# UNIT V

5.1 TORSION

5.1.1 INTRODUCTION:

A shaft is said to be in torsion, when equal and opposite forces are applied at the two ends of the shaft. The torque is equal to the product of the force applied and radius of the shaft. Due to the application of the force at the ends the shaft is subjected to a twisting moment. This causes the shear stress and shear strains in the material of the shaft.

5.1.2 DERIVATION OF SHEAR STRESS PRODUCED IN A CIRCULAR SHAFT SUBJECTED TO TORSION:

Before the derivation of shear stress produced in a circular shaft the following assumption are to be made as:

Assumption made in the Derivation of Shear Stress Produced in a Circular Shaft Subjected to Torsion:

1. The material of the shaft is uniform throughout.
2. The twist along the shaft is uniform.
3. The shaft is uniform circular section throughout.
4. Cross section of the shaft, which are plane before and after twist.
5. All radii which are straight before and after twist.
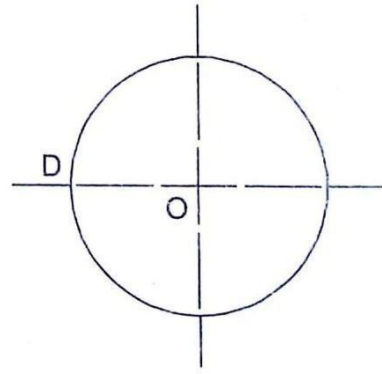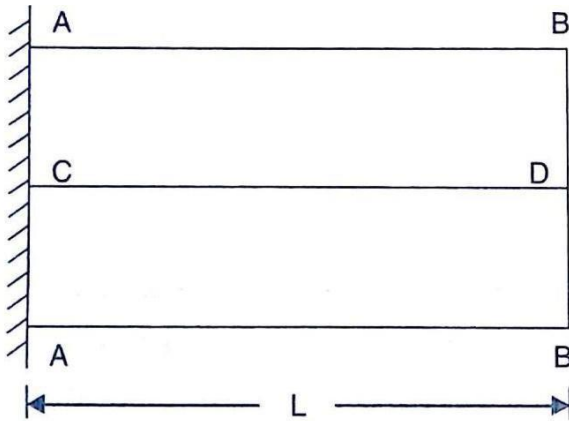
Consider a shaft of length l, radius R fixed at one end and free other end is free is subjected to a torque T as shown in figure.

    Let C = Modulus of rigidity of the material

        τ = Shear stress induced at the surface of the shaft due to torque T

Let 'O' be the centre of the shaft D a point on surface and AB be the line on the shaft parallel to the axis of the shaft.
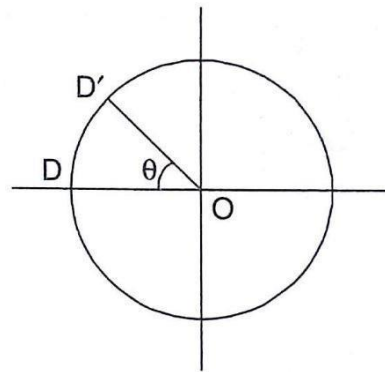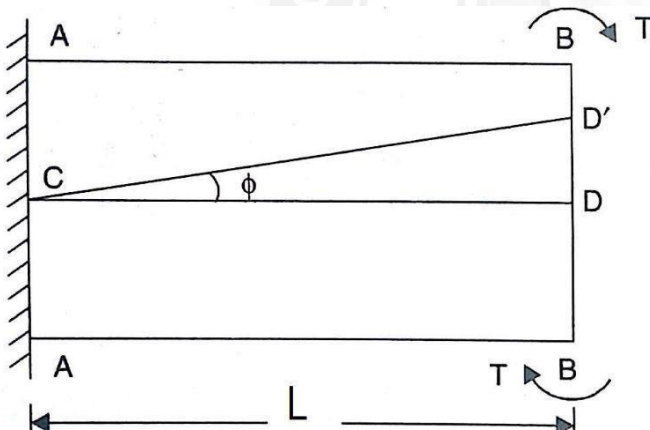
When the shaft is subjected to torque T then D is moved to D^1. If 'ϕ' be the shear strain and 'θ' be the angle of twist in length l then

Shear strain ϕ = Distortion per unit length = $\dfrac{\text{Distortion at the outer surface}}{\text{Length of the shaft}}$

ϕ $= \dfrac{DD^1}{l}$

Then DD^l $= l\,\phi$

But, Arc length DD^l $= R\,\theta = l\,\phi$ ....(3.1)

Shear strain ϕ $= \dfrac{\text{Shear stress}}{\text{Modulus of rigidity}} = \dfrac{\tau}{C}$ …………….... (3.2)

Substitute φ value in eqn 3.1

$$= R\,\theta = l\frac{\tau}{C}$$

$$= \frac{C\theta}{l} = \frac{\tau}{R}$$

.... (3.3)

5.1.3 STRENGTH (OR) MAXIMUM TORQUE TRANSMITTED BY A CIRCULAR SOLID SHAFT

The strength of a shaft means the maximum torque or maximum power the shaft can transmit.

