1.2 Transformation of independent/ dependent variables

1.2.1 Operations performed on the Independent Variable

Operations on the independent variable in signals and systems refer to mathematical modifications applied to the time (for continuous-time signals) or the index (for discrete-time signals), which change the position, speed, or orientation of the signal without altering its shape (amplitude values).

These operations allow us to manipulate the timing and structure of signals for analysis, system response evaluation, and signal processing tasks.

(i) Time Scaling
(ii) Time Reversal (Reflection)
(iii) Time shifting

(i) Time Scaling:

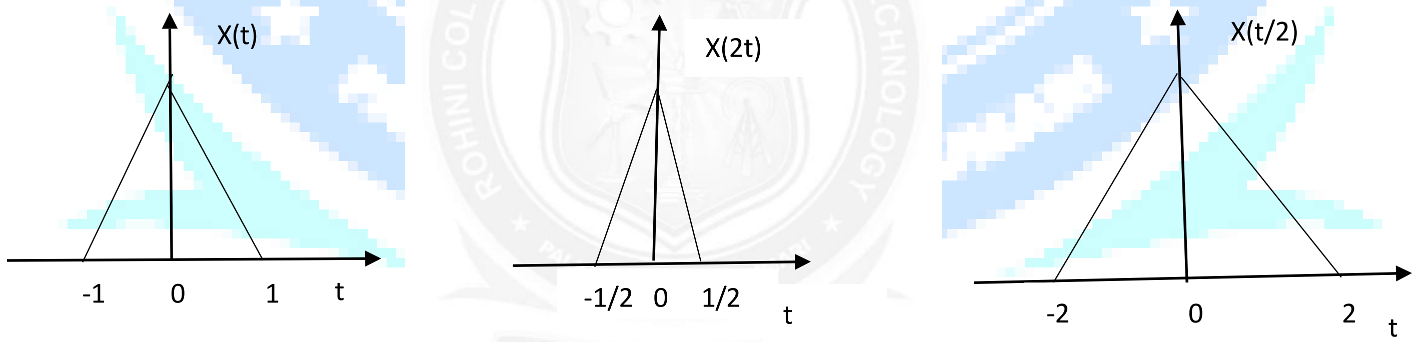
Let $x(t)$ be a continuous Time signal, then

$$y(t) = x(at)$$

Case (i) : If $a > 1,$ the signal $y(t)$ is a compressed version of $x(t)$.

Case(ii) : If $0 < a < 1$, the signal $y(t)$ is an expanded version of $x(t)$.

Example:



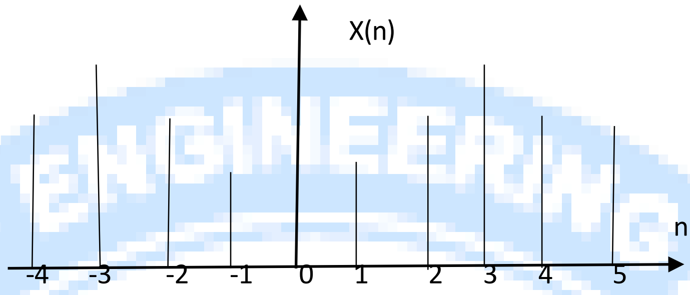
Let $x(n)$ be a Discrete Time signal, then

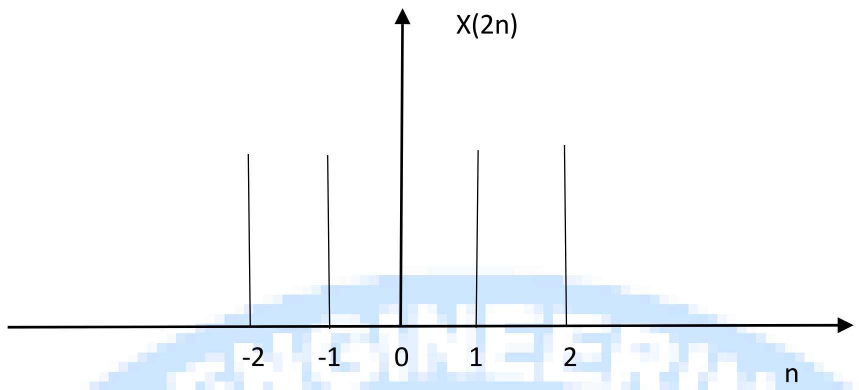
$$y(n) = x(kn)$$

Case (i): If $K > 1$, some values of the Discrete time signal are lost.

*Case (ii):*If $0 < k < 1$, the signal $y(n)$ is an expanded version of $x(n)$.

