MAGNETIC BOUNDARY CONDITIONS

The conditions of the magnetic field existing at the boundary of the two media when the magnetic field passes from one medium to other are called boundary conditions for magnetic field.

$\overline{B}$ and $\overline{H}$ at the boundary are resolved into two components.

a.) Tangential to boundary and
b.) Normal (perpendicular) to boundary.

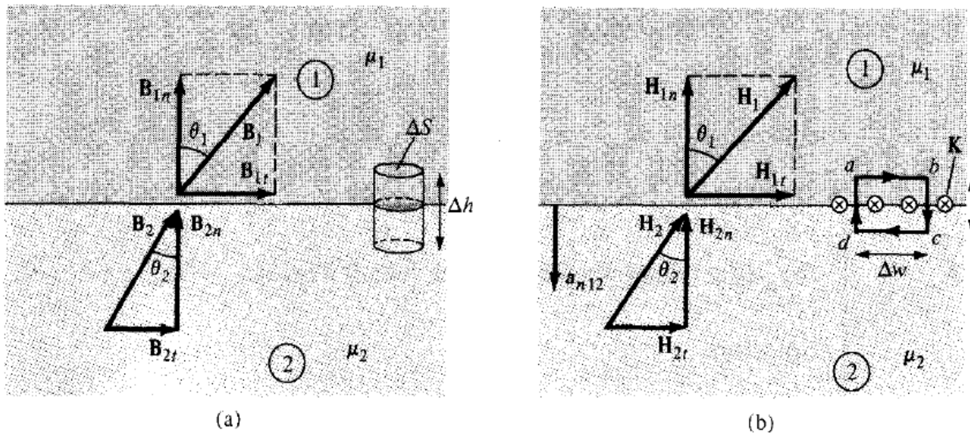

Fig3: boundary between two magnetic materials of different Permeabilities a) for B b) for H

NORMAL COMPONENT:

Consider a Gaussian surface in fig (cylinder). The axis of the cylinder is in the normal direction to the surface.

According to Gauss's law for the magnetic field,

$\oint_s B.ds$=0 ---------------------①

① can be written as ,

$\oint_{lop} \overline{B}.\overline{ds} + \oint_{bottom} \overline{B}.\overline{ds} + \oint_{lateral} \overline{B}.\overline{ds} = 0$----------②

In the boundary conditions, reduce Δh to zero, the cylinder tends to boundary. Let the magnitude of normal component of 'B' be in B_{N1} in medium 1 and B_{N2} in medium 2 respectively.

②→

$\oint_{top} B.ds + \oint_{bottom} B.ds = 0$ -----------③

For top surfaces,

$\oint_{top} \overline{B}.\overline{ds}= B_{N1} \oint ds = B_{N1}\Delta S$--------------④

For bottom surfaces,

$\oint_{bottom} \overline{B}.ds = -B_{N2} \oint ds = -B_{N2}\Delta s$ ----------⑤

For lateral surface

$\oint_{later} B.ds =0$ as $\Delta h \rightarrow 0$

Eqn ③→

$B_{N1}\Delta S - B_{N2}\Delta S = 0$

$B_{N1}\Delta S = B_{N2}\Delta S$ ---------------------------------⑥

$B_{N1} = B_{N2}$

Thus the normal component of $\overline{B}$ is continuous at the boundary.

$$B=\mu H$$

The magnetic field intensity is expressed as ,

$H=\frac{B}{\mu}$, $H_{N1} = \frac{B_{N1}}{\mu_1}$

$$H_{N2} = \frac{B_{N2}}{\mu_2}$$

$\mu_1 H_{N1} = \mu_2 H_{N2}$----------------⑦

$\frac{HN_1}{HN_2} = \frac{\mu_2}{\mu_1}$ ---------------------------⑧

Hence the normal component of $\overline{H}$ is not continuous at the boundary.

TANGENTIAL COMPONENT:

According to Ampere's circuital law ,

$\oint H.dl = I_{enc}$ ----------------------------⑨

Consider a rectangular closed path abcda ,

$\oint H.dl = \int_a^b H.dl + \int_b^c H.dl + \int_c^d H.dl + \int_d^a H.dl = I_{en}$ -------⑩

In the rectangular two sides a-b and c-d are parallel to the boundary surfaces while the other two sides are normal to the boundary.

$\int_a^b H.dl = \int_a^b H \tan 1.dl = H\tan 1\, \Delta w$ ---------------⑪

$\int_b^c H.dl = \int_b^2 HN_1 dl + \int_2^c HN_{2.}dl$= H$N_1 \frac{\Delta h}{2}$ + H$N_2 \frac{\Delta h}{2}$ ------------⑫

$\int_c^d H.dl = \int_c^d H$ tan 2.dl = - Htan 2Δw ------------------⑬

$\int_d^a H.dl = \int_d^1 HN_1$.dl +$\int_1^a HN_2$.dl = - H$N_1 \frac{\Delta h}{2}$ - H$N_2 \frac{\Delta h}{2}$ --------⑭

I_{enc} = I.Δw ----------------------⑮

Substitute all the values in eqn ⑩,

⑩→

Htan 1Δw+ H$N_1 \frac{\Delta h}{2} + HN_2 \frac{\Delta h}{2} - H \tan 2\Delta w - HN_1 \frac{\Delta h}{2} - HN_2 \frac{\Delta h}{2}$ =KΔW

Htan 1Δw - Htan 2Δw = KΔw

Δw (Htan 1- Htan 2)= KΔw

Htan 1- Htan 2 = K ----------⑯

Htan 1- Htan 2 = a_N * K

a_N →unit vector normal to the boundary

$\frac{B \tan 1}{\mu_1} - \frac{B \tan 2}{\mu_2}$ =K ------------⑰

For source free region K=0,

Htan 1- Htan 2 = 0 ------------⑱

Htan 1= Htan 2

$\frac{B \tan 1}{\mu_1} - \frac{B \tan 2}{\mu_2} = 0$

$$\frac{B \tan 1}{\mu_1} = \frac{B \tan 2}{\mu_2}$$

$\frac{B \tan 1}{B \tan 2} = \frac{\mu 1}{\mu 2}$