24AG401 THEORY OF MACHINES

UNIT V NOTES

Four masses A, B, C and D as shown below are to be completely balanced

	A	B	C	D
mass	-	30	50	40
Radius	180	240	120	150

The planes containing masses B and C are 300 mm apart. The angle between planes containing B and C is 90°. B and C make angles of 210° and 120° respectively with D in the same sense. Find :

1. The magnitude and the angular position of mass A ; and

2. The position of planes A and D.

Solution. Given : r_A = 180 mm = 0.18 m ; m_B = 30 kg ; r_B = 240 mm = 0.24 m ; m_C = 50 kg ; r_C = 120 mm = 0.12 m ; m_D = 40 kg ; r_D = 150 mm = 0.15 m ; $\angle BOC = 90°$; $\angle BOD = 210°$; $\angle COD = 120°$

1. *The magnitude and the angular position of mass A*

Let m_A = Magnitude of Mass *A*,

x = Distance between the planes *B* and *D*, and

y = Distance between the planes *A* and *B*.

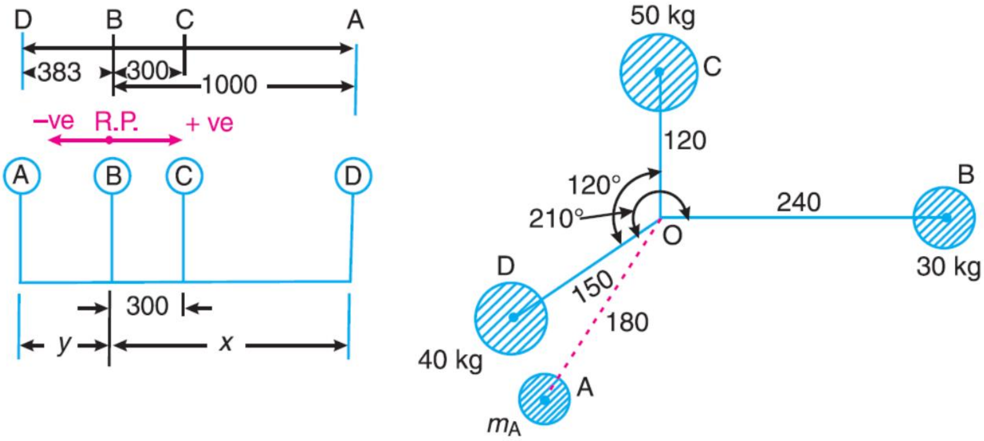
Plane (1)	*Mass (m) kg* (2)	*Radius (r) m* (3)	*Cent.force* $\div \omega^2$ *(m.r) kg-m* (4)	*Distance from plane B (l) m* (5)	*Couple* $\div \omega^2$ *(m.r.l) kg-m*^{2} (6)
A	m_A	0.18	0.08 m_A	$-y$	$-0.18\, m_A.y$
B (R.P)	30	0.24	7.2	0	0
C	50	0.12	6	0.3	1.8
D	40	0.15	6	x	$6x$

The magnitude and angular position of mass *A* may be determined by drawing the force polygon from the data given in Table, as shown in Fig. (*c*), to some suitable scale. Since the masses are to be completely balanced, therefore the force polygon must be a closed figure. The closing side (*i.e.* vector *do*) is proportional to 0.18 *m* A.

By measurement,

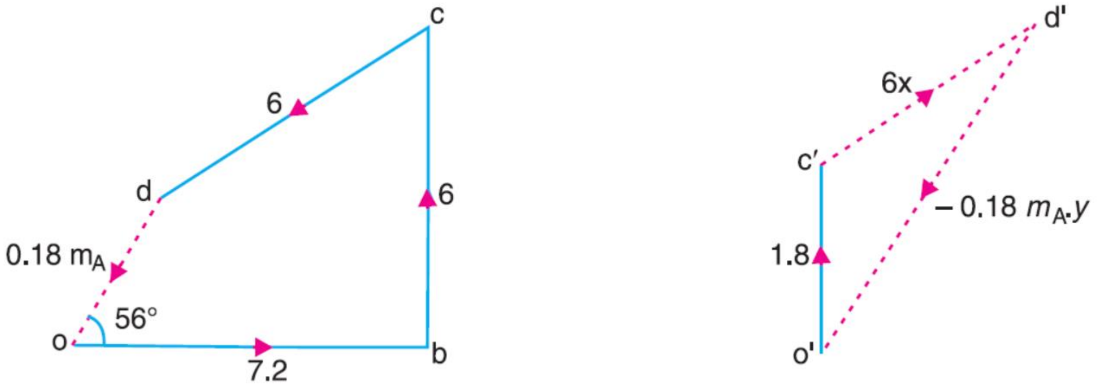
$0.18\, m_A$ = Vector *do* = 3.6 kg-m or m_A = 20 kg

In order to find the angular position of mass *A*, draw *OA* in Fig. (*b*) parallel to vector *do*. By measurement, we find that the angular position of mass *A* from mass *B* in the anticlockwise direction is $\angle AOB$ = **236°**



All dimensions in mm.

