Characterization of Linear Time Invariant(LTI)system

Both continuous time and discrete timelineartimeinvariant(LTI)systemsexhibitoneimportant characteristics that the superposition theoremcanbeappliedtofindtheresponsey(t)toagiveninputx(t).

Hence, following steps may be adoptedtofindtheresponseofaLTIsystemusingsuperpositiontheorem:

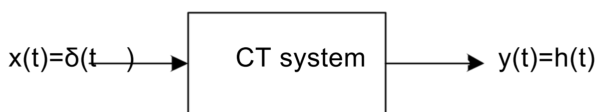
1. Resolve the input functionx(t)intermsofsimplerorbasicfunctionlikeimpulsefunctionforwhich response can be easilyevaluated.

2. Determine individuallytheresponseofLTIsystemforthesimplerinputimpulsefunctions.

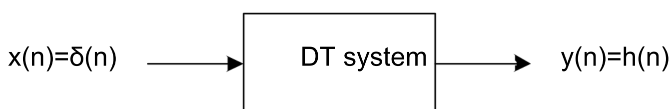
3. Using superpositiontheorem,findthesumoftheindividualresponses,whichwillbecometheoverall response y(t) of functionx(t).

From theabovediscussions,itisclearthattofindtheresponseofaLTIsystemtoanygivenfunction,first we havetofindtheresponseofLTIsysteminputtoanunitimpulsecalledunitimpulseresponseofLTI system.

Hence,theimpulseresponseofacontinuoustimeordiscretetimeLTIsystemistheoutputofthesystem due to anunit impulse input appliedattimet=0or n=0.



Here,δ(t)istheunitimpulseinputincontinuoustimeandh(t)istheunitimpulseresponseofcontinuous timeLTIsystem.Continuoustimeunitimpulseresponseh(t)istheoutputofacontinuoustimesystemwhen appliedinput x(t) is equalto unitimpulse functionδ(t)



Similarly,foradiscretetimesystem,discretetimeimpulseresponseh(n)istheoutputofadiscretetime systemwhenappliedinputx(n)isequaltodiscretetimeunitimpulsefunctionδ(n).Here,δ(n)istheunit impulseinputindiscretetimeandh(n)istheunitimpulseresponseofdiscretetimeLTIsystem.Therefore, any LTIsystemcanbecompletelycharacterizedintermsofitsunitimpulseresponse.

Propertiesof Lineartime invariant (LTI) system:-

The LTI systemhaveanumberofpropertiesnotexhibitedbyothersystems.Thoseareasunder:

CommutativepropertyofLTIsystems
DistributivepropertyofLTIsystems
AssociativepropertyofLTIsystems
Static anddynamicLTIsystems
InvertibilityofLTIsystems
CausalityofLTIsystems
StabilityofLTIsystems
Unit-stepresponseofLTIsystems

Commutative property:

The commutative property is a basic property of convolution in both continuous and discrete time cases, thus, both convolution integral for continuous time LTI systems and convolution sum for discrete time LTI systems are commutative. According to the property, for continuous time LTI system. The output is given by

$$y(t) = x(t) * h(t) = \int_{\tau=-\infty}^{\infty} x(\tau) h(t-\tau) d\tau$$

Or

$$y(t) = h(t) * x(t) = \int_{\tau=-\infty}^{\infty} h(\tau) x(t-\tau) d\tau$$

Thus, we can say that according to this property, the output of a continuous time LTI system having input x(t) and unit impulse h(t) is identical to the output of a continuous time LTI system having input h(t) and the unit impulse response x(t).

Distributive property:

The distributive property states that both convolution integral for continuous time LTI system and convolution sum for discrete time LTI system are distributive.