The maximum torque transmitted by a circular shaft is obtained from the maximum shear stress induced at the outer surface of the solid shaft. ie., $\tau \, \alpha \, R$

Consider a shaft subjected to a torque T. Also consider a small elementary circular ring of thickness dr at a distance r from the center as shown in figure.

Let τ = Shear stress induced at the surface of the shaft due to torque T

      R = Radius of the shaft

      q = shear stress at the radius' r' from the centre.
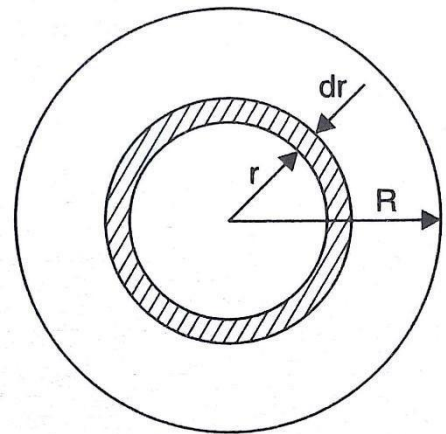
The area of the ring $dA = 2\pi r dr$

If q is the shear stress induced at a radius r from the centre of the shaft then

$$\frac{q}{r} = \frac{\tau}{R}$$

∴Shear stress at the radius r,

$$q = \frac{\tau}{R}r$$

Tuning force on the elementary circular ring

      = Shear stress acting on the ring x area of ring

      = q x dA

      $= \frac{\tau}{R}$ x r x $2\pi r dr$

      $= \frac{\tau}{R}$ x $2\pi r^2 dr$

Now tuning moment due to tuning force on the elementary circular ring

dT = Tuning force x distance of the ring from axis

$$= \frac{\tau}{R} \text{x } 2\pi r^2 dr \text{ x } r$$

$$= \frac{\tau}{R} \text{x } 2\pi r^3 dr$$

Now the total turning moment or torque on the shaft is obtained by integrating the above eqn. ( ) between the limit 0 to R

$$T = \int_o^R dT = \int_0^R \frac{\tau}{R} 2\pi r^3 dr$$

$$= \frac{\tau}{R} \text{x } 2\pi \int_0^R r^3 dr$$

$$= \frac{\tau}{R} \text{x } 2\pi \left[\frac{r^4}{4}\right]_0^R$$

$$= \frac{\tau}{R} \text{x } 2\pi \text{ x } \frac{R^4}{4}$$

$$= \frac{\tau}{2} \text{x } \pi R^3$$

$$= \frac{\tau}{2} \text{x } \pi \text{ x } \left(\frac{D}{2}\right)^3 \qquad \left(R = \frac{D}{2}\right)$$

$$= \frac{\tau}{2} \text{x } \pi \text{ x } \frac{D^3}{8}$$

$$T \qquad = \frac{\pi}{16} \tau D^3 \qquad \qquad \ldots\ldots\ldots\ldots..(3.6)$$

5.1.4 DERIVE THE TORSIONAL EQUTION:

From the eqn (3.3) we know that

$$\frac{C\theta}{l} = \frac{\tau}{R}$$

But from torque transmission on a shaft of eqn. (3.6)

$$\tau = \frac{T \text{ x } 16}{\pi \text{ x} D^3}$$

Substitute the τ value in the eqn (3.3)

$$\frac{C\theta}{l} = \frac{T \text{ x } 16}{\frac{D}{2} \text{x } \pi \text{ x } D^3}$$

$$\frac{C\theta}{l} = \frac{T}{\frac{\pi}{32} D^4}$$

Where $\frac{\pi}{32} D^4$ is the **polar moment of inertia (J)** of the solid shaft. Then the above

equation become $\quad \frac{C\theta}{l} = \frac{T}{J} \qquad \qquad \ldots\ldots\ldots\ldots.$ 3.10

Similarly polar moment of inertia of hollow circular shaft $= \frac{\pi}{32}(D^4 - d^4)$