Example:

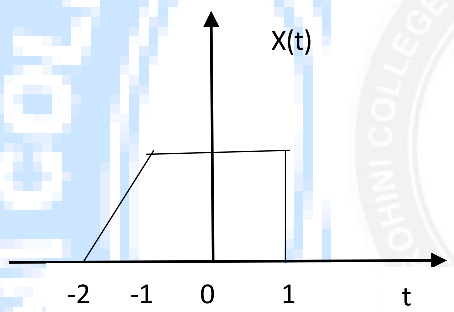


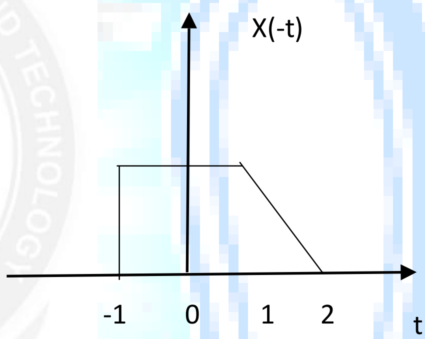


(ii) Time Reversal:

Let $x(t)$ denote a continuous time signal ,then $y(t) = x(-t)$. $y(t)$ is the reflected version of $x(t)$ about $t = 0$.

Example:





(iii) Time shifting:

Let $x(t)$ denote a continuous time signal. Then

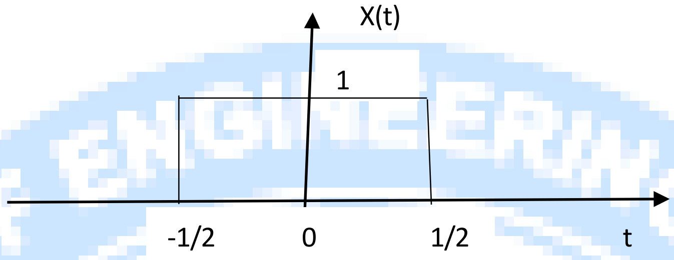
$$y(t) = x(t - t_0).$$

Where, $t_0 \rightarrow Amount\ of\ time\ shift.$

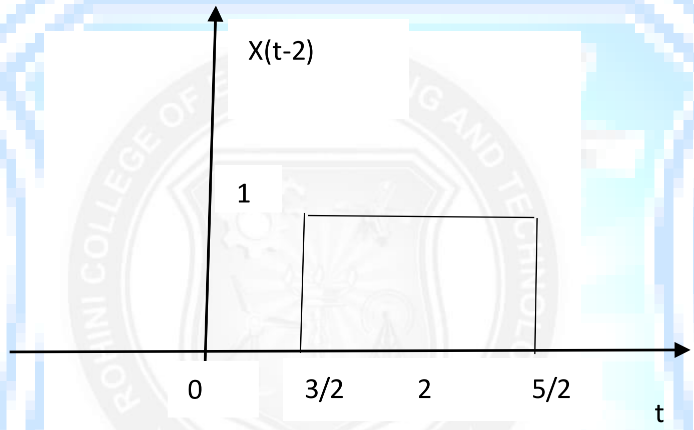
*Case (i):*If $t_0 > 0$, then $x(t)$ is shifted to right (delay).

*Case (ii):*If $t_0 < 0$, then $x(t)$ is shifted to left (advance).

Example: 1 Fig below shows a rectangular pulse $x(t)$ is unit amplitude and unit duration. Find $y(t) = x(t-2)$.



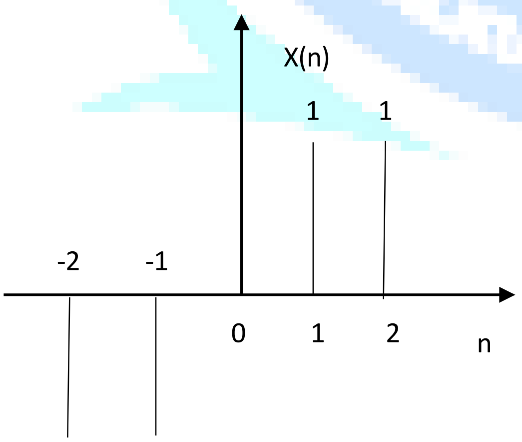
Soln:

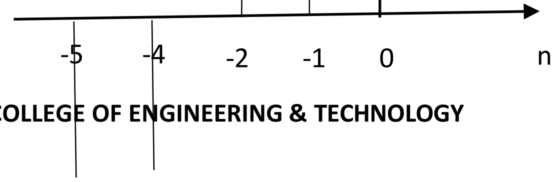


2. The Discrete signal $x(n) = \begin{cases} 1 \;,\; n = 1,2 \\ -1 \;,\; n = -1,-2 \\ 0 \;, n = 0 \;\; and \;|n| > 2 \end{cases}$.

Find $y(n) = x(n+3)$.