For continuous time LTI system, the distributive property is expressed as

Theoutput, $y(t) = x(t) * [h_1(t) + h_2(t)]$

Or

$$y(t) = x(t) * h_1(t) + x(t) * h_2(t)$$

Thus, the two continuous time LTI systems, with impulse responses h1(t) and h2(t), have identical inputs and outputs are added as

$$y_1(t) = x(t) * h_1(t), \quad y_2(t) = x(t) * h_2(t)$$

The output $y(t) = y_1(t) + y_2(t) = x(t) * h_1(t) + x(t) * h_2(t)$

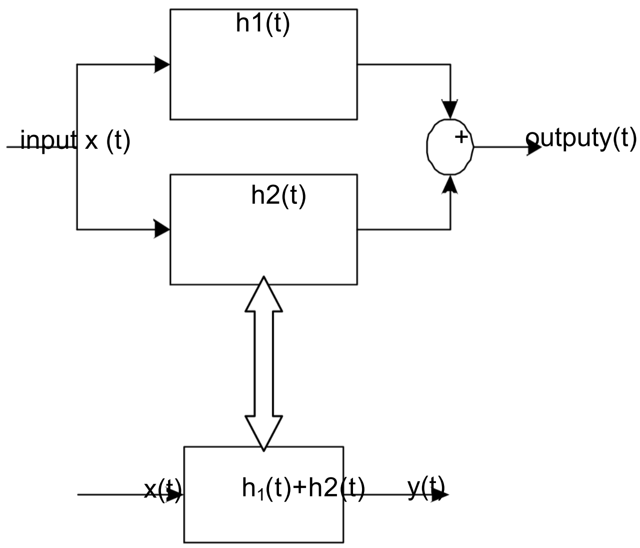


Fig3.1Thedistributivepropertyofconvolutionintegralfor a parallel interconnection of continuous time LTI system

Associative Property of LTI system:

According to associative property, both convolution integral for continuous time LTI systems and convolution sum for discrete time LTI systems are associative.

For continuous time LTI system, according to associative property,

The output

$$y(t) = [x(t) * h_1(t)] * h_2(t)$$

or

$$y(t) = x(t) * [h_1(t) * h_2(t)]$$

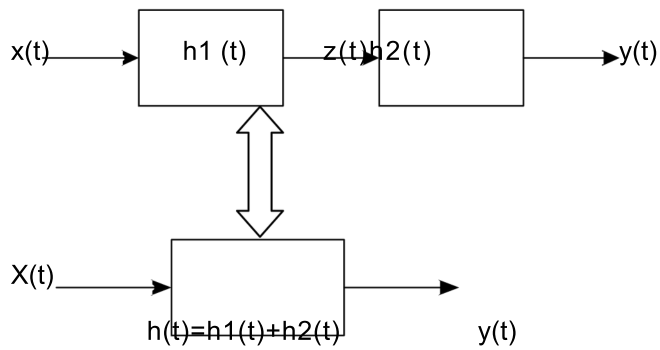


Fig 3.2:The associative property of convolution integral for a cascade interconnection of continuous time LTI systems

Here we have y(t)=z(t) *h2(t)But z(t)=x(t)*h1(t). Therefore y(t)=[x(t)*h1(t)*h2(t)]

Static and Dynamic property:

Static systems are also known as memory less systems. A system is known as static if its output at any time depends only on the value of the input at the same time. A continuous time system is memory less if its unit impulse response h(t) is zero for $t \neq 0$. These memory less LTI systems are characterized by y (t) =Kx (t) where K is constant.And its impulse response h(t)=Kδ(t) .If K=1 ,then these systems are called identity systems.

Invertibility of LTI systems:

A system is known as invertible only if an inverse system exists which, when cascade with the original system, produces and output equal to the input at first system. If an LTI system is invertible then it will have a LTI inverse system. This means that we have a continuous time LTI system with impulse response h(t) and its inverse system with impulse response h1(t) which results in an output equal to x(t). Cascade interconnection of original continuous time LTI system with its inverse system is given as identity system.