Where D = outer diameter and d = inner diameter of hollow shaft

From eqn. (3.3) and eqn. (3.10)

$$\frac{T}{J} = \frac{C\theta}{l} = \frac{\tau}{R}$$

5.1.5 TORQUE TRANSMITTED BY A HOLLOW CIRCULAR SHAFT:

Consider a hollow circular shaft of outer and inner radius are Ro and Ri is subjected to a torque T. Take an elementary circular ring of thickness 'dr' at a distance r from the centre as shown in figure

Let q = shear stress induced on the elementary ring.

dA = 2πrdr area of the elementary circular ring   shear stress at the elementary ring is obtained from shear stress ratio

$$\frac{q}{r} = \frac{\tau}{R_o}$$

$$q = \frac{\tau}{R_o}r$$

Tuning force on the elementary circular ring

= Shear stress acting on the ring x area of ring

= q x dA

$$= \frac{\tau}{R_o} \times r \times 2\pi rdr \qquad\qquad = \frac{\tau}{R_o} \times$$

$2\pi r^2 dr$

Now tuning moment due to tuning force on the elementary circular ring

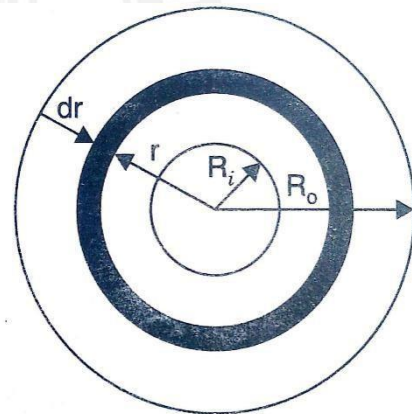dT = Tuning force x distance of the ring from axis

$$= \frac{\tau}{R_o} \times 2\pi r^2 dr \times r$$

$$= \frac{\tau}{R_o} \times 2\pi r^3 dr$$

Now the total turning moment or torque on the shaft is obtained by integrating the above eqn. between the limit Ri to Ro

$$T =\int_{R_i}^{R_o} dT = \int_{R_i}^{R_o} \frac{\tau}{R_o} \times 2\pi r^3 dr$$

$$= \frac{\tau}{R_o} \times 2\pi \int_{R_i}^{R_o} r^3 dr$$

$$= \frac{\tau}{R_o} \times 2\pi \left[\frac{r^4}{4}\right]_{R_i}^{R_o}$$

$$= \frac{\tau}{R_o} \times 2\pi \times \left[\frac{R_o{}^4 - R_i{}^4}{4}\right]$$

$$= \frac{\tau}{2} \times \pi \times \left[\frac{R_o{}^4 - R_i{}^4}{R_o}\right]$$

$$= \frac{\tau}{2} \times \pi \times \left[\frac{\left(\frac{D_o}{2}\right)^4 - \left(\frac{D_i}{2}\right)^4}{\frac{D_o}{2}}\right] \qquad \left(R_o = \frac{D_o}{2} \; ; R_i = \frac{D_i}{2}\right)$$

$$= \frac{\tau}{2} \times \pi \times \left[\frac{D_o{}^4 - D_i{}^4}{16} \times \frac{2}{D_o}\right]$$

$$\boxed{T = \frac{\pi}{16} \tau \left[\frac{D_o{}^4 - D_i{}^4}{D_o}\right]}$$

5.1.6 TORSIONAL RIGIDITY:

The product of modulus of rigidity and polar moment of inertia of a circular shaft is known as torsional rigidity. It is denoted by (K).

Torsional rigidity (K) = C x J

Torsional rigidity is also defined as the torque required to produce a twist of radian per unit length of the shaft.

From the torsional equation $\frac{T}{J} = \frac{C\theta}{l} = \frac{\tau}{R}$ gives $CJ = \frac{Tl}{\theta}$

If l = 1metre and θ = 1radian then Torsional rigidity = Torque

Since C, J and l are constant for a given shaft, the angle of twist θ is directly proportional to the torque (T). The term CJ is known as torsional rigidity.

5.1.7 POLAR MODULUS:

It is the ratio between the polar moment of inertia and the radius of the shaft. It is denoted by (Z).Its unit is mm^3. It is also called torsional section modulus.

