{-------- (11)}$$

$$\sigma_I^2 \quad \frac{4-\pi}{2}\sigma_0^2. \qquad \text{----------- (12)}$$

Equations (11) and (12) can be used to estimate the value of σ_0^2 in an image from measurements in background regions where the magnitude is almost zero.