Soln:





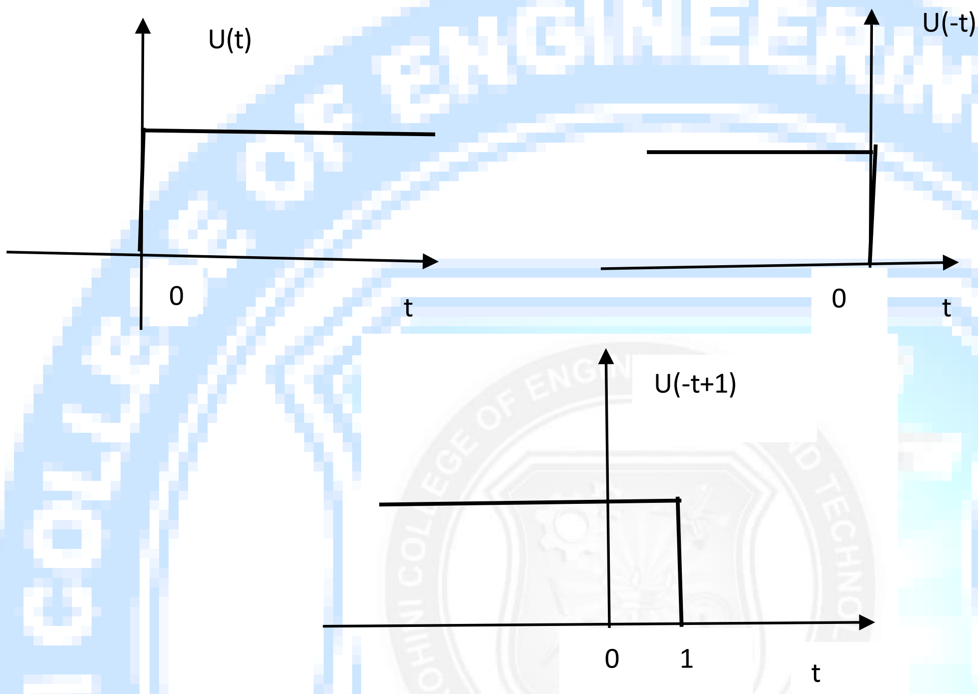
-1 -1

3. Sketch the following signals.

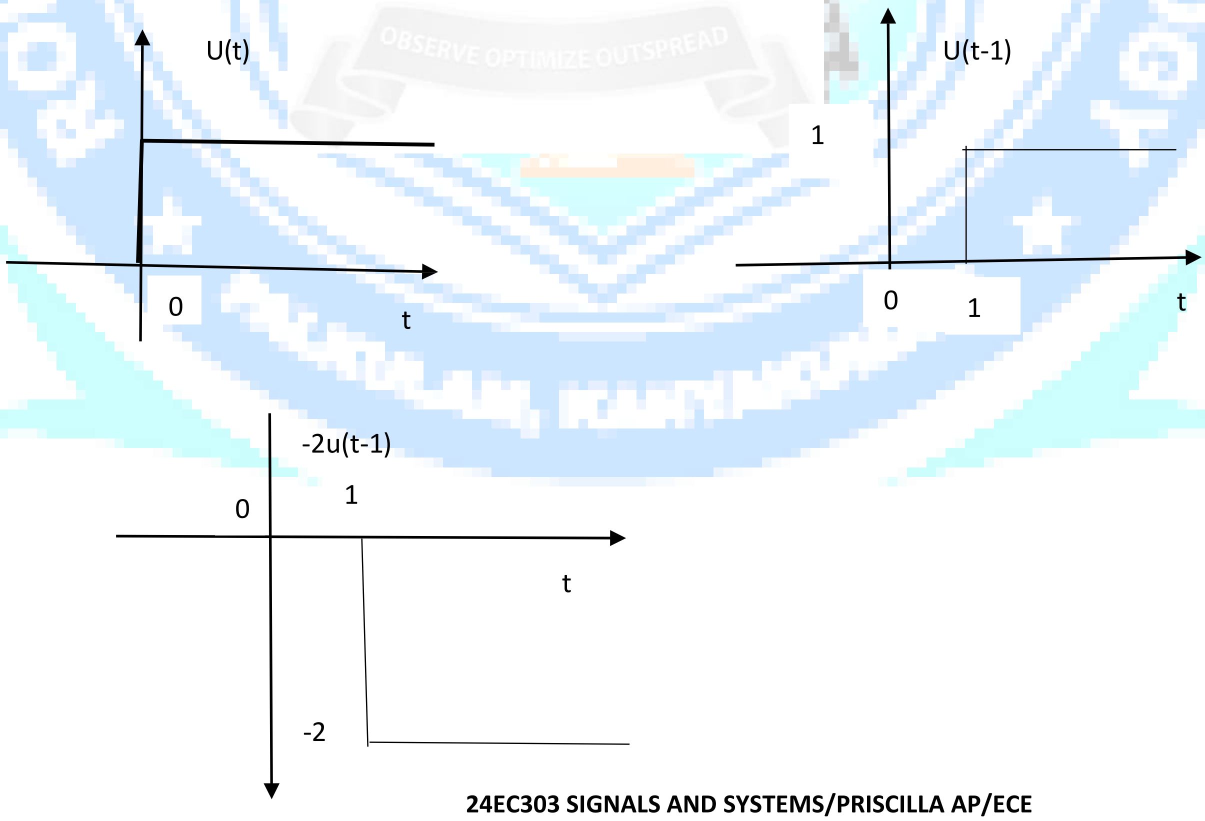
(i) $u(-t+1)$ (ii) $-2u(t-1)$ (iii) $r(-t+2)$ (iv) $\pi(t+3)$

