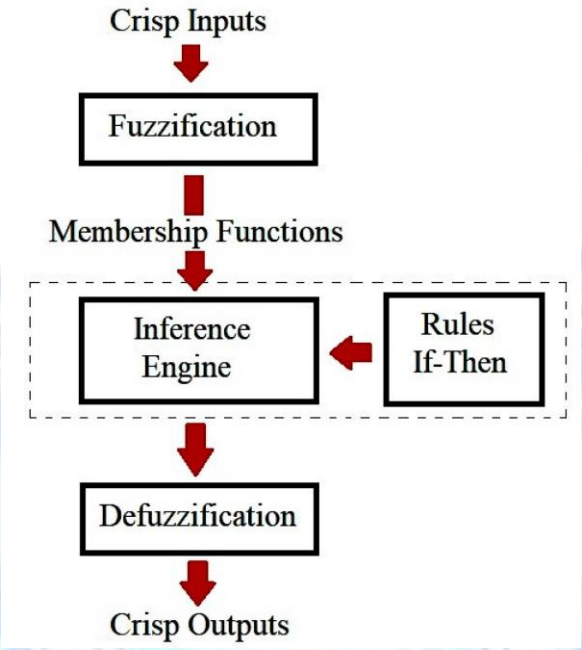
2.2 Fuzzification

Fuzzification is the process of converting a **crisp input value** into one or more **fuzzy membership values** using membership functions.

The input is usually a precise numerical value, while the output represents linguistic concepts such as **Low, Medium, High**.



Example – Temperature Control

Suppose the input temperature is:

$T = 30^{\circ}C$

Fuzzy Set	Membership Value
Low	0.0
Medium	0.4
High	0.8

The temperature can belong to different fuzzy sets:This means **30°C** is:

- 0% Low
- 40% Medium
- 80% High

The same input can therefore partially belong to **more than one fuzzy set**.

Steps in Fuzzification:

1. **Input Values**:Fuzzification starts with real, crisp inputs (e.g., temperature, speed, height, etc.) that are measured from the environment or system.

2. **Membership Functions**:A membership function (MF) defines how each input value corresponds to a fuzzy set. A fuzzy set is a collection of elements, each of which has a degree of membership between 0 and 1.

- For example, a temperature input of 35°C might belong to the fuzzy sets "cold," "warm," and "hot" with varying degrees of membership (e.g., 0.2 for cold, 0.5 for warm, and 0.8 for hot).

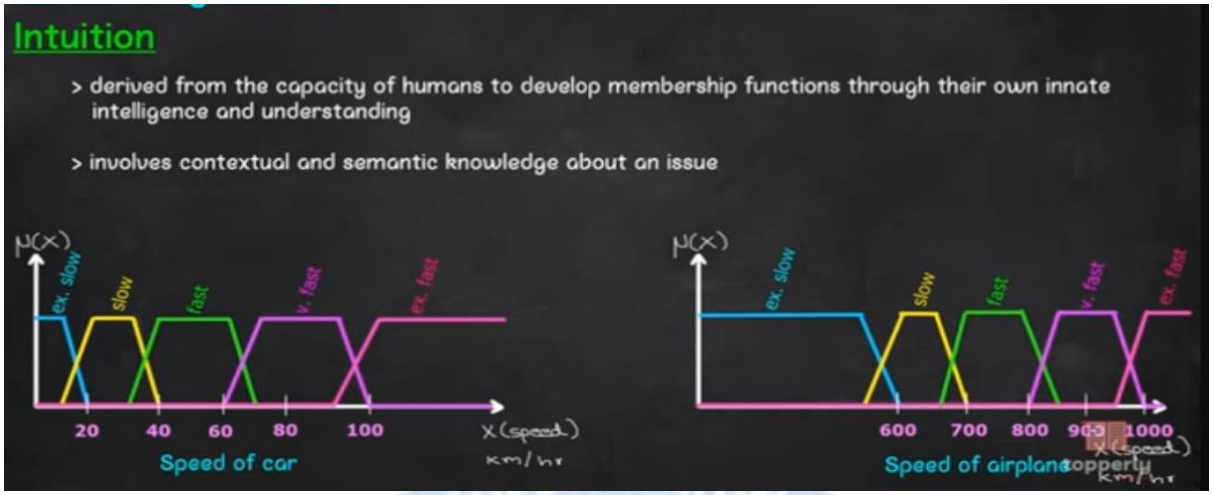
3.**Fuzzy Sets**:Fuzzy sets represent categories or linguistic variables. These sets could represent concepts like "low," "medium," "high," or more specific terms like "very hot," "cold," etc.

