Ampere's circuital law

The line integral of magnetic field intensity H around a closed path is exactly equal to the direct current enclosed by that path.

The mathematical representation of Amperes circuit law is

$$\oint \bar{H}\, d\bar{l} = I$$

Proof:

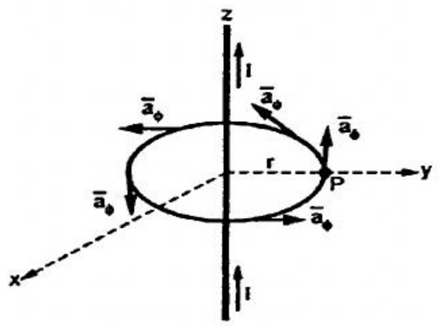

Fig 9: Long straight conductor carrying current I placed along Z- axis.

Consider a long straight conductor carrying direct current I placed along z-axis as shown in fig. Consider a closed circular path of radius 'r' which encloses the straight conductor carrying direct current I. The point P is at perpendicular distance 'r' from the conductor.

Consider dl at point P which is aΦ direction tangential in circular path at point P.

$$d\bar{l} = r\, d\emptyset\, \overline{a\emptyset}$$

While H obtained at point P from Biot-Savart law due to infinitely long conductor,

$$\bar{H} = \frac{I\, a\emptyset}{2\pi r}$$

$$\bar{H}.\overline{dl} = \frac{I}{2\pi r}\overline{a\emptyset}\, r\, d\emptyset\, \overline{a\emptyset} \;=\; \frac{I}{2\pi}\, d\emptyset$$

Integrating $\bar{H}\,.\overline{dl}$ over the entire closed path,

$$\oint \bar{H}\,.\, \overline{dl} = \int_0^{2\pi} \frac{I}{2\pi}\, d\emptyset = \frac{I}{2\pi}[\emptyset]_0^{2\pi} = \frac{I}{2\pi} 2\pi$$

$\oint \bar{H}\,.\overline{dl} = I$ I → current carried by a conductor.

Applications of ampere's circuital law:

$\overline{H}$ due to infinitely long straight conductor

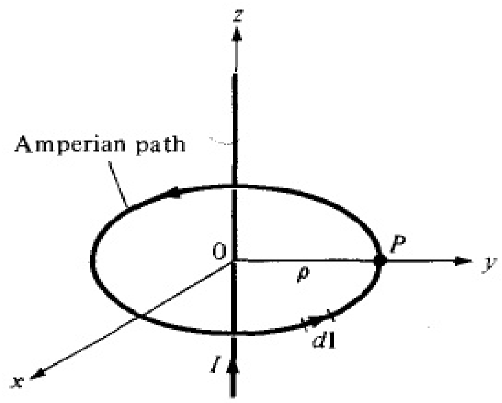

Fig 10: Ampere's law applied to infinitely long filamentary line current

Consider an infinitely long straight conductor placed along z-axis carrying direct current I. The path is Amperian closed path. Consider a point P on the closed path at which $\overline{H}$ is to be obtained. The radius of the path 'r' and hence 'p' is at a perpendicular distance 'r' from the conductor. The magnitude of $\overline{H}$ depends on r and the direction is always tangential to the closed path. ie., aΦ. So H has only component in aΦ direction

$$\overline{H} = H\emptyset\,\overline{a\emptyset}\,\&\quad d\overline{l} = r\,d\emptyset\,\overline{a\emptyset}$$

$$\overline{H}.d\overline{l} = H\emptyset\,.\overline{a\emptyset}\ .r\,d\emptyset\,a\emptyset \quad = \quad H\emptyset\,r\,d\emptyset$$

According to Amperes circuital law,

$$\oint H.dl = I$$

$$\int_0^{2\pi} H\emptyset\,r\,d\emptyset = I$$

$$H\emptyset r \int_0^{2\pi} d\emptyset = I$$

$$H\emptyset\,2\pi r = I\,,\qquad H\emptyset = \frac{I}{2\pi r}$$

$$\overline{H} = H\emptyset\,a\emptyset = \frac{I}{2\pi r}a\emptyset \quad A/m.$$

$\bar{H}$ due to co-axial cable

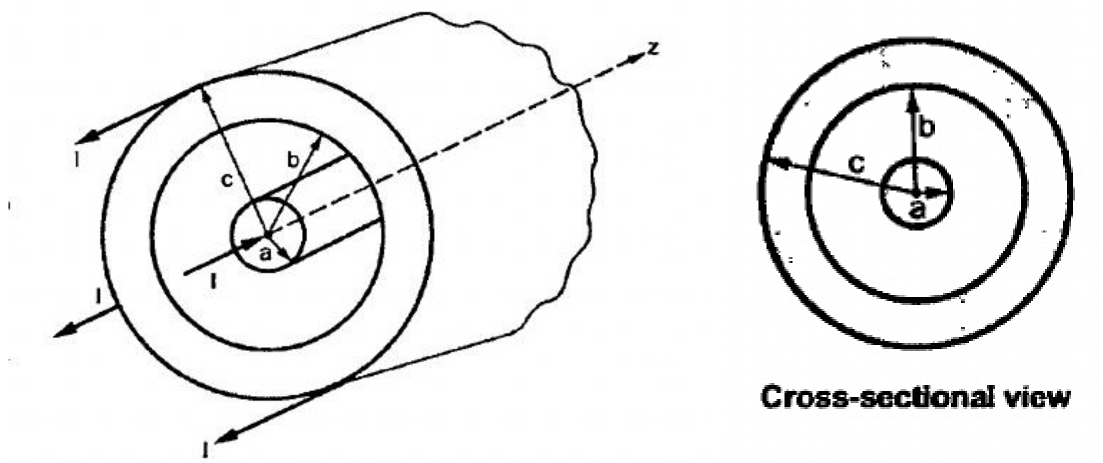

Fig 11: Co-axial cable

- Consider a co-axial cable its inner conductor is solid with radius' a' carrying I current outer conductor is in the from of concentric cylinder whose inner radius b & outer radius c.
- The cable is placed along Z - axis
- The calculation contains various regions