Polar Modulus $(Z) = \frac{Polar\ moment\ of\ inertia}{Radius\ of\ shaft} = \frac{J}{R}$

For solid shaft $(J) = \frac{\pi}{32}D^4$ then $Z = \frac{\frac{\pi}{32}D^4}{\frac{D}{2}} = \frac{\pi}{16}D^3$

For hollow shaft $(J) = \frac{\pi}{32}(D^4 - d^4)$ then $Z = \frac{\frac{\pi}{32}(D^4 - d^4)}{D} = \frac{\pi}{16}\left[\frac{D^4 - d^4}{D}\right]$

2

5.1.8 POWER TRANSMITTED BY SHAFT:

Once the torque (T) for a solid or a hollow shaft is obtained, power transmitted by the shaft can be determined,

Let N = Speed of shaft in rpm

T = mean torque transmitted in Nm   Power   P   =

$\frac{2\pi N T_{mean}}{60}$ Watts   or   P = T x ω   where ω = $\frac{2\pi N}{60}$

5.1.9 SAVING OF MATERIAL AND WEIGHT OF SOLID AND HOLLOW

SHAFT: (i) Percentage of saving in Material:

$$= \frac{\text{Area of solid shaft} - \text{Area of hollow shaft}}{\text{Area of solid shaft}} \times 100$$

Let D = diameter of solid shaft

l_S = length of solid shaft
ρ_S = density of solid shaft

l_H = length of hollow shaft

D_H = Outer diameter of hollow shaft

d_H = inner diameter of hollow shaft

ρ_H = density of Hollow shaft

$$= \frac{\frac{\pi \times D^2}{4} - \frac{\pi \times \left(D_H^2 - d_H^2\right)}{4}}{\frac{\pi \times D^2}{4}} \times 100 = \frac{D^2 - \left(D_H^2 - d_H^2\right)}{D^2} \times 100$$

(ii) Percentage of saving in weight:

$$= \frac{\text{Weight of solid shaft} - \text{Weight of hollow shaft}}{\text{Weight of solid shaft}} \times 100$$

Weight of solid shaft = density x volume = density x area x length

$$= \rho_S \times \frac{\pi \times D^2}{4} \times l_S$$

Weight of hollow shaft = $\rho_H \times \frac{\pi \times \left(D_H^2 - d_H^2\right)}{4} \times l_H$

Then Percentage of saving in material

$$= \frac{\rho_S \times \frac{\pi \times D^2}{4} \times l_S - \rho_H \times \frac{\pi \times \left(D_H^2 - d_H^2\right)}{4} \times l_H}{\rho_S \times \frac{\pi \times D^2}{4} \times l_S} \quad \text{x 100}$$

For same material and same length $\rho_S = \rho_H$ and $l_S = l_H$ then

Percentage of saving in material $= \dfrac{D^2 - \left(D_H^2 - d_H^2\right)}{D^2}$ x 100

Problem 5.1.1 A solid shaft of is to transmit a torque of 25kNm. If the shearing stress is not to exceed 60 Mpa. Find the minimum diameter of the shaft.

Given Data:

   Torque transmitted $T = 25kNm = 25 \times 10^6 Nmm$

   Shear stress $\tau = 60$ Mpa $= 60$ N/mm^2 **To**

find:

Diameter of the shaft D = ?

Solution:

   Wkt Torque transmitted $T = \dfrac{\pi}{16} D^3$ then $D = \sqrt[3]{\dfrac{T \times 16}{\pi}}$

$$= \sqrt[3]{\dfrac{25 \times 10^6 \ x \ 16}{\pi}} = \textbf{128.5 mm}$$

Result:

   Diameter of the shaft D = **128.5 mm**

Problem 5.1.2. A hollow circular shaft of external diameter 50mm and internal diameter 40mm transmit a torque of 10 kNm. Find the maximum shear induced in the shaft.

Given Data: 6Nmm

 External diameTorque transmitted T = 10kNm = 10er D = 50mm   × 10

   Internal diameter d = 40mm

To find:

   Shear stress τ =?

Solution:

Wkt   Torque transmitted T $= \frac{\pi}{16} \tau \left[\frac{D^4-d^4}{D}\right]$

$$10 \times 10^6 \qquad = \frac{\pi}{16} \tau \left[\frac{50^4-40^4}{50}\right]$$

$$\tau = 690.1 \text{N/mm}^2$$

Result:

Shear stress τ = **690.1N/mm²**

Problem 5.1.3. Find the power that can be transmitted by a shaft of 50mm diameter at a speed of 120 rpm. If the shear stress is 60 N/mm² **Given Data:**

Diameter   D = 50mm

Speed   N = 120 rpm

Shear stress τ = 60 N/mm² **To**

find:

Power   P =?

Solution:

Wkt   Power transmittedP $= \frac{2\pi N T_{mean}}{60}$

But T_{mean} $= \frac{\pi}{16} \tau D^3 \quad = \frac{\pi}{16} 60 \times 50^3$

T_{mean} = 1472621.5 Nmm   = 1.472× 10³ Nm

Then   P $= \frac{2 \times \pi \times 120 \times 1.472 \times 10^3}{60} =$ 18476.5 W

= **18.476 kW**

Result:

Power   P = **18.476 kW**

Problem 5.1.4. A solid circular shaft transmits85kW power at 200 rpm. Find the shaft diameter if the shear stress is 50 MN/m².

