

PROVING LANGUAGES NOT TO BE REGULAR

Theorem

Let L be a regular language. Then there exists a constant ' c ' such that for every string w in L –
 $|w| \geq c$

We can break w into three strings, $w = xyz$, such that –

- $|y| > 0$
- $|xy| \leq c$
- For all $k \geq 0$, the string xy^kz is also in L .

Applications of Pumping Lemma

Pumping Lemma is to be applied to show that certain languages are not regular. It should never be used to show a language is regular.

- If L is regular, it satisfies Pumping Lemma.
- If L does not satisfy Pumping Lemma, it is non-regular.

Method to prove that a language L is not regular

- At first, we have to assume that L is regular.
- So, the pumping lemma should hold for L .
- Use the pumping lemma to obtain a contradiction –
 - Select w such that $|w| \geq c$
 - Select y such that $|y| \geq 1$
 - Select x such that $|xy| \leq c$
 - Assign the remaining string to z .
 - Select k such that the resulting string is not in L .

Hence L is not regular.

Problem

Prove that $L = \{a^i b^i \mid i \geq 0\}$ is not regular.

Solution

- At first, we assume that L is regular and n is the number of states.
- Let $w = a^n b^n$. Thus $|w| = 2n \geq n$.
- By pumping lemma, let $w = xyz$, where $|xy| \leq n$.
- Let $x = a^p$, $y = a^q$, and $z = a^r b^n$, where $p + q + r = n$, $p \neq 0$, $q \neq 0$, $r \neq 0$. Thus $|y| \neq 0$.
- Let $k = 2$. Then $xy^2z = a^p a^{2q} a^r b^n$.
- Number of a 's $= (p + 2q + r) = (p + q + r) + q = n + q$
- Hence, $xy^2z = a^{n+q} b^n$. Since $q \neq 0$, xy^2z is not of the form $a^n b^n$.
- Thus, xy^2z is not in L . Hence L is not regular.