Thus, the overall impulse response of a system with impulse response h(t) cascaded with its inverse system with Impulse response h1(t) is given as h(t)*h1(t)=δ(t)

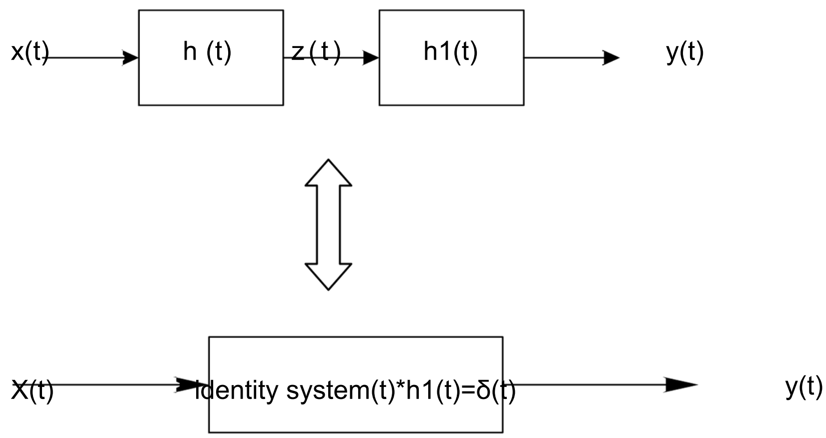


Fig3.3: An inverse system for continuous time LTI system

Causality for LTI System

This property says that the output of a causal system depends only on the present and past values of the input to the system.

A continuous time LTI system is called causal system if its impulse response h(t) is zero t<0.

For a causal continuous time LTI system, convolution integral is given as

$$y(t) = \int_{\tau=-\infty}^{t} x(\tau)h(t-\tau)\,d\tau = \int_{\tau=0}^{\infty} h(\tau)x(t-\tau)\,d\tau$$

For pure time shift with unit impulse response h(t)=δ(t-t0) is a causal continuous timeLTI system for t≥0. In this case time shift is known as a delay.

Stability for LTI systems

A stable system is a system which produces bounded output for every bounded input.

Condition of Stability for continuous time LTI system:

Let us consider an input x(t) that is bounded in magnitude $|x(t)|<M$ for all values of t

Now, we apply this input to an continuous time LTI system with unit impulse response h(t).

Output of this LTI system is determined by convolution integral and is given by

$$y(t) = \int_{\tau=-\infty}^{\infty} h(\tau)x(t-\tau)\,d\tau$$

Magnitude of output y(t) is given as

$$|y(t)| = \left|\int_{\tau=-\infty}^{\infty} h(\tau)x(t-\tau)\,d\tau\right| \le \int_{\tau=-\infty}^{\infty} |h(\tau)||x(t-\tau)|\,d\tau$$

Substituting the value $|x(t-\tau)| \le M_x$ for all values of τ and t, we get

$$|y(t)| \le M_x \int_{\tau=-\infty}^{\infty} |h(\tau)|\,d\tau$$

$|y(t)| \le M_y$ Or

$\int_{\tau=-\infty}^{\infty} |h(\tau)|\,d\tau < \infty$ for all values of t

Fromtheaboveequationwecanconcludethatiftheimpulseresponse h(t) is absolutely integerable then outputofacontinuoustimeLTIsystemisboundedinmagnitude, and thus, the system is bounded input, bounded output(BIBO) stable.

Unit step responseof an LTI system:

UnitstepresponseistheoutputofaLTIsystemforinputisequal to unit step function or sequence. Unit stepresponseofcontinuoustimeLTIsystemisfoundbyconvolution integral of u(t) with unit impulse responseh(t) and is expressedas

$$g(t)=u(t)*h(t)=h(t)*u(t)$$

accordingtocommutativeproperty.Therefore,unitstepresponse g(t) may be viewed as the response to the inputh(t)ofacontinuoustimeLTIsystemwithunitimpulseresponse u(t).