Given Data:

Power   P = 85kW = 85 × 10³W

Speed   N = 200 rpm

Shear stress τ = 50 MN/m^2 = 50 N/mm^2 **To**

find:

Shaft diameterD =?

Solution:

Wkt Power transmitted P $= \dfrac{2\pi NT}{60}$

Then $85 \times 10^3 = \dfrac{2 \times \pi \times 200 \times T}{60}$

T $= 4.05 \times 10^3$Nm $= 4.05 \times 10^6$Nmm

But Wkt, $T = \dfrac{\pi}{16}\,\tau\,D^3 = \dfrac{\pi}{16} \times 50 \times D^3$

D $=$ **74.4 mm**

Result:

Shaft diameterD = **74.4 mm**

Problem 5.1.5. A hollow shaft is to transmit 200kW at 80 rpm. If the shear stress is not to exceed 70 MN/m^2. and internal diameter is 0.5 of the external diameter. Find the external and internal diameters assuming that maximum torque is 1.6 times the mean.

Given Data:

Power P = 200kW = 200 $\times 10^3$W

Internal Diameter d = 0.5 D

Speed N = 80 rpm

Shear stress τ = 70 MN/m^2 = 70$\times 10^6$ N/m^2= 70 N/mm^2

Max. Torque T_{max}= 1.6 T_{mean} **To find:**

External and internal diameter D, d =?

Solution:

Wkt Power transmitted P $= \dfrac{2\pi NT}{60}$

$200 \times 10^3 = \dfrac{2 \times \pi \times 80 \times T}{60}$

T = T$_{mean}$= 23.87$\times 10^3$Nm = 23.87$\times 10^6$ Nmm

In given data T$_{max}$ = 1.6 T_{mean}

Then T$_{max}$ = 1.6×23.87$\times 10^6$= 38.19 $\times 10^6$ Nmm

But
$$T_{max} = \frac{\pi}{16} \tau \left[\frac{D^4 - d^4}{D}\right]$$

$$T_{max} = \frac{\pi}{16} \tau \left[\frac{D^4 - (0.5\,D)^4}{D}\right] \qquad (\because d = 0.5\ D)$$

$$38.19 \times 10^6 = \frac{\pi}{16} \times 70 \times D^4 \left[\frac{1 - 0.5^4}{D}\right]$$

$$D = 143.6 \text{ mm}$$

$$\Rightarrow \qquad d = 0.5 \times 143.6 = 71.82 \text{ mm}$$

Result:

Outer diameter D = **143.6 mm**

Inner diameter d = **71.82 mm**

Problem 5.1.6. Find the maximum torque that can be safely applied to a shaft of 120 mm diameter. If the allowable twist is 3° in a length of 1.5m. Take C = $1 \times^5$ N/mm10 ^{2}

Given Data:

Diameter   D = 120 mm

Angle of twist $\theta = 3° = 3 \times \frac{\pi}{180} = 0.05$ rad

Length $l = 1.5\text{m} = 1.5 \times 10^3$ mm

Modulus of rigidity C = 1×10^5 N/mm^2 **To find:**

Maximum torque transmitted   T =?

Solution:

From the torsional equation $\frac{T}{J} = \frac{C\theta}{l}$

Where, J = polar moment of inertia = $\frac{\pi}{32} \times D^4$

$$J = \frac{\pi}{32} \times 120^4 \qquad = 20.3 \times 10^6 \text{mm}^4$$

Substitute J value in the torsion equation, then

$$T = \frac{1 \times 10^5 \times 0.05}{1.5 \times 10^3} \times 20.3 \times 10^6$$

$$T = 67.6 \times 10^6 \text{ Nmm}$$

Result:

Torque transmitted T = **67.6 × 10⁶ Nmm**

5.1.7   A solid shaft of diameter 100mm is require d to transmit 150kW at 120

rpm. If the length of the shaft is 4m and modulus of rigidity for shaft is 75 Gpa, find the angle of twist.