(*a*) Position of planes. (*b*) Angular position of masses.



(*c*) Force polygon. (*d*) Couple polygon.

2. Position of planes A and D

The position of planes *A* and *D* may be obtained by drawing the couple polygon, as shown in Fig. (*d*), from the data given in Table (column 6). The couple polygon is drawn as discussed below :

1. Draw vector $o'c'$ parallel to *OC* and equal to 1.8 kg-m^2, to some suitable scale.

2. From points c' and o', draw lines parallel to *OD* and *OA* respectively, such that they intersect at point d'. By measurement, we find that

$$6x = \text{vector } c'd' = 2.3 \text{ kg-m}^2 \text{ or } x = 0.383 \text{ m}$$

We see from the couple polygon that the direction of vector $c'd'$ is opposite to the direction of mass *D*. Therefore the plane of mass *D* is 0.383 m or 383 mm towards left of plane *B* and not towards right of plane *B* as already assumed. **Ans.**

Again by measurement from couple polygon,

$$-0.18\, m_A.y = \text{vector } o'd' = 3.6 \text{ kg-m}^2$$

$$-0.18 \times 20\, y = 3.6 \quad \text{or} \quad y = -1 \text{ m}$$

The negative sign indicates that the plane *A* is not towards left of *B* as assumed but it is 1 m or 1000 mm towards right of plane *B*. **Ans.**

A shaft is supported in bearings 1.8 m apart and projects 0.45 m beyond bearings at each end. The shaft carries three pulleys one at each end and one at the middle of its length. The mass of end pulleys is 48 kg and 20 kg and their centre of gravity are 15 mm and 12.5 mm respectively from the shaft axis. The centre pulley has a mass of 56 kg and its centre of gravity is 15 mm from the shaft axis. If the pulleys are arranged so as to give static balance, determine :

1. relative angular positions of the pulleys, and 2. dynamic forces produced on the bearings when the shaft rotates at 300 r.p.m.

Solution. Given : m_A = 48 kg ; m_C = 20 kg ; r_A = 15 mm = 0.015 m ; r_C = 12.5 mm = 0.0125 m ; m_B = 56 kg ; r_B = 15 mm = 0.015 m ; *N* = 300 *r.p.m.* or $\omega = 2\pi \times 300/60$ = 31.42 rad/s

1. Relative angular position of the pulleys

The position of the shaft and pulleys is shown in Fig. (*a*).

Let m_L and m_M = Mass at the bearings *L* and *M*, and

r_L and r_M = Radius of rotation of the masses at *L* and *M* respectively.

Plane (1)	*Mass (m) kg* (2)	*Radius (r) m* (3)	*Cent. force* $\div \omega^2$ *(m.r) kg-m* (4)	*Distance from plane L(l)m* (5)	*Couple* $\div \omega^2$ *(m.r.l) kg-m*^{2} (6)
A	48	0.015	0.72	– 0.45	– 0.324
L(R.P)	m_L	r_L	$m_L.r_L$	0	0
B	56	0.015	0.84	0.9	0.756
M	m_M	r_M	$m_M.r_M$	1.8	1.8 $m_M.r_M$
C	20	0.0125	0.25	2.25	0.5625

First of all, draw the force polygon to some suitable scale, as shown in Fig. (*c*), from the data given in Table (column 4). It is assumed that the mass of pulley *B* acts in vertical direction. We know that for the static balance of the pulleys, the centre of gravity of the system must lie on the axis of rotation. Therefore a force polygon must be a closed figure. Now in Fig. (*b*), draw *OA* parallel to vector *bc* and *OC* parallel to vector *co*. By measurement, we find that

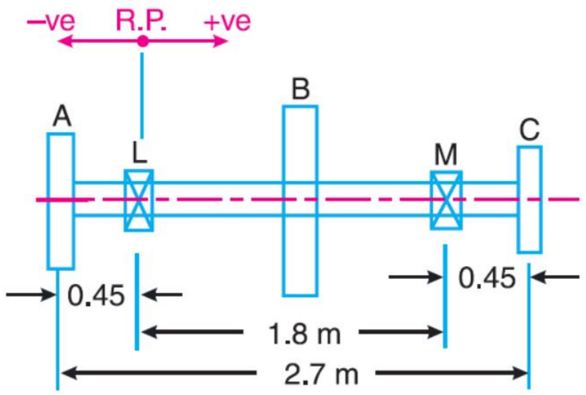
Angle between pulleys *B* and *A* = 161° **Ans.**