Soln:

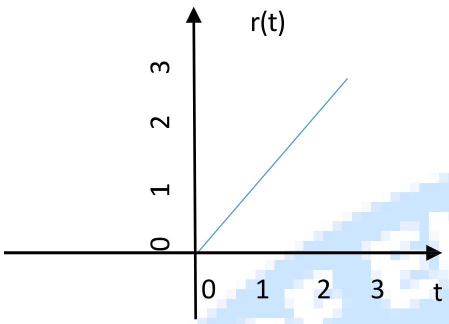
(i) $u(-t+1)$

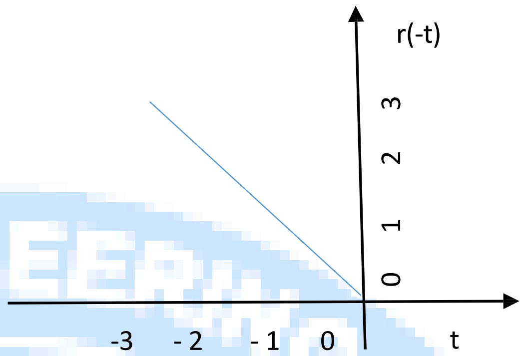


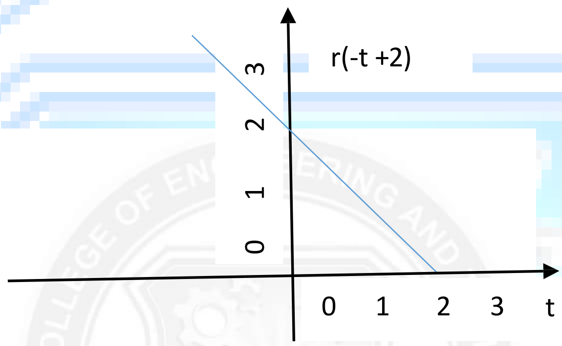
(ii) $-2u(t-1)$



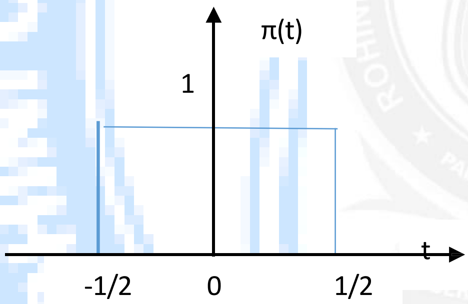
(iii) $r(-t+2)$

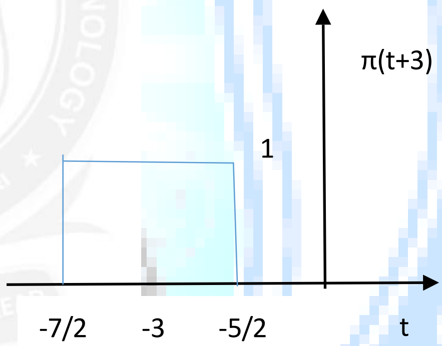






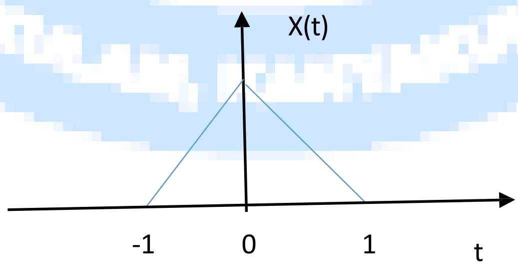
(iv) $\pi(t+3)$





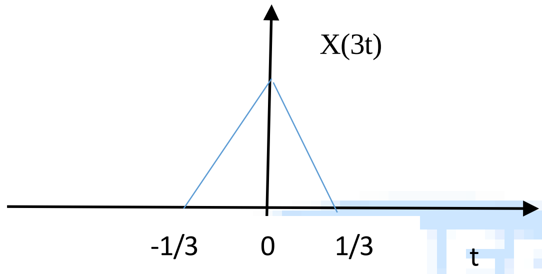
4. A triangular pulse signal $x(t)$ is shown below. Sketch the following signals
a) $x(3t)$ b) $x(3t+2)$ c) $x(-2t+1)$ d)$x(2(t+2))$ e) $x(2(t-2))$