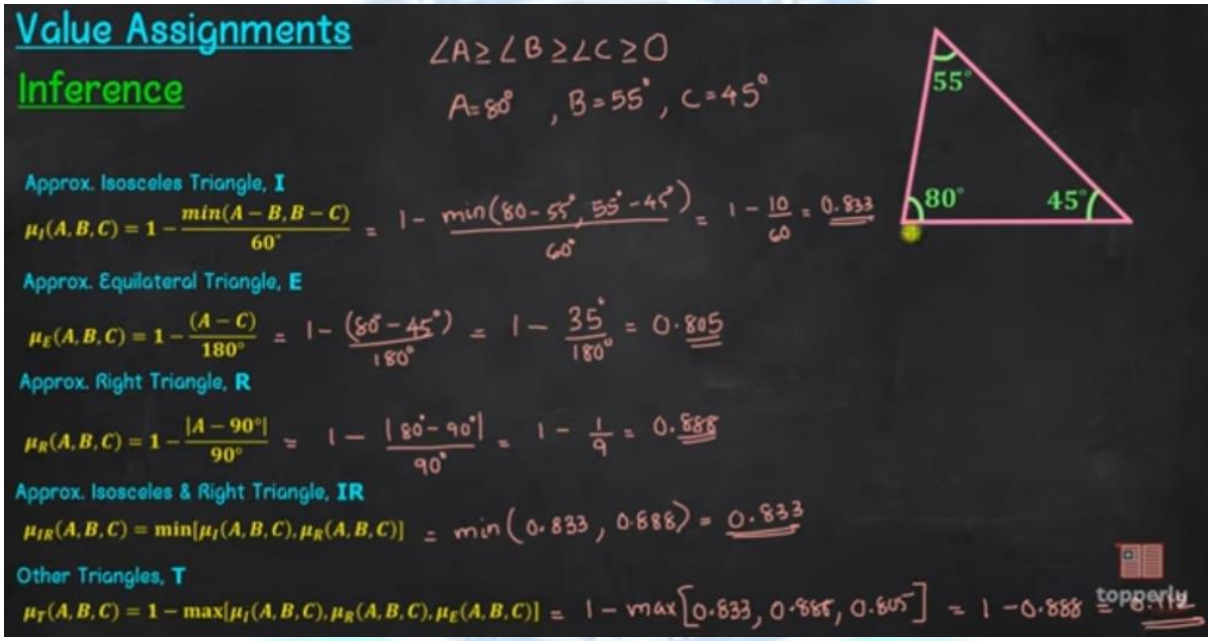
- The fuzzification process calculates the degree to which each crisp input belongs to each fuzzy set.