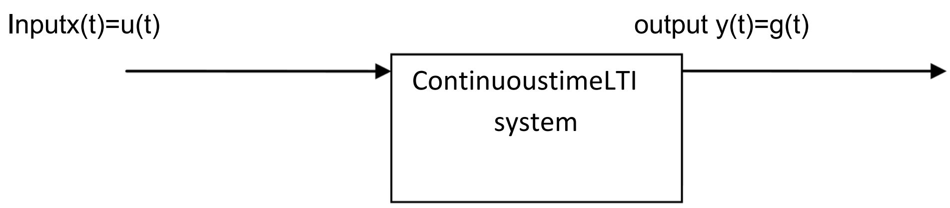


Fig 3.4: Continuous time LTI system

Classification of CT LTI system:-

The systems are classified into two types: continuous time and discrete time systems, Now these two broad types of systems are further classified on the basis of system properties as under:

Causal system and non causal system Time invariant and time variant system Stable and unstable system Linear and Non-linear system Static and Dynamic systems Invertible and noninvertible system

Causal systems and Non-causal systems

A system is causal if the response or output does not begin before the input function is applied. This means that if input is applied at t=t0,then for causal system, output will depend on values of input x(t) for t≤t0. Mathematically,

y(t0)=f[x(t),t≤t0].

In other words we can say that, the response or output of the causal system to an input does not depend on future values of that input but depends only on the present or past values of the input.This means that all the real-time systems are also causal systems since these systems cannot know the future values of the input signal when it constructs output signal.Thus, causal systems are physically realizable. For example a resister is a continuous time causal systembecause voltage across it is given by the expression v(t)=R.i(t) and output v(t),i.e., voltage depends only on the input i(t) i.e., current at the present time.

Time invariant and time variant system

A system is said to be time invariant if its input –output characteristics do not change with time. H{x(t)}=y(t) implies that ,H{x(t-t0)}=y(t-t0) for every input signal x(t) and every time shift t0A system is said to be time variant if its input- output characteristics changes with time.

Procedure to Test for Time Invariance:-

1. Delay the input signal by t0 units of time and determine the response of the system for this delayed input signal. Let this response be y(t-t0).

2. Delay the response of the system for undelayed input by t0 units of time. Let this delayed response be yd(t).

3. Check whether y(t-t0)=yd(t). If they are equal then the system is time invariant. Otherwise the system is time variant.

Stableandunstable system

A systemiscalled bounded input, bounded output(BIBO) stable if and only if every bounded input results in a boundedoutput. The output of such a system does not diverge or does not grow unreasonably large.

ConditionofStability for continuous time LTI system:

|Let u|s consider an input x(t) that is bounded in magnitude

<M<∞ for all values of t

Now, we apply this input to an continuous time LTI system with unit impulse response h(t).

Output of this LTI system is determined by convolution integral and is given by

$$\int = y(t) = \int_{\tau=-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau$$

Magnitude of output y(t) is given as

$$|y(t)| = \left|\int_{\tau=-\infty}^{\infty} h(\tau)x(t-\tau)\,d\tau\right| = \int_{\tau=-\infty}^{\infty} |h(\tau)||x(t-\tau)|\,d\tau$$

Substituting the value $|x(t-\tau)| < M_x$ for all values of τ and t, we get

$$|y(t)| \leq M_x \int_{\tau=-\infty}^{\infty} |h(\tau)|\,d\tau$$

$|y(t)| \leq$ Or

$\int_{\tau=-\infty}^{\infty} |h(\tau)|\,d\tau < \infty$ for all values of t

From the above equation we can conclude that if the impulse response h(t) is absolutely integerable then output of a continuous time LTI system is bounded in magnitude, and thus, the system is bounded input, bounded output(BIBO) stable.

The systems not satisfying the above conditions are unstable.

Linear and nonlinear system

A linear system is one that satisfies the superposition principle. The principles of superposition requires that the response of the system to a weighted sum of the signals is equal to the corresponding weighted sum of the responses of the system to each of the individual input signals.