f)) $x(3t) + x(3t+2)$

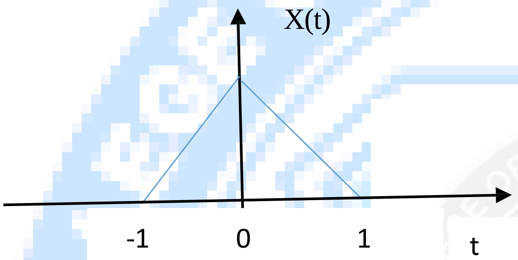


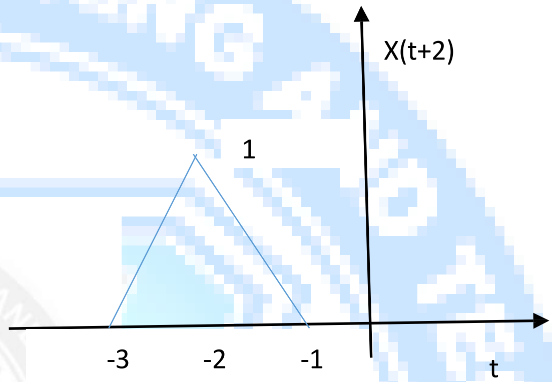
Soln:

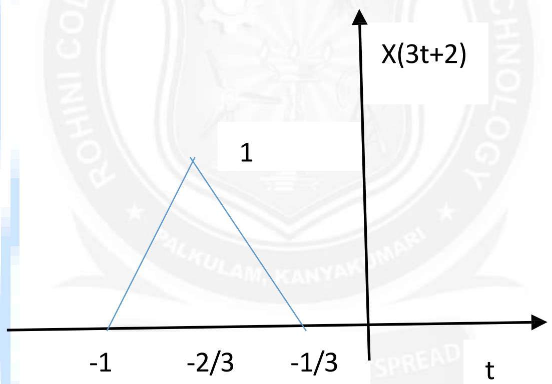
a) $x(3t)$



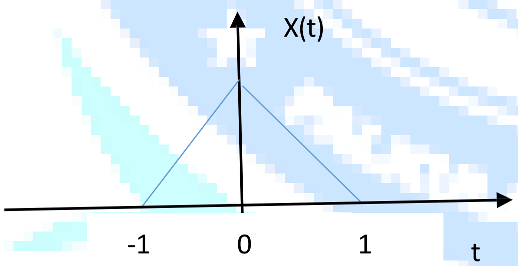
b) $x(3t + 2)$

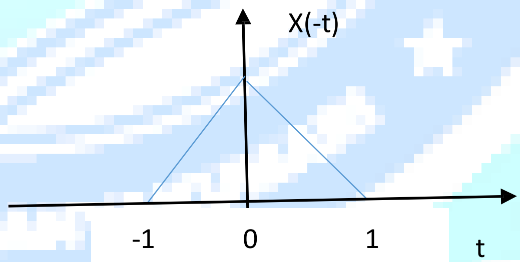


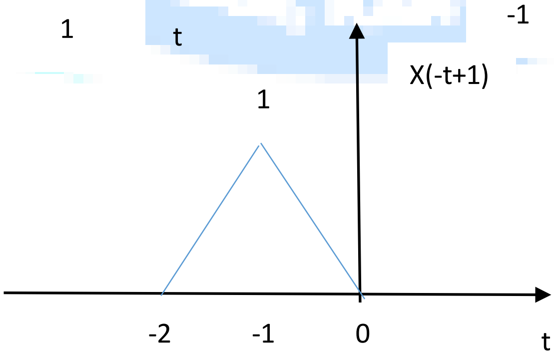


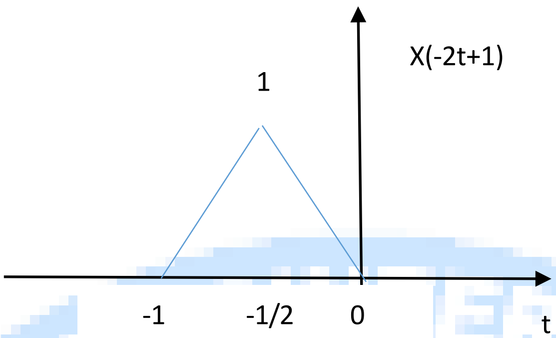


c) $x(-2t + 1)$