Region 1

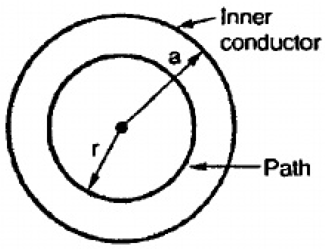

Fig 11 a: Amperian path radius r < a (inner conductor radius)

- Amperian path is With in the inner conductor r< a
- consider a closed path having r<a . Hence it encloses only part of the conductor.
- The area of the cross section enclosed is πr^2 m^2.
- The total current flowing is I trough the area πa^2. Hence the current enclosed by the closed path is

$$I' = \frac{\pi r^2}{\pi a^2}.I = \frac{r^2}{a^2}.I$$

$H = H\varphi\ \overline{a\varphi}$

dL in the $\overline{a\varphi}$ direction which is r dφ

$\overline{dl} = rd\varphi\ \overline{a\varphi}$

$\overline{H}.dl = \overline{H\varphi}\ a\varphi\ .\ r\ d\varphi\ a\varphi\ = \overline{H\varphi}\ r\ .\ d\varphi$

$\oint\ \overline{H}.\ dl\ =\ I'$

$\oint\ \overline{H\varphi}\ r\ d\varphi\ =\ \frac{r^2}{a^2} I$

$\int_0^{2\pi} \overline{H\varphi}\, r\, d\varphi = \frac{r^2}{a^2} .I$

$\overline{H\varphi}\ = \frac{r^2}{2\pi r\ a^2} I$

$\overline{H\varphi} = \frac{rI}{2\pi\ a^2}\ \overline{a\varphi}$ A/M

Region 2:

- Within a < r < b consider a circular path which encloses the inner conductor carrying direct current I. This is the case of infinitely long conductor along Z axis
- $\overline{H}\ = \frac{I}{2\pi r} \overline{a\varphi}$ A/M

$\oint \overline{H} dl = I,\ \ \oint_0^{2\pi} \overline{H}\, r d\varphi = I,\ \ \overline{H} = \frac{I}{2\pi r}$

Region 3:

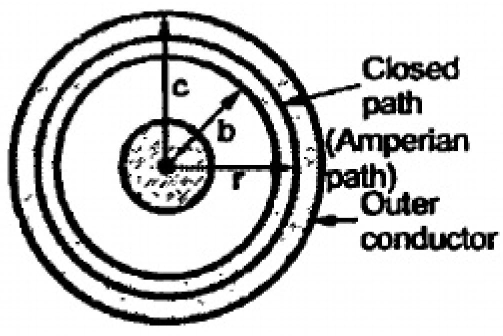

Fig 11b : Amperian path radius c < r <b

Amperian path is With in outer conductor b < r < c

- Consider a closed path as shown in fig. The current I flowing through the cross section $\pi(c^2 - b^2)$ while the closed path encloses the cross section $\pi(r^2 - b^2)$

- Hence the current enclosed by the closed path of outer conductor is

$$I' = \frac{\pi(r^2-b^2)}{\pi(c^2-b^2)}(-I) = -\frac{r^2-b^2}{c^2-b^2} I$$

$I'' = I \rightarrow$ current in inner conductor enclosed.

Total current is

$$I = I' + I'' = -\frac{r^2-b^2}{c^2-b^2} I + I$$

$$= I\,\frac{-r^2+b^2+c^2-b^2}{c^2-b^2}$$

$$= I\,\frac{c^2-r^2}{c^2-b^2}$$

$$\oint \bar{H}.dL = I_{enc}$$

$\bar{H} = \bar{H}\varphi\, a\varphi$ & $dl = r\, d\varphi\, \overline{a\varphi}$

$\bar{H}.\,dL = \overline{H\varphi}.\, a\varphi\, .\, r\, d\varphi\, a\varphi$

$= \overline{H\varphi}\, r.\, d\varphi$

$$\oint_0^{2\pi} \overline{H\varphi}\, r\, d\varphi = I_{enc}$$

$$\overline{H\varphi} r\,[2\pi] = I\left[\frac{c^2-r^2}{c^2-b^2}\right]$$

$$\overline{H\varphi} = \frac{I}{2\pi r}\left[\frac{c^2-r^2}{c^2-b^2}\right]$$

$$\bar{H} = \overline{H\varphi}\, a\varphi = \frac{I}{2\pi r}\left[\frac{c^2-r^2}{c^2-b^2}\right] a\varphi \text{ A/M}$$

Region 4:

Outside of cable $r > c$

- Both current +I & -I flowing through the region

Thus total current enclosed is

$$I_{enc} = +I - I = 0A$$

$$\oint H.\overline{dl} = 0$$

$$H = 0 \text{ A/M}$$

The magnetic flux does not exist outside the cable