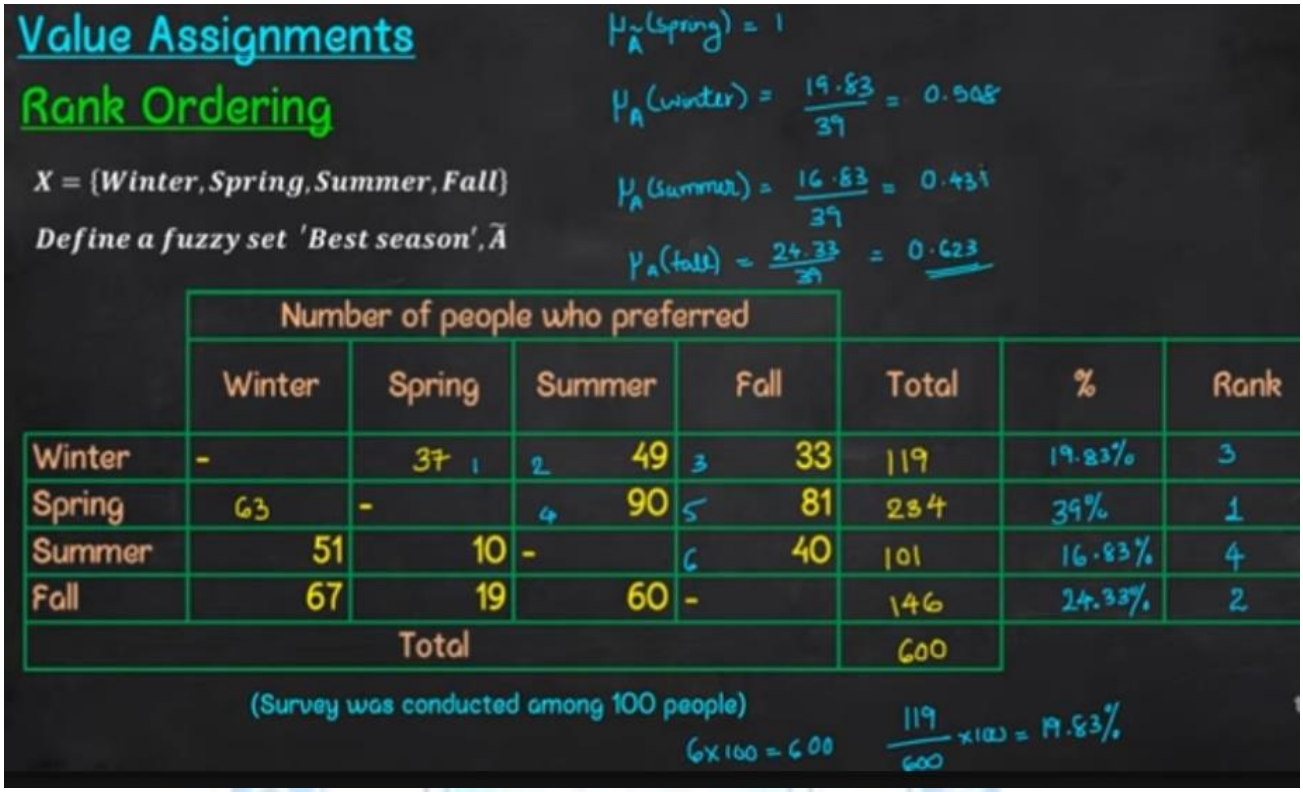
METHODS OF MEMBERSHIP VALUE ASSIGNMENTS

- **There are several ways to assign membership values to fuzzy variable in comparison with the probability density functions to random variables. Methods are as follow :**

1) **Intuition**
2) **Inference**
3) **Rank Ordering**
4) **Angular fuzzy sets**
5) **Neural Networks**
6) **Genetic Algorithm**
7) **Inductive Reasoning**


Intuition
> derived from the capacity of humans to develop membership functions through their own innate intelligence and understanding
> involves contextual and semantic knowledge about an issue
μ(x)
ex. slow
slow
fast
v. fast
ex. fast
20
40
60
80
100
X(speed)
km/hr
Speed of car
μ(x)
ex. slow
slow
fast
v. fast
ex. fast
600
700
800
900
1000
X(speed)
km/hr
Speed of airplane
topperly


Value Assignments
Inference
∠A ≥ ∠B ≥ ∠C ≥ 0
A = 80°, B = 55°, C = 45°
55°
80°
45°
Approx. Isosceles Triangle, I
$\mu_I(A,B,C) = 1 - \frac{min(A-B, B-C)}{60^\circ} = 1 - \frac{min(80-55^\circ, 55^\circ-45^\circ)}{60^\circ} = 1 - \frac{10}{60} = 0.833$
Approx. Equilateral Triangle, E
$\mu_E(A,B,C) = 1 - \frac{(A-C)}{180^\circ} = 1 - \frac{(80^\circ-45^\circ)}{180^\circ} = 1 - \frac{35^\circ}{180^\circ} = 0.805$
Approx. Right Triangle, R
$\mu_R(A,B,C) = 1 - \frac{|A-90^\circ|}{90^\circ} = 1 - \frac{|80^\circ-90^\circ|}{90^\circ} = 1 - \frac{1}{9} = 0.888$
Approx. Isosceles & Right Triangle, IR
$\mu_{IR}(A,B,C) = \min[\mu_I(A,B,C), \mu_R(A,B,C)] = \min(0.833, 0.888) = 0.833$
Other Triangles, T
$\mu_T(A,B,C) = 1 - \max[\mu_I(A,B,C), \mu_R(A,B,C), \mu_E(A,B,C)] = 1 - \max[0.833, 0.888, 0.805] = 1 - 0.888 = 0.112$
topperly



Real-World Example – Washing Machine

For a washing machine, suppose:

Dirt level = 70

The fuzzification process may produce:

- Low dirt = **0.1**
- Medium dirt = **0.5**
- High dirt = **0.9**

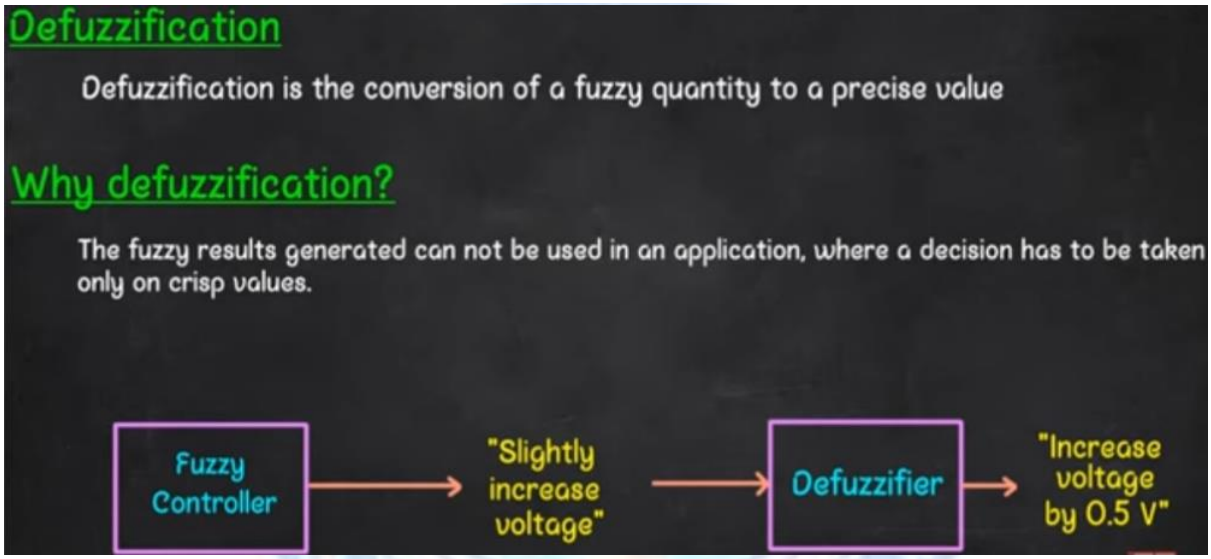
These fuzzy values are then given to the **fuzzy rule base**.

For example:

> **IF** dirt is High **THEN** washing time is Long.

DEFUZZIFICATION

- **The defuzzification process has the capability to reduce a fuzzy set into a crisp single-valued quantity or into a crisp set; to convert fuzzy matrix into a crisp matrix; or to convert fuzzy number into crisp number.**
- **Mathematically also known as "rounding it off".**
- **Various methods are available in literature, although no method has proved to b always advantageous than the others.**
- **Selection of the method depends on the experience of the designer.**



Example – Washing Machine

Suppose the fuzzy system determines the washing time as:

- Short = 0.2
- Medium = 0.6
- Long = 0.8

After combining these fuzzy outputs, the defuzzification process may produce:

Washing Time=35 minutes

Thus, the fuzzy result is converted into an actual value that the washing machine can use.

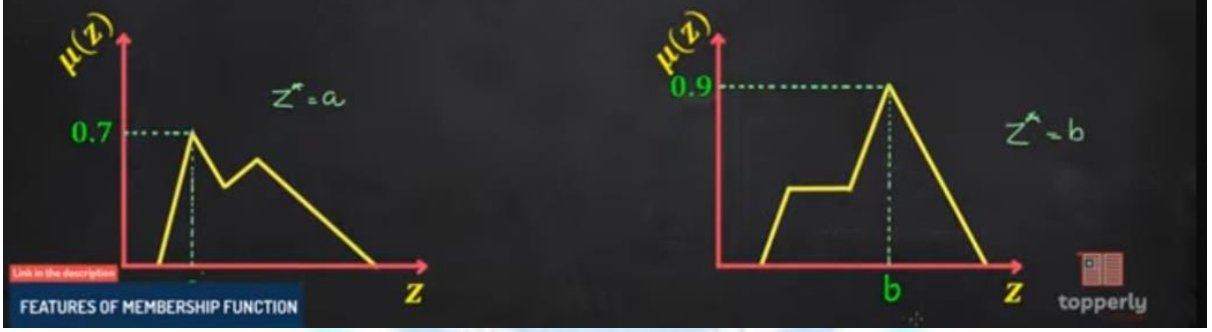
Types of Defuzzification Method

- > Max Membership Principle
- > Centroid Method
- > Weighted Average
- > Mean-Max Membership
- > Centre of Sums
- > Centre of Largest Area
- > First(or last) of Maxima

Max Membership Principle

As per this method, the defuzzified value or scalar value z* is chosen for a fuzzy set $\tilde{A}$ such that,

$$z^* | \mu_{\tilde{A}}(z^*) \geq \mu_{\tilde{A}}(z) \quad \forall z \in \tilde{A}$$



Weighted Average Method

Fuzzy set $\tilde{A}$(which is the output of the fuzzy controller/fuzzy inference engine) is the union of M fuzzy sets.