Given Data:

Diameter $\quad\quad$ D = 100 mm

Power $\quad\quad$ P = 150 kW = 150×10^3 W

Speed $\quad\quad$ N = 120 rpm

Length $\quad\quad$ l = 4 m = 4×10^3 mm

Modulus of rigidity C = 75 Gpa = 75×10^9 pa = 75×10^9 N/m²

$\quad\quad\quad\quad = 75 \times 10^3$ N/mm²

To find:

Angle of twist $\quad\quad \theta = ?$

Solution:

From the torsional equation $\dfrac{T}{J} = \dfrac{C\theta}{l}$

Where, J = polar moment of inertia = $\dfrac{\pi}{32} \times D^4$

$$J = \frac{\pi}{32} \times 100^4 \quad = 9.81 \times 10^6 \quad mm^4$$

Wkt, $\quad\quad$ Power transmitted P $= \dfrac{2\pi NT}{60}$

$$150 \times 10^3 = \frac{2 \times \pi \times 100 \times T}{60}$$

$\gg \quad\quad\quad\quad\quad\quad\quad\quad$ T $= 11.93 \times 10^3$ Nm $= 11.93 \times 10^6$ Nmm

Substitute J and T value in the torsion equation, then

$$\frac{11.93 \times 10^6}{9.81 \times 10^6} = \frac{75 \times 10^6 \times \theta}{4 \times 10^3}$$

$\gg \quad\quad\quad\quad\quad \theta = 0.06$ rad

$$\theta = 0.06 \times \frac{180}{\pi} = 3.7_o$$

Result:

Angle of twist θ = **3.7º**

5.1.7 A hollow shaft of 120mm external diameter and 80mm internal diameter is required to transmit 200kW at 120 rpm. If the angle of twist is not to exceed 3º find the length of the shaft. Take modulus of rigidity for shaft is 80 Gpa.

Given Data:

External Diameter D = 120 mm

Internal diameter   d = 80 mm

Power   P = 200 kW = 200 × 10³W

Speed   N = 120 rpm

Angle of twist $\theta = 3° = 3 \times \dfrac{\pi}{180} = 0.05$ rad

Modulus of rigidity C = 80 Gpa = 80 × 10⁹ pa = 80 × 10⁹N/m²

= 80 × 10³N/mm² **To**

find:

Length of shaft l = ?

Solution:

From the torsional equation $\dfrac{T}{J} = \dfrac{C\theta}{l}$

Where, J = polar moment of inertia $= \dfrac{\pi}{32} \times [D^4 - d^4]$

$$J = \frac{\pi}{32} \times [120^4 - 80^4]$$

$$= 16.3 \times 10^6 \ \ mm^4$$

Wkt,   Power transmitted P $= \dfrac{2\pi NT}{60}$

$$200 \times 10^3 = \frac{2 \times \pi \times 120 \times T}{60}$$

»   T = 15.93× 10³Nm = 11.93× 10⁶Nmm

Substitute J and T value in the torsion equation, then

$$\frac{15.93 \times 10^6}{16.3 \times 10^6} = \frac{80 \times 10^3 \times 0.05}{l}$$

» l = 4264.6 mm

Result: Length of shaft l = **4264.6 mm**

5.1.9: Find the maximum torque that can be safely applied to a shaft of 120mm diameter. The permissible shear stress and allowable twist are 200 N/mm² and 3° respectively. Take C = 75Gpa and length of shaft = 4m.

Given data:

Diameter   D = 120 mm

Shear stress τ = 200 N/mm²

Angle of twist $\theta = 3° = 3 \times \dfrac{\pi}{180} = 0.052$ rad

Modulus of rigidity $C = 75$ Gpa $= 75 \times 10^9$ pa $= 75 \times 10^9$ N/m^2

$$= 75 \times 10^3 \text{N/mm}^2$$

Length of shaft $\quad l = 4\text{m} = 4 \times 10^3$ mm

To find:

Maximum torque$T_{max} = $?

Solution:

Consider based on shear stress

Torque $\qquad T = \frac{\pi}{16} \tau D^3$

$$= \frac{\pi}{16} \times 200 \times 120^3 = 67.8 \times 10^6 \text{ Nmm}$$

Considering angle of twist

From the torsional equation$\dfrac{T}{J} = \dfrac{C\theta}{l}$

Where, J = polar moment of inertia $= \frac{\pi}{32} \times D^4 = \frac{\pi}{32} \times 120^4 = 20.35 \times 10^6$ mm^4

Then torsion equation become

$$\frac{T}{20.35 \times 10^6} = \frac{75 \times 10^3 \times 0.052}{4 \times 10^3}$$

Then $\qquad\qquad T = 19.8 \times 10^6$ Nmm

From the above two torque value we have to find the maximum value that can be safely applied on the shaft is take the minimum value as 19.8×10^6 Nmm.