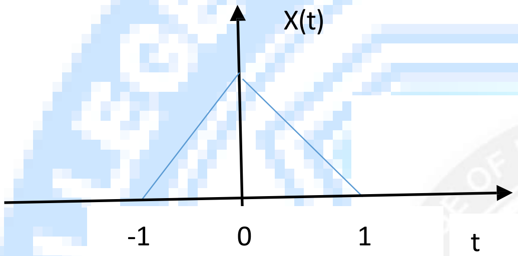


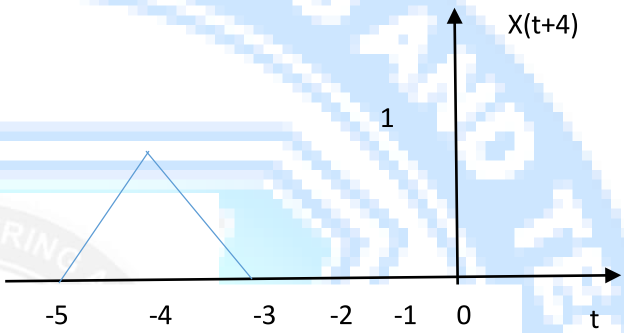


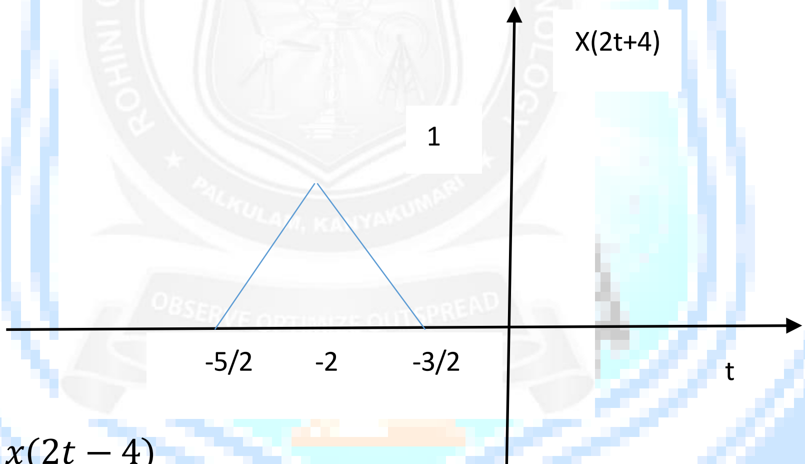




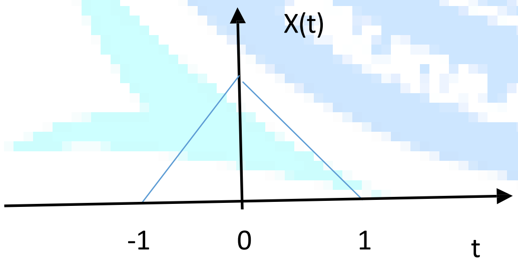
d)$x(2(t+2)) = x(2t+4)$

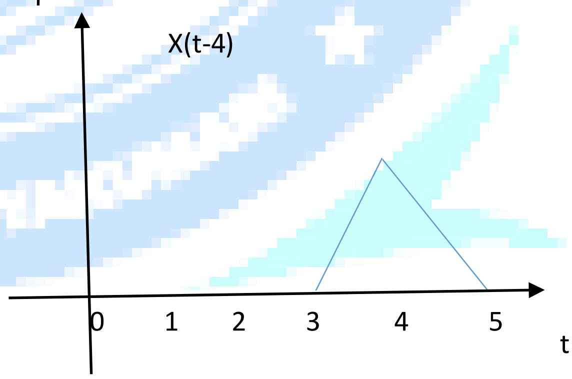


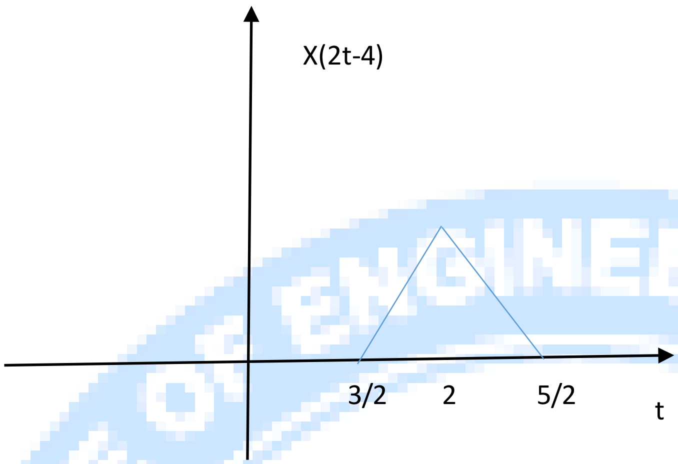




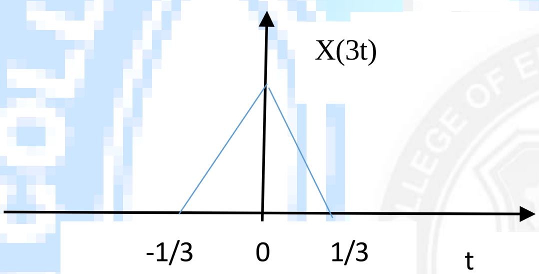
e) $x(2(t-2)) = x(2t-4)$

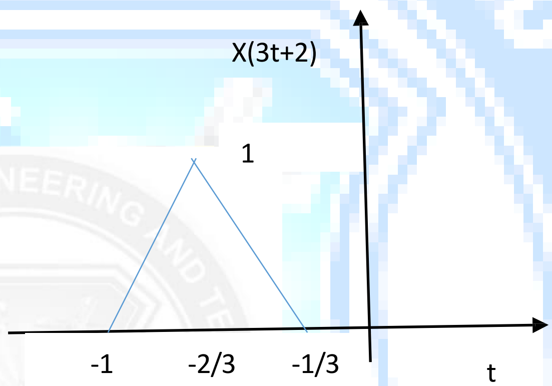