This method is restricted to symmetrical output membership functions

$$z^* = \frac{\sum_{i=1}^{M} \mu_{\tilde{A}}(\bar{z}_i) \cdot \bar{z}_i}{\sum_{i=1}^{M} \mu_{\tilde{A}}(\bar{z}_i)}$$

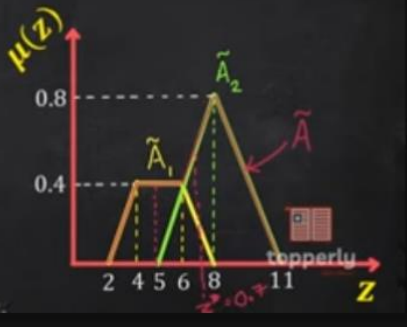
$\sum$ respresents algebraic sum

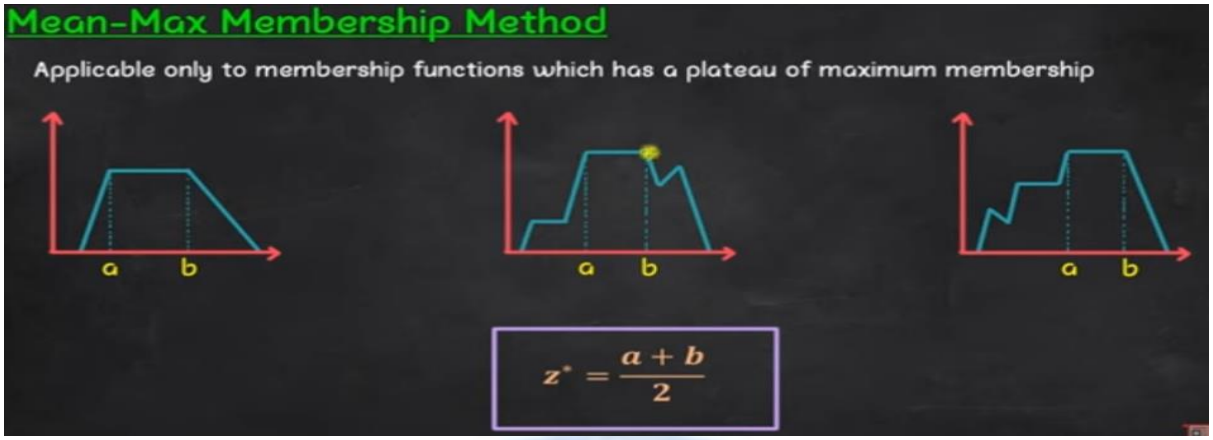
$\bar{z}_i$ respresents centroid of each symmetric function

$\bar{z}_{A_1} = 5$ and $\mu_{\tilde{A}}(5) = 0.4$

$\bar{z}_{A_2} = 8$ & $\mu_{\tilde{A}}(8) = 0.8$

$$z^* = \frac{(0.4 \times 5) + (0.8 \times 8)}{0.4 + 0.8} = \frac{7}{}$$





S.No.	Comparison	Fuzzification	Defuzzification
1.	Basic	Precise data is converted into imprecise data.	Imprecise data is converted into precise data.
2.	Definition	Fuzzification is the method of converting a crisp quantity into a fuzzy quantity.	Defuzzification is the inverse process of fuzzification where the mapping is done to convert the fuzzy results into crisp results.
3.	Example	Like, Voltmeter	Like, Stepper motor and D/A converter

4.	Methods	Intuition, inference, rank ordering, angular fuzzy sets, neural network, etcetera.	Maximum membership principle, centroid method, weighted average method, center of sums, etcetera.
5.	Complexity	It is quite simple.	It is quite complicated.
6.	Use	It can use IF-THEN rules for fuzzifying the crisp value.	It uses the center of gravity methods to find the centroid of the sets.