A system is linear if

H{a1x1(t)+a2x2(t)}=a1H{x1(t)}+a2H{x2(t)} for any arbitrary input sequences x1(t) and x2(t) and for any arbitrary constants a1 and a2.

If a relaxed system does not satisfy the super position principle as given by the above definition, then the system is nonlinear.

Static and dynamic system

A continuous time system is called static or memory less if its output at any instant t depends on present input but not on the past or future samples of the input. These systems contain no energy storage elements. This means that the equation relating its output signal to its input signal contains no derivative, integrals or signal delays.

As an example, consider the system described by the following relationship

$Y(t)=x^2(t)$ this system is memory less because the value of the output signal y(t) at time t depends only on the present value of the input signal x(t).In any other case the system is said to be dynamic or to have memory. Dynamic systems have one or more energy storage elements. Input output relationship of a dynamic continuous time system is described by its differential equation.

Invertible and non invertible system

A system is said to be invertible if there is a one to one correspondence between its input and output signals. If a system is invertible, then an inverse system exists. The cascading of and invertible system and its inverse system is equivalent to the identity system.

The frequency response of an inverse system is basically reciprocal of the frequency response of the original system or invertible system.

An example of an invertible continuous time system is given by

$y(t)=3x(t)$ and its inverse system will be given by $x(t)=\frac{1}{3}y(t)$

Convolution integral:-

The output of any general input may be found by convolving the given input signal x(t) with the LTI systems unit impulse response h(t).

$$y(t)=x(t)*h(t)=\int_{\tau=-\infty}^{\infty} x(\tau)h(t-\tau)\,d\tau$$

Or

$$y(t)=h(t)*x(t)=\int_{\tau=-\infty}^{\infty} h(\tau)x(t-\tau)\,d\tau$$

Properties of convolution integral:

- Commutative property
- Distributive property
- Associative property

Commutative property:

The commutative property is a basic property of convolution in both continuous and discrete time cases, thus, both convolution integral for continuous time LTI systems and convolution sum for discrete time LTI systems are commutative. According to the property, for continuous time LTI system. The output is given by

$$y(t)=x(t)*h(t)=\int_{\tau=-\infty}^{\infty} x(\tau)h(t-\tau)\,d\tau$$

Or

$$y(t)=h(t)*x(t)=\int_{\tau=-\infty}^{\infty} h(\tau)x(t-\tau)\,d\tau$$

Thus, we can say that according to this property, the output of a continuous time LTI system having input x(t) and unit impulse h(t) is identical to the output of a continuous time LTI system having input h(t) and the unit impulse response x(t).

The distributive property:

The distributive property states that both convolution integral for continuous time LTI system and convolution sum for discrete time LTI system are distributive.

For continuous]ทีeL ทีsystem,ทีeดีstributivepropertyisexpressedas

Theoutput, hดก [ทีပ

Or

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Thus, the two continuous time LTI systems, with impulse responses h1(t) and h2(t), have identical inputs and outputs are added as

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The output ကပကฺทีที=ပด ပกပกีทีပที+∗

+ ပกပที∗hฝกပที

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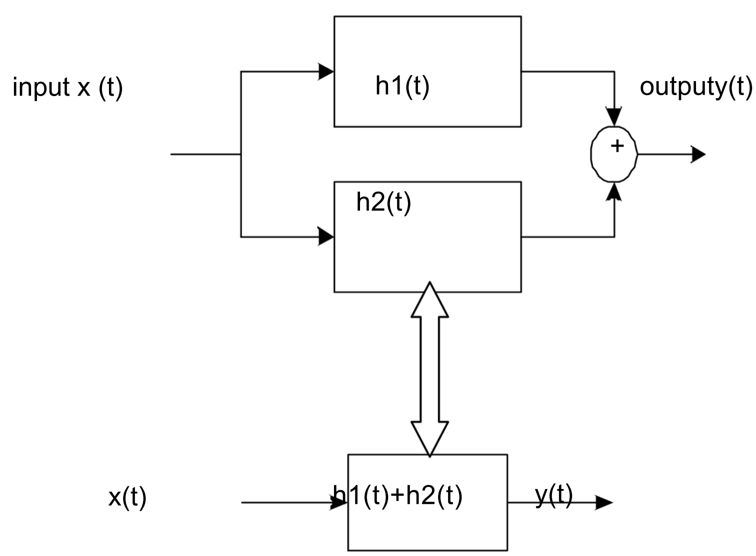

