

Viscoelasticity

Viscoelasticity is a material property that combines both viscous and elastic characteristics. Many materials have such viscoelastic properties. Especially materials that consist of large molecules show viscoelastic properties. Polymers are viscoelastic because their macromolecules can make temporary entanglements with neighbouring molecules which causes elastic properties. After some time these entanglements will disappear again and the macromolecules will flow into other positions where new entanglements will be made (viscous properties).

A viscoelastic material will show elastic properties on short time scales and viscous properties on long time scales. These materials exhibit behavior that depends on the time and rate of applied forces, allowing them to both store and dissipate energy.

Viscoelasticity has been studied since the nineteenth century by researchers such as James Clerk Maxwell, Ludwig Boltzmann, and Lord Kelvin.

Several models are available for the mathematical description of the viscoelastic properties of a substance:

- Constitutive models of linear viscoelasticity assume a linear relationship between stress and strain. These models are valid for relatively small deformations only.
- Constitutive models of non-linear viscoelasticity are based on a more realistic non-linear relationship between stress and strain. These models are valid for relatively large deformations.

The viscoelastic properties of polymers are highly temperature dependent. From low to high temperature the material can be in the glass phase, rubber phase or the melt phase. These phases have a very strong effect on the mechanical and viscous properties of the polymers.

Typical viscoelastic properties are:

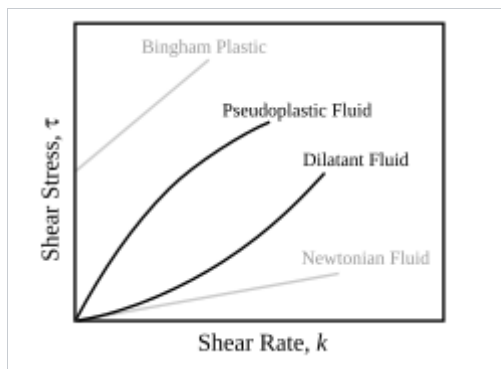
- A time dependant stress in the polymer under constant deformation (strain).
- A time dependant strain in the polymer under constant stress.
- A time and temperature dependant stiffness of the polymer.
- Viscous energy loss during deformation of the polymer in the glass or rubber phase (hysteresis).
- A strain rate dependant viscosity of the molten polymer.
- An ongoing deformation of a polymer in the glass phase at constant load (creep).

The viscoelasticity properties are measured with various techniques, such as tensile testing, dynamic mechanical analysis, shear rheometry and extensional rheometry.

Background

In the nineteenth century, physicists such as James Clerk Maxwell, Ludwig Boltzmann, and Lord Kelvin researched and experimented with creep and recovery of glasses, metals, and rubbers. Viscoelasticity was further examined in the late twentieth century when synthetic polymers were engineered and used in a variety of applications.

Viscoelasticity calculations depend heavily on the viscosity variable, η . The inverse of η is also known as fluidity, ϕ . The value of either can be derived as a function of temperature or as a given value (i.e. for a dashpot).



Different types of responses (σ) to a change in strain rate ($d\varepsilon/dt$)

Depending on the change of strain rate versus stress inside a material, the viscosity can be categorized as having a linear, non-linear, or plastic response:

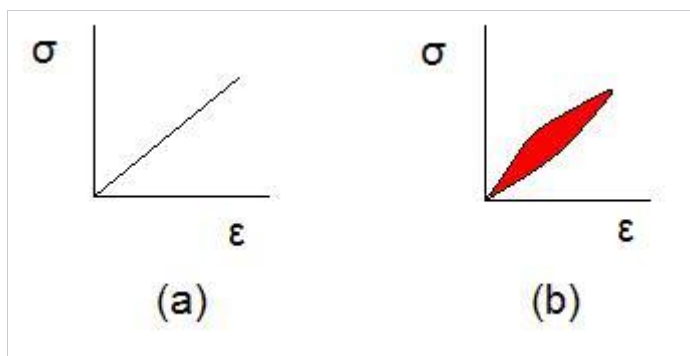
- When a material exhibits a linear response it is categorized as a Newtonian material. In this case the stress is linearly proportional to the strain rate.
- If the material exhibits a non-linear response to the strain rate, it is categorized as non-Newtonian fluid.
- There is also an interesting case where the viscosity decreases as the shear/strain rate remains constant. A material which exhibits this type of behavior is known as thixotropic.
- In addition, when the stress is independent of this strain rate, the material exhibits plastic deformation. Many viscoelastic materials exhibit rubber like behavior explained by the thermodynamic theory of polymer elasticity.

Some examples of viscoelastic materials are amorphous polymers, semicrystalline polymers, biopolymers, metals at very high temperatures, and bitumen materials. Cracking occurs when the strain is applied quickly and outside of the elastic limit. Ligaments and tendons in the human body are viscoelastic, so the extent of the potential damage to them depends on both the rate of the change of their length and the force applied.

A viscoelastic material has the following properties:

- hysteresis is seen in the stress–strain curve
- stress relaxation occurs: step constant strain causes decreasing stress
- creep occurs: step constant stress causes increasing strain
- its stiffness depends on the strain rate $\dot{\varepsilon}$ or the stress rate $\dot{\sigma}$

Elastic versus viscoelastic behavior



Stress–strain curves for a purely elastic material (a) and a viscoelastic material (b). The red area is a hysteresis loop and shows the amount of energy lost (as heat) in a loading and unloading cycle. It is equal to $\oint \sigma d\epsilon$, where σ is stress and ϵ is strain.

Unlike purely elastic substances, a viscoelastic substance has an elastic component and a viscous component. The viscosity of a viscoelastic substance gives the substance a strain rate dependence on time. Purely elastic materials do not dissipate energy (heat) when a load is applied, then removed. However, a viscoelastic substance dissipates energy when a load is applied, then removed. Hysteresis is observed in the stress–strain curve, with the area of the loop being equal to the energy lost during the loading cycle. Since viscosity is the resistance to thermally activated plastic deformation, a viscous material will lose energy through a loading cycle. Plastic deformation results in lost energy, which is uncharacteristic of a purely elastic material's reaction to a loading cycle.

Specifically, viscoelasticity is a molecular rearrangement. When a stress is applied to a viscoelastic material such as a polymer, parts of the long polymer chain change positions. This movement or rearrangement is called creep. Polymers remain a solid material even when these parts of their chains are rearranging to accommodate the stress, and as this occurs, it creates a back stress in the material. When the back stress is the same magnitude as the applied stress, the material no longer creeps. When the original stress is taken away, the accumulated back stresses will cause the polymer to return to its original form. The material creeps, which gives the prefix visco-, and the material fully recovers, which gives the suffix -elasticity.

Linear viscoelasticity and nonlinear viscoelasticity

Linear viscoelasticity is when the function is separable in both creep response and load. All linear viscoelastic models can be represented by a Volterra equation connecting stress and strain:

$$\varepsilon(t) = \frac{\sigma(t)}{E_{\text{inst,creep}}} + \int_0^t K(t-t')\dot{\sigma}(t')dt'$$

or
$$\sigma(t) = E_{\text{inst,relax}}\varepsilon(t) + \int_0^t F(t-t')\dot{\varepsilon}(t')dt'$$
 where

- t is time
- $\sigma(t)$ is stress
- $\varepsilon(t)$ is strain
- $E_{\text{inst,creep}}$ and $E_{\text{inst,relax}}$ are instantaneous elastic moduli for creep and relaxation
- $K(t)$ is the creep function
- $F(t)$ is the relaxation function

Linear viscoelasticity is usually applicable only for small deformations.