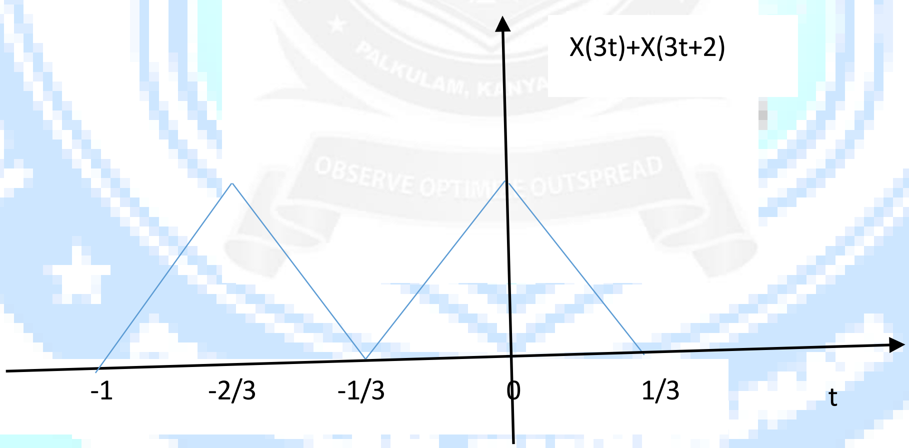




f)) $x(3t) + x(3t + 2)$





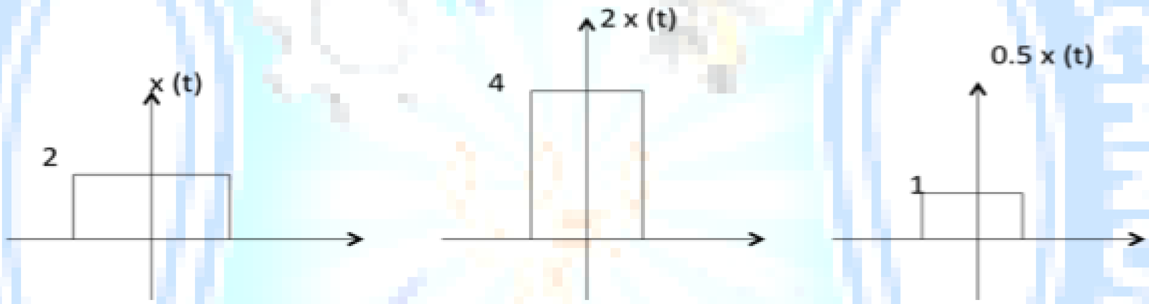


1.2.2 Operations performed on the Dependent Variable:

These transformations change how the signal behaves in amplitude, affecting its energy, distortion, or suitability for further processing like filtering, transmission, or analysis.