Result:

Maximum torque$T_{max} = \mathbf{19.8 \times 10^6}$ **Nmm**

5.1.10: A solid circular shaft transmit 70kW power at 175 rpm. Calculate the shaft diameter if the twistin the shaft is not to exceed $2°$ in 2 meter length of shaft and shear stress is limited to $50^3 \times \text{kN/m}10^2$. Take $C = 100 \times 10^3 \text{MN/m}^2$.

Given data:

Power $\qquad\qquad\qquad$ P = 70 kW = 70×10^3W

Speed $\qquad\qquad$ N = 175 rpm

Angle of twist $\qquad \theta = 2° = 2 \times \frac{\pi}{180} = 0.034$ rad

Length of shaft $\quad l = 2\text{m} = 2 \times 10^3$ mm

Shear stress $\qquad \tau = 50 \times 10^3 \text{kN/m}^2 = 50$ N/mm^2

Modulus of rigidity $C = 100 \times 10^3 \text{MN/m}^2 = 100 \times 10^9 \text{N/m}^2$

$$= 100 \times 10^3 \text{ N/mm}^2$$

To find:

Diameter of shaft   D = ?

Solution:

Wkt, Power $P = \dfrac{2\pi NT}{60}$

$70 \times 10^3 = \dfrac{2 \times \pi \times 175 \times T}{60}$

» Torque $T = 3.81 \times 10^3 \text{ Nm} = 3.81 \times 10^6 \text{ Nmm}$

Consider based on shear stress

Torque $T = \dfrac{\pi}{16} \ \tau \ D^3$

$3.81 \times 10^6 = \dfrac{\pi}{16} \times 200 \times D^3$

» $D = 72.9$ mm

Considering angle of twist

From the torsional equation $\dfrac{T}{J} = \dfrac{C\theta}{l}$

Where, J = polar moment of inertia $= \dfrac{\pi}{32} \times D^4 = 0.098 D^4$

Then torsion equation become

$\dfrac{3.81 \times 10^6}{0.098 D^4} = \dfrac{100 \times 10^3 \times 0.034}{2 \times 10^3}$

Then $D = 69.12$ mm

From the above two cases, we have to find the suitable diameter for the shaft is take the greatest value as 72.9 = 73 mm.

Result:

Shaft diameter **D = 73 mm**

Problem 5.1.11: A hollow shaft is to transmit 300kW power at 80 rpm. If the shear stress is not to exceed 50MN/m^2 and diameter ratio is 3/7. Find the external and internal diameter if the twist of shaft is 1.2º in 2 meter length. Assuming maximum torque is 20% greater than mean. Take C = 80 GN/m^2.

Given data:

Power $\qquad$ P = 300 kW = 300×10^3 W

Speed $\qquad$ N = 80 rpm

Shear stress $\qquad$ τ = 50 MN/m² = 50 N/mm²

Diameter ratio $\quad$ d/D = 3/7 $\gg$ d = 0.428D

Angle of twist $\qquad$ $\theta = 1.2^\circ = 1.2 \times \frac{\pi}{180} = 0.020$ rad

Length of shaft $\quad l = 2m = 2 \times 10^3$ mm

Maximum torque T_{max} = 20% greater than T_{mean} = (100% +20%)T_{mean}

$$=1.2 T_{mean}$$

Modulus of rigidity C = Modulus of rigidity C = 80 GN/m² = 80×10^9 N/m²

$$= 80 \times 10^3 \text{N/mm}^2$$

To find:

External and internaldiameter of shaft $\quad$ D,d =?

Solution:

Wkt, Power $\qquad$ $P = \dfrac{2\pi NT}{60}$

$$300 \times 10^3 = \frac{2\times\pi\times80\times T}{60}$$

$\gg$ $\quad$ Torque T= T_{mean} = 35.8×10^3 Nm = 35.8×10^6 Nmm

In our data, T_{max} = 1.2 T_{mean} = 1.2×35.8× 10^6 = 42.96× 10^6 Nmm

Consider based on shear stress

$$\text{Torque } T_{max} = \frac{\pi}{16}\,\tau\,\left[\frac{D^4-d^4}{D}\right]$$

$$42.96\times 10^6 = \frac{\pi}{16} \times 50 \times \left[\frac{D^4-(0.428D)^4}{D}\right]$$

$$42.96\times 10^6 = \frac{\pi}{16} \times 50 \times D^3[1-(0.428)^4]$$

$\gg$ $\qquad\qquad$ D = 165.4 mm, then d = 0.428×165.4 = 70.8 mm

Considering angle of twist

From the torsional equation $\dfrac{T}{J} = \dfrac{C\theta}{l}$

Where, J = polar moment of inertia = $\dfrac{\pi}{32} \times [D^4 - d^4] = \dfrac{\pi}{32} \times [D^4 - (0.428D)^4]$

$$= \frac{\pi}{32} \times D^4[1 - (0.428)^4] = 0.095D^4$$

Then torsion equation become

$$\frac{42.96 \times 10^6}{0.095 D^4} = \frac{80 \times 10^3 \times 0.02}{2 \times 10^3}$$