Angle between pulleys *A* and *C* = 76° **Ans.**

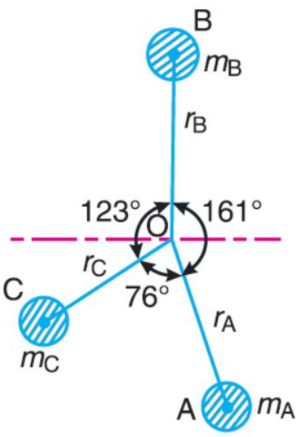
and Angle between pulleys *C* and *B* = 123° **Ans.**

2. *Dynamic forces at the two bearings*

In order to find the dynamic forces (or reactions) at the two bearings *L* and *M*, let us first calculate the values of $m_L.r_L$ and $m_M.r_M$ as discussed below :

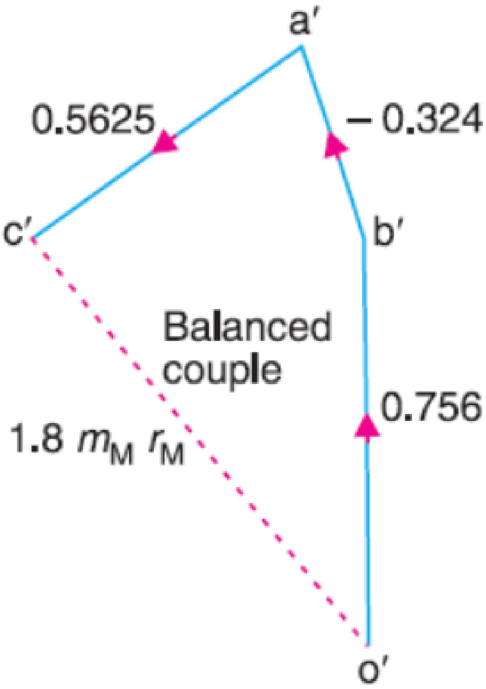


(*a*) Position of shaft and pulleys.

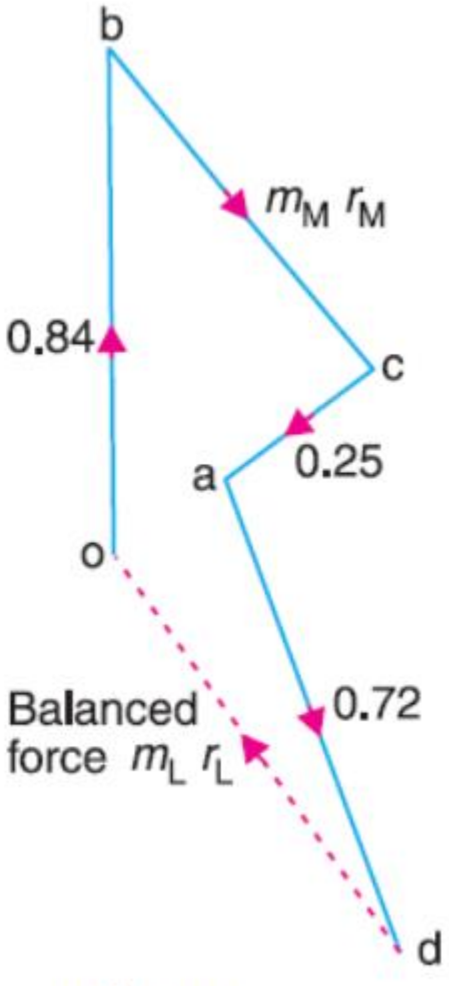


(*b*) Angular position of pulleys.

(*c*) Force polygon.



(*d*) Couple polygon.



(e) Force polygon.

1. Draw the couple polygon to some suitable scale, as shown in Fig. (*d*), from the data given in Table (column 6). The closing side of the polygon (vector $c'o'$) represents the balanced couple and is proportional to 1.8 $m_M.r_M$. By measurement, we find that

$$1.8\ m_M.r_M = \text{vector } c'o' = 0.97 \text{ kg-m}^2 \quad \text{or} \quad m_M.r_M = 0.54 \text{ kg-m}$$

∴ Dynamic force at the bearing M

$$= m_M.r_M.\omega^2 = 0.54\ (31.42)^2 = 533 \text{ N} \textbf{ Ans.}$$

2. Now draw the force polygon, as shown in Fig. (*e*), from the data given in Table (column 4) and taking $m_M.r_M = 0.54$ kg-m. The closing side of the polygon (vector *do*) represents the balanced force and is proportional to $m_L.r_L$. By measurement, we find that

$$m_L.r_L = 0.54 \text{ kg-m}$$

$\therefore$ Dynamic force at the bearing L

$$= m_L.r_L.\omega^2 = 0.54\ (31.42)^2 = 533 \text{ N } \textbf{Ans.}$$