1. Amplitude Scaling
2. Amplitude Shift
3. Signal Addition
4. Signal Multiplication

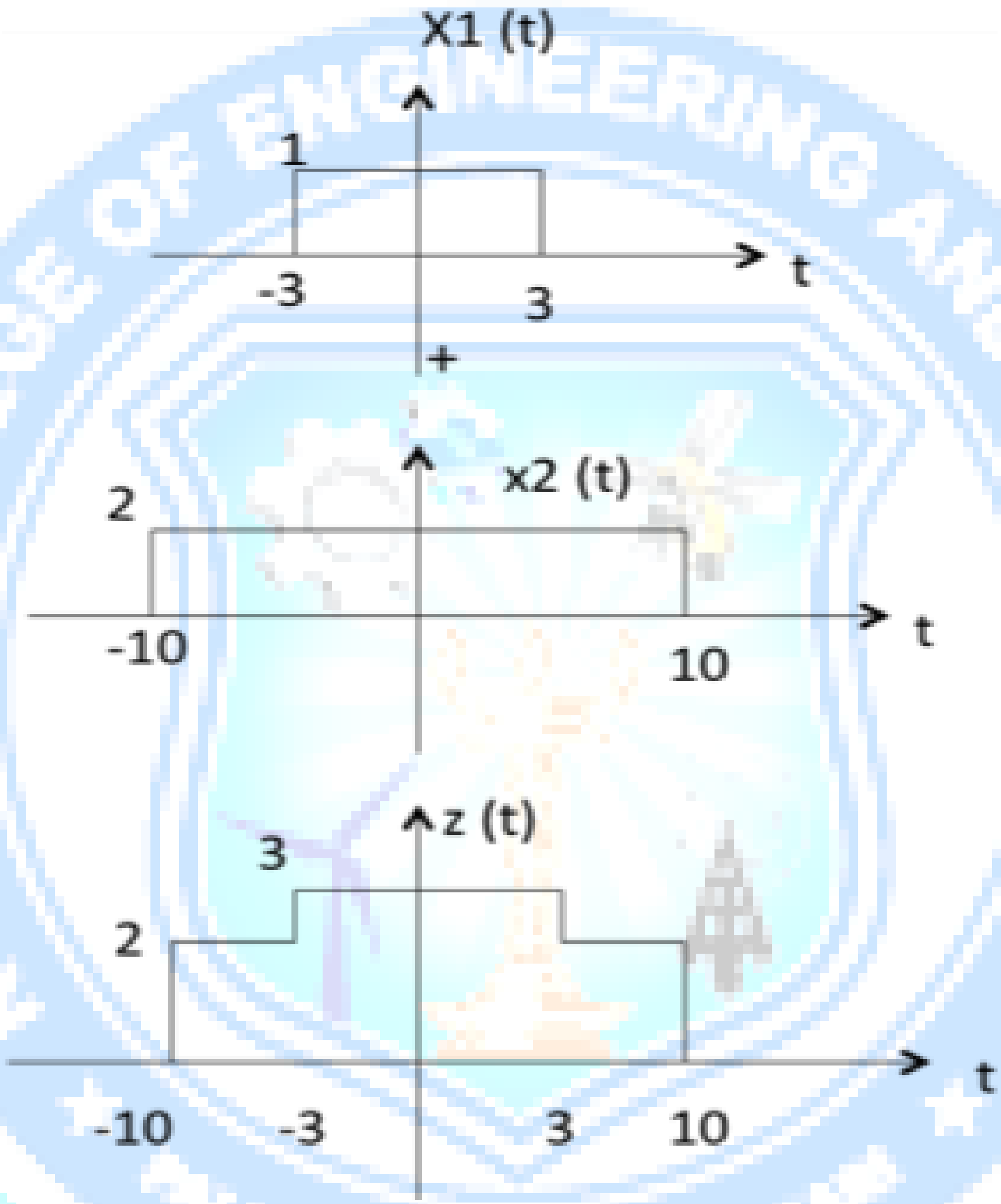
1. Amplitude Scaling: y(t)= a x(t), where a is a real (or possibly



complex) constant. C x(t) is an amplitude scaled version of x(t) whose amplitude is scaled by a factor C.

2. Amplitude Shift: y(t)= x(t)+ b, where b is a real (or possibly complex) constant

3. Signal Addition: y(t)= x1(t) + x2(t)



As seen from the diagram above,

-10 < t < -3 amplitude of z(t) = x1(t) + x2(t) = 0 + 2 = 2

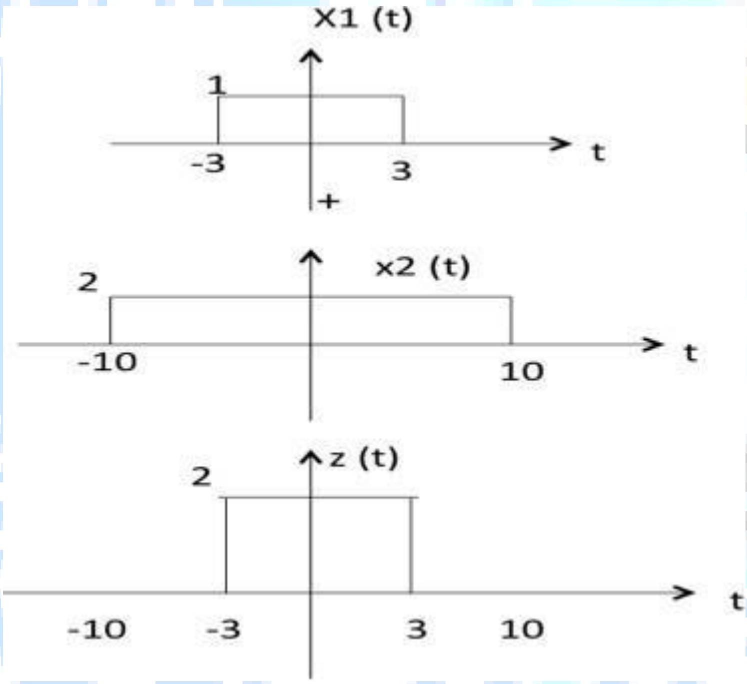
-3 < t < 3 amplitude of z(t) =

x1(t) + x2(t) = 1 + 2 = 3 3 < t

< 10 amplitude of z(t) = x1(t)

+x2(t) = 0 + 2 = 2

4.Signal Multiplication: y(t)= x1(t). x2(t)



As seen from the diagram above,

-10 < t < -3 amplitude of z (t) = x1(t) ×x2(t) = 0 ×2 = 0

-3 < t < 3 amplitude of z (t) = x1(t) ×x2(t) = 1 ×2 = 2

3 < t < 10 amplitude of z (t) = x1(t) × x2(t) = 0 × 2 = 0