» D = 154.21mm, then d = 0.428×154.21= 66mm

From the above two cases, we have to find the suitable diameter for the shaft is take the greatest value as

External diameter D = 165.4mm and Internal diameter d = 70.8mm **Result:**

External diameter of shaft D = **165.4mm**

Internal diameter of shaft d = **70.8mm**

5.10 REPLACING SHAFT PROBLEMS:

Problem 5.1.12: A solid shaft of 50mm diameter is to be replaced by a hollow steel shaft whose internal diameter is 0.5 times outer diameter. Find the diameter of the hollow shaft and percentage of saving in weight and material, the maximum shearing stress being the same.

Given data:

Solid shaft diameter D = 50mm

Hollow shaft internal diameter d = $0.5 D_H$

Shear stress τ = same for solid and hollow shaft **To**

find:

External and internaldiameter of shaft D_H,d =?

Solution:

Wkt, the torque transmitted by the solid shaft should be equal to that hollow shaft when solid shaft is replaced by hollow shaft.

Torque transmitted by the solid shaft

$$T = \frac{\pi}{16} \tau D^3 = \frac{\pi}{16} \times \tau \times 50^3 = 24543.6 \tau \text{ Nmm}$$

Torque transmitted by the solid shaft

$$T = \frac{\pi}{16} \tau \left[\frac{D_H{}^4 - d^4}{D_H} \right]$$

$$= \frac{\pi}{16} \times \tau \times \left[\frac{D_H{}^4 - (0.5 D_H)^4}{D_H} \right]$$

$$= \frac{\pi}{16} \times \tau \times D_H{}^3 [1 - (0.5)^4]$$

» $T = 0.184 \tau D_H{}^3 \text{Nmm},$

Toque transmitted in both shafts are same,

>> $24543.6 \, \tau = 0.184 \, \tau D_H^3$

Based on given data shear stress are same (not given assume same), then

>> $D_H = \sqrt[3]{\dfrac{24543.6 \, \tau}{0.184 \, \tau}} = 51.09$ mm

Now internal diameter of hollow shaft $d = 0.5 D_H = 0.5 \times 51.09 = 25.54$ mm

Percentage of saving in weight =

$= \dfrac{\text{Weight of solid shaft} - \text{Weight of hollow shaft}}{\text{Weight of solid shaft}} \times 100$

Weight of solid shaft = density × Area × length

$= \rho_S \times \dfrac{\pi \times D^2}{4} \times l_S = \rho_S \times \dfrac{\pi \times 50^2}{4} \times l_S$

$= 1963.4 \, \rho_S . l_S$

Weight of hollow shaft $= \rho_H \times \dfrac{\pi \times \left(D_H^2 - d^2\right)}{4} \times l_H$

$= \rho_H \times \dfrac{\pi \times \left(51.09^2 - 25.54^2\right)}{4} \times l_H = 1537.5 \, \rho_H . l_H$

>> % of saving in weight $= \dfrac{1963.4 \, \rho_S . l_S - 1537.5 \, \rho_H . l_H}{1963.4 \, \rho_S . l_S} \times 100$

For same material and same length $\rho_S = \rho_H . l_H = l_S$ then

>> % of saving in weight $= \dfrac{1963.4 - 1537.5}{1963.4} \times 100 = 21.7\%$

Percentage of saving in material =

$= \dfrac{\text{Area of solid shaft} - \text{Area of hollow shaft}}{\text{Area of solid shaft}} \times 100$

Area of solid shaft $= \dfrac{\pi \times D^2}{4} = \dfrac{\pi \times 50^2}{4} = 1963.4 \, \text{mm}^2$

Area of hollow shaft $= \dfrac{\pi \times \left(D_H^2 - d^2\right)}{4} = \dfrac{\pi \times (51.09^2 - 25.54^2)}{4} = 1537.5 \, \text{mm}^2$

>> % of saving in material $= \dfrac{1963.4 - 1537.5}{1963.4} \times 100 = 21.7\%$

Result:

External diameter of hollow shaft $D_H = $ **51.09mm**

Internal diameter of hollow shaft d = **25.54mm**

% of saving in weight $\quad$ = **21.7%**

% of saving in material $\quad$ = **21.7%**