Fig3.1:Thedistributivepropertyofconvolutionintegral for a parallel interconnection of continuous time LTI system

Associative Propertyof LTIsystem:

Accordingtoassociativeproperty,bothconvolution integral for continuous time LTI systems and convolution sumfordiscretetimeLTIsystemsare associative.

For continuoustimeLTIsystem,accordingtoassociative property, The output

$$y(t)=[x(t)*h_1(t)]*h_2(t)$$

or

$$y(t)=x(t)*[h_1(t)*h_2(t)]$$

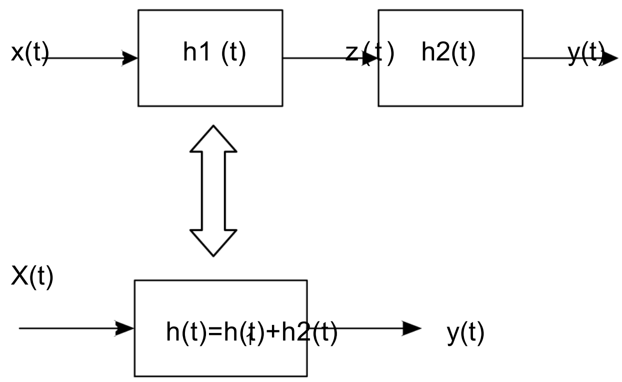

Fig 3.2:The associative propertyofconvolutionintegral for a cascade interconnection of continuous time LTI systems

Here we have y(t)=z(t) *h2(t). But z(t)=x(t)*h1(t). Therefore y(t)=[x(t)*h1(t)*h2(t)]

Linear constant coefficient differential equation:

The continuous time linear time invariant (LTI) systems are described by their linear constantcoefficient differential equations. For this, let us consider a first order differential equation as under $\frac{dy(t)}{dt} + a\,y(t) = x(t)$

Where x (t) and y(t) are the input and output of the continuous time LTI system. A is a constant value. The first order differential equation can be extended for higher order differential equations. A general Nth order linear constant coefficient differential equation can be given by

$$\sum_{k=0}^{N} a_k \frac{d^k y(t)}{dt^k} = \sum_{k=0}^{M} b_k \frac{d^k x(t)}{dt^k}$$

The complete solution of differential equation consists of thesum of particular solution yp(t) and homogenous solution yh(t).

The homogeneous solution of a differential equation is possibleby substituting

$$\sum_{k=0}^{N} a_k \frac{d^k y(t)}{dt^k} = 0$$

This solution to differential equation is also known as natural response of the system.

A particular case of differential equation is determined by putting N=0, we obtain

[illegible] x (t)

Transfer function: Transfer functions are commonly used in the analysis of systems such as single-input single-output filters,
typically within the fields of signal processing, communication theory, and control theory. The term is often used exclusively to refer to linear, time-invariant systems (LTI). Most real systems have non-linear input/output characteristics, but many systems, when operated within nominal parameters have behavior that is close enough to linear that LTI system theory is an acceptable representation of the input/output behavior.

The descriptions below are given in terms of a complex variable, $s = \sigma + j \cdot \omega$ which bears a brief explanation. In many applications, it is sufficient to define $\sigma = 0$ (and $s = j \cdot \omega$), which reduces the Laplace transforms with complex arguments to Fourier transforms with real argument ω. The applications where this is common are ones where there is interest only in the steady-state response of an LTI system, not the fleeting turn-on and turn-off behaviors or stability issues. That is usually the case for signal processing and communication theory.

Thus, for continuous-time input signal x(t) and output y(t0, the transfer function H(s) is the linear mapping of the Laplace transform of the input, X(s)=L{x(t)}, to the Laplace transform of the output Y(s)=L{y(t)}:

Y(s)=H(s)X(s)

[illegible]

Conditions required for transfer function:

(i) System should be in unloaded condition (initial conditions are zero)

(ii) The system should be linear time invariant.

Impulse Response:

In signal processing, the impulse response, of a dynamic system is its output when presented with a brief input signal, called an impulse. More generally, an impulse response refers to the reaction of any dynamic system in response to some external change. In both cases, the impulse response describes the reaction of the system as a function of time. In all these cases, the dynamic system and its impulse response may be actual physical objects, or may be mathematical systems of equations describing such objects. Since the impulse function contains all frequencies, the impulse response defines the response of a linear time-invariant system for all frequencies. The impulse can be modeled as a Dirac delta function for continuous-time systems, or as. The Dirac delta represents the limiting case of a pulse made very short in time while maintaining its area or integral. While this is impossible in any real system, it is a useful idealization. In Fourier theory, such an impulse comprises equal portions of all possible excitation frequencies, which makes it a convenient test probe. Any system in a large class known as linear, time-invariant (LTI) is completely characterized by its impulse response. That is, for any input, the output can be calculated in terms of the input and the impulse response. The impulse response of a linear transformation is the image of Dirac's delta function under the transformation, analogous to the fundamental solution of a partial differential operator. It is usually easier to analyze systems using transfer functions as opposed to impulse responses.Thetransferfunction is theLaplace transform of the impulse response. The Laplace transform of a system'soutputmaybedeterminedbythe multiplication of the transfer function with the input's Laplace transforminthecomplexplane, alsoknown as the frequency domain. An inverse Laplace transform of this result willyieldtheoutputinthe timedomain. To determine an output directly in the time domain requires the convolutionoftheinputwiththe impulseresponse. When the transfer function and the Laplace transform of the inputareknown,thisconvolutionmay be more complicated than the alternative of multiplying two functionsinthedomain.Itisobtained bytaking inverse Laplace transform of transfer function H(s).

$$h(t) = \mathcal{L}^{-1}\{H(s)\} = \mathcal{L}^{-1}\left\{\frac{Y(s)}{X(s)}\right\}$$

FrequencyResponse:

Frequencyresponse is thequantitative measure of the output spectrum of a system or device in response to a stimulus,and is used tocharacterize the dynamics of the system. It is a measure of magnitude and phase of the outputas a functionoffrequency, in comparison to the input.It is obtained from the transfer function by substitutings=jω in transferfunction.

$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)}$$

Systems respond differently to inputs of different frequencies. Some systems may amplify components of certain frequencies, and attenuate components of other frequencies. The way that the system output is related to the system input for different frequencies is called the ***frequency response*** of the system.

The frequency response is the relationship between the system input and output in the Fourier Domain.

In this system, X(jω) is the system input, Y(jω) is the system output, and H(jω) is the frequency response. We can define the relationship between these functions as:

$$Y(j\omega)=H(j\omega)X(j\omega)$$

$$\frac{Y(j\omega)}{X(j\omega)}=H(j\omega)$$

Since the frequency response is a complex function, we can convert it to polar notation in the complex plane. This will give us a magnitude and an angle.

Amplitude Response:

For each frequency, the magnitude represents the system's tendency to amplify or attenuate the input signal.

$$A(\omega) = |H(j\omega)|$$

Phase Response:

The phase represents the system's tendency to modify the phase ofthe input sinusoids.

$$\phi(\omega) = \angle H(j\omega)$$

The phase response, or its derivative the group delay, tells us howthe system delays the input signal as a function of frequency.